University of Cihan-Sulaimaniya
Engineering Faculty
Architectural Engineering Department

ENGINEERING MECHANICS

Chapter 4: Force System Resultants

2nd Grade- Fall Semester 2024-2025

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Chapter Description

- Aims
  - To explain the Moment of Force (2D-scalar formulation & 3D-Vector formulation)
  - To explain the Principle Moment
  - To explain the Moment of a Couple
  - To explain the Simplification of a Force and Couple System
  - To explain the Reduction of Simple Distributed Loading
- Expected Outcomes
  - Able to solve the problems of MOF and COM in the mechanics applications by using principle of moments
- References
  - Russel C. Hibbeler. Engineering Mechanics: Statics & Dynamics, 14th Edition

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Chapter Outline

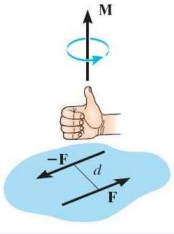
1. Moment of Force (MOF) –Part I
2. Principle of Moment –Part II
3. Moment of Couple (MOC) Part III
4. Simplification of a Force and Couple System
5. Reduction of Simple Distributed Loading- part IV

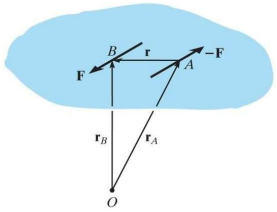
4.3 Moment of a Couple

A couple is defined as two parallel forces with the same magnitude but opposite in direction separated by a perpendicular distance "d."

- Resultant force = 0
- Tendency to **rotate in specified direction**
- Couple moment is a **free vector**
- It can be compute d by **any point**
- Choose the line action of **one of the force** in the couple
- A resultant couple moment = sum of the couple moments of the system
- $\mathbf{M_R = M_1 + M_2}$

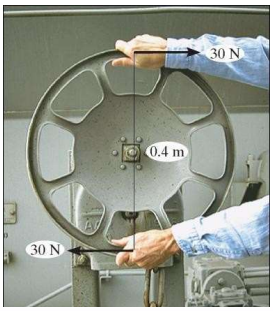
- The moment of a couple is defined as

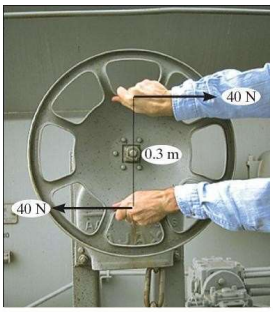




$M_O = F\ d$ (using a scalar analysis)

Application (Moment of a Couple)



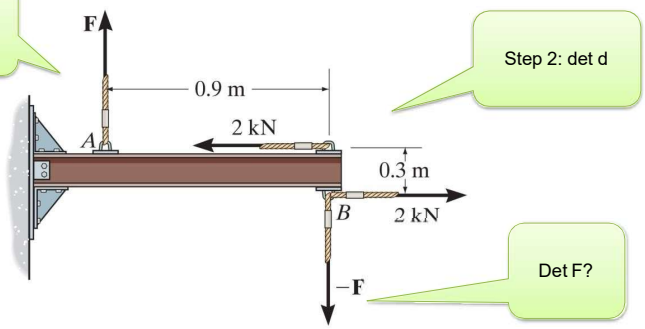


A torque or moment of 12 N·m is required to rotate the wheel. Why does one of the two grips of the wheel above require less force to rotate the wheel?

Example 4.7

Two couples act on the beam with the geometry shown. Determine the magnitude of F so that the resultant couple moment is 1.5 kN.m clockwise

Step 1: FBD (Sketch outline shape)



Step 2: det d

use formula: scalar analysis

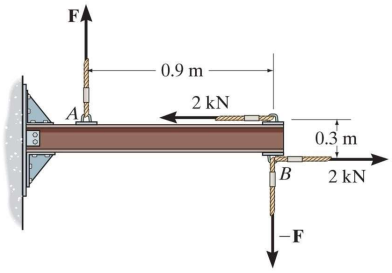
$M_O = Fd$

Det F?

Solution Example 4.7

Net moment = 1.5 kN.m

$M_O = Fd$



- The net moment is equal to:

$+\Sigma M = -F(0.9) + (2)(0.3)$

$= -0.9F + 0.6$

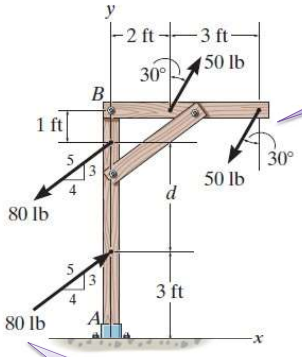
$-1.5\ \text{kN·m} = -0.9F + 0.6$

- Solving for force F,

F = 2.33 kN

Example 4.8

Two couples act on the beam with the geometry shown and d = 4 ft. Determine the resultant couple.



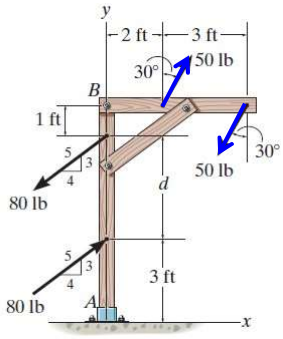
First , Resolve 50 lb into x y

2nd , Resolve 80 lb into x y

1) Resolve the forces in x and y-directions so they can be treated as couples.

