Operation Research

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2025- 2026

Chapter one

Introduction to Operation Research

Definitions:

1. **Operation Research**: Is the systematic application of quantitative methods , techniques and tools to the analysis of problems involving the operation of systems.
2. **Operation Research**: In the most general sense , can be characterized as the application of scientific methods , techniques and tools ,to problems involving the operation of systems so as to provide those in control of the operations with optimum solutions to the problems.
3. **Operation Research**: Operations Research uses mathematical models to represent real-life problems and solve them optimally under given constraints.

1.1 The methodology of OR

OR is applied to problems that concern how to conduct and coordinate operations (i.e. activities) with in an organization.

The major phases of a typical OR study are the following:

1. Define the problem and gather relevant data.

2. Formulate a mathematical model to represent the problem.

3. Select a Suitable Alternative and Present the Results and Conclusions of the Study.

1.2 Basic or Concepts

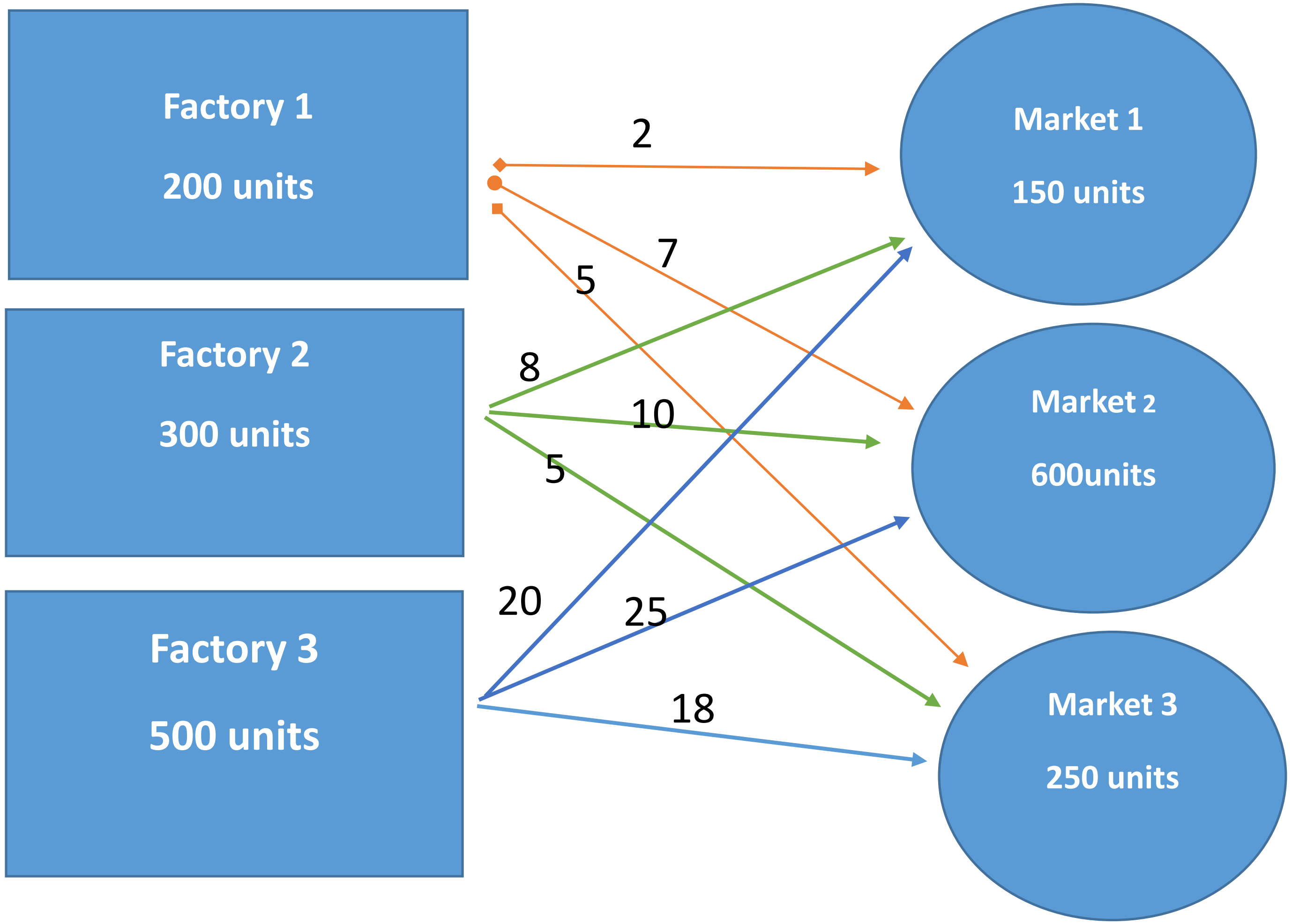
We can define a mathematical model of OR as consisting of:

