

University of Cihan-Sulaimaniya
Engineering Faculty
Architectural Engineering Department



ENGINEERING MECHANICS

Chapter 4: Force System Resultants

2nd Grade- Fall Semester 2024-2025

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Chapter Description

- Aims
 - To explain the Moment of Force (2D-scalar formulation & 3D-Vector formulation)
 - To explain the Principle Moment
 - To explain the Moment of a Couple
 - To explain the Simplification of a Force and Couple System
 - To explain the Reduction of Simple Distributed Loading
- Expected Outcomes
 - Able to solve the problems of MOF and COM in the mechanics applications by using principle of moments
- References
 - Russel C. Hibbeler. Engineering Mechanics: Statics & Dynamics, 14th Edition

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Chapter Outline

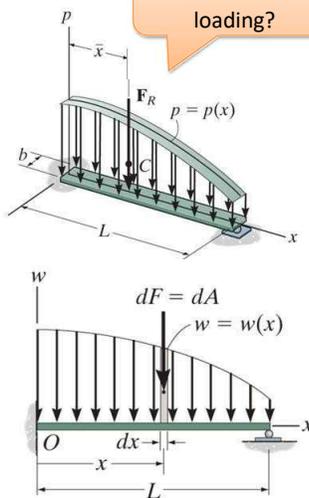
1. Moment of Force (MOF) –Part I
2. Principle of Moment –Part II
3. Moment of Couple (MOC) Part III
4. Simplification of a Force and Couple System
5. Reduction of Simple Distributed Loading- part IV



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4.5 Reduction of a simple distributed loading

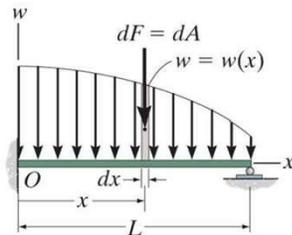
What is distributed loading?



- In many situations, a surface area of a body is subjected to a **distributed load**
- Such forces are caused by **winds, fluids, or the weight** of items on the body's surface
- Distributed loadings are defined by using a loading function $w = w(x)$ that indicates the intensity of the loading along the length of the member
- Intensity is measured in N/m
- The external effects caused by a coplanar **distributed load** acting on a body can be represented by a **single resultant force**
- The **magnitude** of the **resultant force** is equal to the **total area under the distributed loading diagram** $w = w(x)$
- The **location** of the **resultant force** is given by the fact that its line of action passes through the **centroid** or **geometric center** of this area

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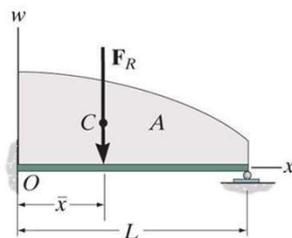
Magnitude of resultant force



Consider an element of length dx .

The force magnitude dF acting on it is given as

$$dF = w(x) dx$$



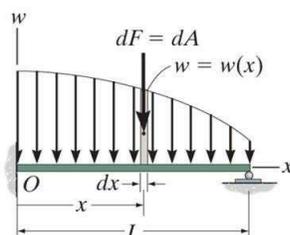
The net force on the beam is given by

$$+\downarrow F_R = \int_L dF = \int_L w(x) dx = A$$

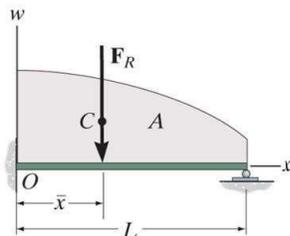
Here A is the area under the loading curve $w(x)$.

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Location of the resultant force



The force dF will produce a moment of $(x)(dF)$ about point O .

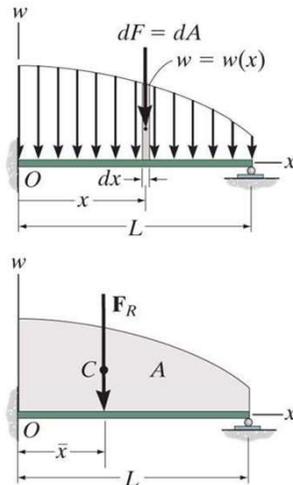


Assuming that F_R acts at \bar{x} , it will produce the moment about point O as

$$\curvearrowright + M_{RO} = (\bar{x})(F_R) = \int_L w(x) dx$$

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Location of the resultant force



Comparing the last two equations, we get

$$\bar{x} = \frac{\int_L xw(x) dx}{\int_L w(x) dx} = \frac{\int_A x dA}{\int_A dA}$$

You will learn more detail later, but F_R acts through a point "C," which is called the geometric center or centroid of the area under the loading curve $w(x)$.