2) Add these two couples to find the resultant couple.

Solution Example 4.8



The x and y components of the upper-left 50 lb force are:

50 lb (cos 30°) = 43.30 lb vertically up

50 lb (sin 30°) = 25 lb to the right

Do both of these components form couples with their matching components of the other 50 force?

No! Only the 43.30 lb components create a couple. Why?

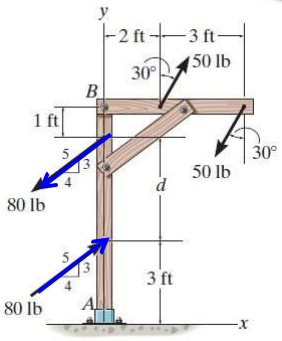
Solution Example 4.8

Do both of these components create a couple with components of the other 80 force?

Now resolve the lower 80 lb force:

(80 lb) (3/5), acting up

(80 lb) (4/5), acting to the right



The net moment is equal to:

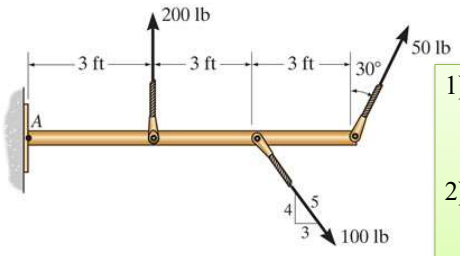
$+ \quad \Sigma M = -(43.3 \text{ lb})(3 \text{ ft}) + (64 \text{ lb})(4 \text{ ft})$

$= -129.9 + 256$

$= 126 \text{ ft·lb} \quad \text{CCW}$

Example 4.9

A 2-D force system with the geometry shown. Determine the equivalent resultant force and couple moment acting at A and then the equivalent single force location measured from A



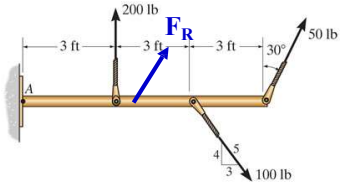
1) Sum all the x and y components of the forces to find F_{RA}.
2) Find and sum all the moments resulting from moving each force component to A.
3) Shift F_{RA} to a distance d such that $d = M_{RA}/F_{Ry}$

Solution Example 4.9

$+\rightarrow \Sigma F_{Rx} = 50(\sin 30) + 100(3/5)$
$= 85$ lb

$+\uparrow \Sigma F_{Ry} = 200 + 50(\cos 30) - 100(4/5)$
$= 163.3$ lb

$+ \quad M_{RA} = 200\,(3) + 50\,(\cos 30)\,(9)$
$- 100\,(4/5)\,6 = 509.7$ lb·ft CCW



$F_R = (85^2 + 163.3^2)^{1/2} = 184$ lb

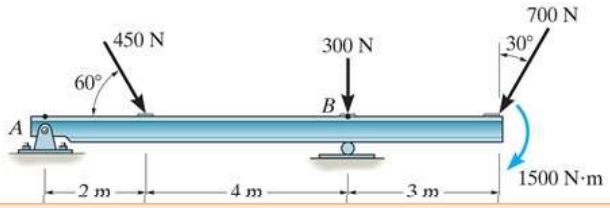
$\theta = \tan^{-1} (163.3/85) = 62.5°$

The equivalent single force F_R can be located at a distance d measured from A.

$d = M_{RA}/F_{Ry} = 509.7 / 163.3 = 3.12$ ft

Example 4.10

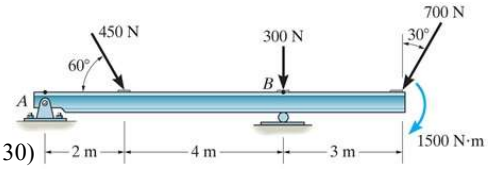
A 2-D force and couple system as shown. Determine the equivalent resultant force and couple moment acting at A.



1) Sum all the x and y components of the two forces to find F_{RA}.
2) Find and sum all the moments resulting from moving each force to A and add them to the 1500 N·m free moment to find the resultant M_{RA} .

Solution Example 4.10

Summing the force components:



$+\rightarrow \Sigma F_x = 450\ (\cos 60) - 700\ (\sin 30)$

$= -125$ N

$+\uparrow \Sigma F_y = -450\ (\sin 60) - 300 - 700\ (\cos 30)$

$= -1296$ N

Now find the magnitude and direction of the resultant.

$F_{RA} = (125^2 + 1296^2)^{1/2} = \underline{1302\ \text{N}}$

And $\theta = \tan^{-1}(1296/125) = \underline{84.5°}$

$+\circlearrowleft M_{RA} = 450\ (\sin 60)\ (2) + 300\ (6) + 700\ (\cos 30)\ (9) + 1500$

$= \underline{9535\ \text{N·m}}$

Conclusion of The Chapter 4

- Conclusions
  - The Moment of couple has been identified
  - The scalar and vector analysis have been implemented to solve Moment problems in specified axis

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Thank you

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