1. Decision variables, which are the unknowns to be determined by the solution to the model.
2. Constraints to represent the physical limitations of the system
3. An objective function
4. non-negativity, i.e. decision variables must be greater than or equal to zero

Transportation : moving amounts of good from places to another places.

From: factories \warehouses\origins\sources\etc.

To: retailers\destinations\distributors\etc.



Formulating Transportation problems

The Transportation problem deals with a situation in which a single product is to be transported from several sources (also called origin, supply or capacity centers) to several sinks (also called destination, demand or requirement centers) in general, let there be m sources $S_1, S_2, \ldots, S_m$ having a_i (i = 1, 2, …, m) units of supplies or capacity respectively to be transported among n destinations. $D_1, D_2, \ldots, D_n$ with b_j (j = 1, 2, …, n) units of requirements, respectively. Let C_{ij} be the cost of shipping one unit of the commodity from source i to destination j for each route. If X_{ij} represents the number of units shipped per route from source i to destination j, then the problem is to determine the transportation schedule so as to minimize the total transportation cost satisfying supply and demand conditions. Mathematically the problem may be stated as follows:

$$Minimize Z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}$$

Subject to

$$\sum_{i=1}^{n} x_{ij} = a_i \quad \text{i} = 1, 2, \ldots, \text{m} \text{ (supply constraints)}$$

$$\sum_{j=1}^{m} x_{ij} = b_j \quad j = 1, 2, \ldots., n \ (demand\ constraints)$$

$$x_{ij} \geq 0 \quad \text{for all i and j}$$

Transportation Problem:-

A special class of linear programming problem is Transportation Problem, where **the objective is to minimize the cost** of distributing a product from a number of sources (e.g. factories) to a number of destinations (e.g. warehouses) while satisfying both the supply limits and the demand requirement.

When the total supply of all the sources is equal to the total demand of all destinations, the problem is a **balanced** transportation problem. But if the total supply of all the sources is not equal to the total demand of all destinations, the problem is an **unbalanced** transportation problem. **The unbalanced problem** can be balanced by adding a dummy supply center **(row)** or a dummy demand center **(dummy column)** as the need arises.

The balanced transportation problem

To / From	Market 1	Market 2	Market 3	Supply
Factory 1	2	7	5	200
Factory 2	10	8	5	300
Factory 3	20	25	18	500
Demand	150	600	250	1000 / 1000

DEMAND>SUPPLY

PRODUCT → / OFFICE ↓	P	Q	SUPPLY
A	10	15	20
B	20	30	50
DEMAND	30	60	70 / 90

PRODUCT → / OFFICE ↓	P	Q	SUPPLY
A	10	15	20
B	20	30	50
DUMMY	0	0	20
DEMAND	30	60	90 / 90

SUPPLY>DEMAND

PRODUCT → OFFICE ↓	P	Q	SUPPLY
A	10	15	30
B	20	30	60
DEMAND	20	50	90 70

PRODUCT → OFFICE ↓	P	Q	DUMMY	SUPPLY
A	10	15	0	20
B	20	30	0	50
DEMAND	20	50	20	90 90

Solution of the Transportation Problem

In this section we introduce the details for solving the T.M the method uses the steps of the simplex method directly and differs only in the details of implementing the optimality