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Example 4.11

Determine the **concentrated** loads (which is a common name for the **resultant of the distributed load**)



The rectangular load: *find the area of rectangular*

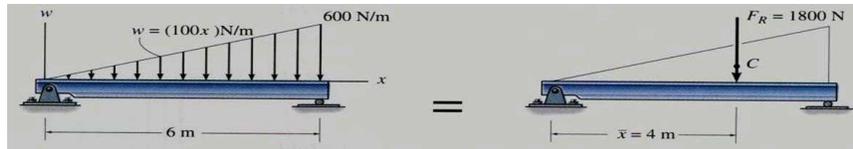
$$F_R = 400 \times 10 = 4,000 \text{ lb}$$

$$\bar{x} = \frac{10}{2} = 5 \text{ ft} \quad \textit{location is the centroid of rectangular}$$

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Example 4.12

Determine the **concentrated** loads (which is a common name for the **resultant of the distributed load**)



The triangular loading: *find the area of triangular*

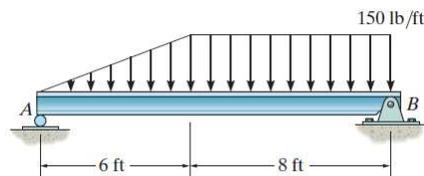
$$F_R = \frac{1}{2} (600) (6) = 1,800 \text{ N} \quad \text{and} \quad \bar{x} = 6 - (1/3) 6 = 4 \text{ m}$$

Please note that the centroid of a **right triangle** is at a distance one third the width of the triangle as **measured from its base**

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Example 4.13

The loading on the beam as shown. Determine equivalent force and its location from point A.



- 1) The distributed loading can be divided into two parts. (one rectangular loading and one triangular loading).
- 2) Find F_R and its location for each of the distributed loads.
- 3) Determine the overall F_R of the point loadings and its location.

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Solution Example 4.13

For the triangular loading of height 150 lb/ft and width 6 ft,

$$F_{R1} = (0.5)(150)(6) = 450 \text{ lb}$$

and its line of action is at $\bar{x}_1 = (2/3)(6) = 4 \text{ ft from A}$

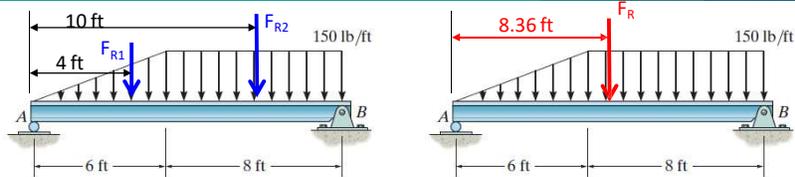
For the rectangular loading of height 150 lb/ft and width 8 ft,

$$F_{R2} = (150)(8) = 1200 \text{ lb}$$

and its line of action is at $\bar{x}_2 = 6 + (1/2)(8) = 10 \text{ ft from A}$

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Solution Example 4.13



The equivalent force and couple moment at A will be

$$F_R = 450 + 1200 = 1650 \text{ lb}$$

$$+ \uparrow M_{RA} = 4(450) + 10(1200) = 13800 \text{ lb-ft}$$

Since $(\bar{x} * F_R)$ has to equal M_{RA} : $1650 * \bar{x} = 13800$

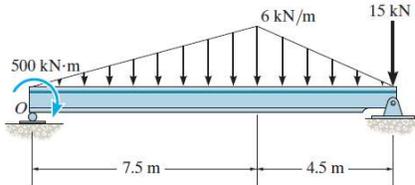
Solve for \bar{x} to find the equivalent force's location.

$$\bar{x} = 8.36 \text{ ft from A.}$$

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Example 4.14

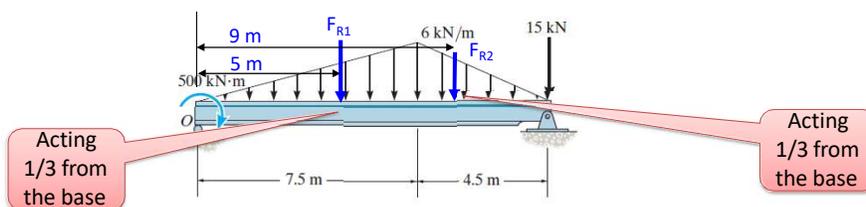
The loading on the beam as shown. Determine equivalent force and couple moment acting at point O



- 1) The distributed loading can be divided into two parts-two triangular loads
- 2) Find F_R and its location for each of these distributed loads
- 3) Determine the overall F_R of the point loadings and couple moment at point O

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Solution Example 4.14



For the triangular loading (right) of height 6 kN/m and width 7.5 m,

$$F_{R1} = (0.5)(6)(7.5) = 22.5 \text{ kN}$$

and its line of action is at $\bar{x}_1 = (2/3)(7.5) = 5 \text{ m from O}$

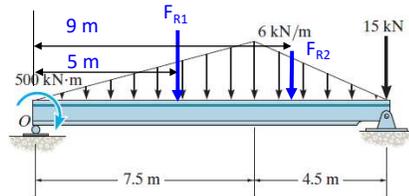
For the triangular loading (left) of height 6 kN/m and width 4.5 m,

$$F_{R2} = (0.5)(6)(4.5) = 13.5 \text{ kN}$$

and its line of action is at $\bar{x}_2 = 7.5 + (1/3)(4.5) = 9 \text{ m from O}$

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Solution Example 4.14



For the combined loading of the **three forces**, add them.

$$F_R = 22.5 + 13.5 + 15 = 51 \text{ kN}$$

The couple moment at point O will be

$$+\curvearrowleft M_{RO} = 500 + 5(22.5) + 9(13.5) + 12(15) = 914 \text{ kN}\cdot\text{m}$$

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Conclusion of The Chapter 4

- Conclusions
 - The reduction of force simple loading has been identified
 - The external effects caused by a coplanar **distributed load** acting on a body can be represented by a **single resultant force**
 - The reduction of force analysis have been implemented to solve resultant force and moment problems in specified axis



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Thank you

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