Methods to finding the basic feasible solution:

There are several methods available to obtain an initial basic feasible solution. Here we shall discuss only three different methods:

1) North West Corner Method **(NWCM).**
2) Least Cost Method **(LCM) or Minimum Cost Method.**
3) Vogel's Approximation Method **(VAM).**

1. North West Corner Method

Most systematic and easiest method for obtaining initial feasible basic solution.

The method can be summarized as follows:

i. Select the North-west (i.e., **upper left**) corner cell of the table and allocate the maximum possible units between the supply and demand requirements. During allocation, the transportation cost is completely discarded (not taken into consideration).
ii. Delete that row or column which has no values (fully exhausted) for supply or demand.
iii. Now, with the new reduced table, again select the North-west corner cell and allocate the available values.
iv. Repeat steps (ii) and (iii) until all the supply and demand values are zero.
v. Obtain the initial basic feasible solution.

Example1 : Use the North west corner method to find initial feasible solution for the following transportation

To / From	Market 1	Market 2	Market 3	Supply
Factory 1	2	7	5	200
Factory 2	10	8	5	300
Factory 3	20	25	18	500
Demand	150	600	250	1000 / 1000

Answer the following:-

1. Factory 1 will shipped ....units to Market 2.

2. Market 3 will received…………..Units from Factory 2.

3. Market 2 will shipped………….Units to Factory 3

4. Total transportation cost of units shipped from factory 3 ……………………….

5. Total transportation cost of units shipped to Market 2 ……………………………

6. Total transportation cost of units shipped to Market 3 ……………………………

7. Total transportation cost ……………………………………………………………….

Example2 : Use the North west corner method to find initial feasible solution for the following transportation

To / From	P1		P 2		P3		Supply
W1	2		7		5		150
W 2	10		8		5		175
W 3	20		25		18		275
Demand	200		100		300		600 / 600

Example3 : Use the North west corner method to find initial feasible solution for the following transportation and answer the following

Market 3 will received…………..Units .

units shipped from warehouse 2 to market 3 is ………..

To / From	M1		M 2		M3		Supply
W1	6		8		8		500
W 2	7		9		5		300
W 3	11		7		10		420
Demand	450		430		390		

Example 4:- AThe following table shows the **transportation cost per unit** from three plants (P1, P2, P3) to three warehouses (W1, W2, W3). The supply available at each plant and the demand required at each warehouse are also given.

Required:
Determine an **initial basic feasible solution** using the **North-West Corner Method (NWCM)**.
Prepare the **allocation table**.
Calculate the **total transportation cost** of the solution.

waerhouse / Plant	W1		W2		W3		Supply
P1	7		6		9		20
P2	5		7		3		28
P3	4		5		8		17
Demand	21		25		19		

Example 5: Find final transportation cost by using north-west corner method.

	A		B		C		Supply
1	5		0		2		25
2	3		6		1		10
3	10		14		10		33
Demand	23		15		30		

Example 6:- A firm have 3 factories located at A, E, and K which produce the same product. There are four major product district centers situated at B, C, D, and M. Find the initial feasible solutionby using the North West corner method

Factories	B	C	D	M	Supply
A	6	8	8	5	30
E	5	11	9	7	40
K	8	9	7	13	50
Demand	35	28	32	25	

Example 7:-

Find the initial feasible solution for the following transportation problem by using the North West corner method

Companies	A		B		C		D		Supply
S1	4		1		2		3		100
S2	6		1		3		5		50
S3	7		5		8		7		30
Demand	20		80		50		50		

Example 8: Find starting feasible solution using north west corner methods

From \ To	P1	P 2	P3	Supply
F1	5	4	3	250
F 2	8	4	3	300
F 3	9	7	5	300
Demand	300	200	200	