University of Cihan-Sulaimaniya
Engineering Faculty
Architectural Engineering Department

ENGINEERING MECHANICS

Chapter 3: Equilibrium of a Particle

2nd Grade- Fall Semester 2025-2026

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Chapter Description

- Aims
  - To explain the Equilibrium Equation
  - To explain the Free Body Diagram
  - To apply the Equations of Equilibrium to solve particle equilibrium problems in Coplanar Force System (2-D)

- Expected Outcomes
  - Able to solve the problems of a particle or rigid body in the mechanics applications by using Equilibrium Equation

- References
  - Russel C. Hibbeler. Engineering Mechanics: Statics & Dynamics, 14th Edition

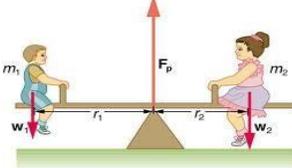*Communitising Technology*

Chapter Outline

1. Equilibrium Equation
2. Free Body Diagram
3. Coplanar Force Systems (2-D)
4. Example

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3.1 Equilibrium Equation

What is Equilibrium?

- **Equilibrium** means the **forces are balanced** but not necessarily equal
- In physic, it means equal balance which the **opposing forces or tendencies neutralize each other**

How to know the body is in Equilibrium?

- A body at rest or in uniform motion (velocity) is in equilibrium

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Condition for the Equilibrium of a Particle

How to know the body is in Equilibrium?

(a)

(b)

T_B = 4.90kN
T_D = 4.25kN

2.452 kN

- Particle at *equilibrium* if
  - At rest
  - Moving at constant a constant velocity
- Newton's first law of motion

$$\sum \mathbf{F} = 0$$

where $\sum \mathbf{F}$ is the vector sum of all the forces acting on the particle

$+\rightarrow \sum F_x = 0; \quad T_B\cos30° - T_D = 0$
$+\uparrow \sum F_y = 0; \quad T_B\sin30° - 2.452kN = 0$

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Condition for the Equilibrium of a Particle

- Newton's second law of motion

$$\sum \mathbf{F} = m\mathbf{a}$$

- When the force fulfill Newton's first law of motion,
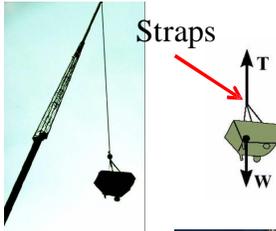
$$m\mathbf{a} = 0$$
$$\mathbf{a} = 0$$

therefore, the particle is moving in constant velocity or at rest

Static Equilibrium is when the body at rest

If the Dynamic Equilibrium, the body move and continue to move

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Application of Equilibrium Equation

Straps

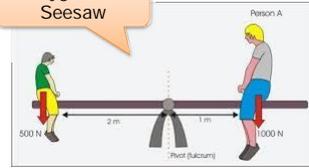Construction-lifting the material
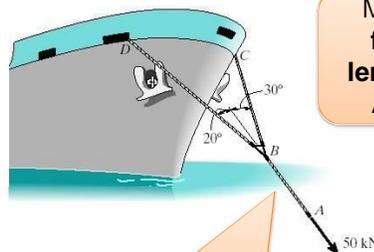
Day life activity-Paddle the boat

Support Oil Rig

Playground-Seesaw

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Application of Equilibrium Equation

Measure the **forces** and **length** of cable AB and AC

Measure the **forces, direction and size** of cable AB

Measure the **forces** to make sure the rigging doesn't fail
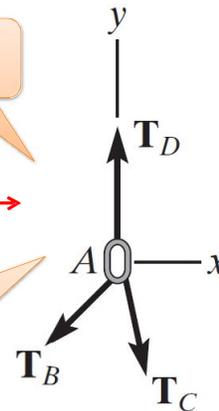
50 kN

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3.2 Free Body Diagram (FBD)

What is FBD?

FBD is a sketch to show only the **forces** acting on **selected** body

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3.2 Free Body Diagram (FBD)

- Best representation of all the unknown forces ($\sum \mathbf{F}$) which acts on a body
- A sketch showing the particle "free" from the surroundings with all the forces acting on it

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Spring

- Linear elastic spring: **change** in **length** is **directly proportional** to the **force** acting on it
- *spring constant or **stiffness k***: defines the ***elasticity*** of the spring
- **Magnitude of force** when spring is **elongated** or **compressed**

$$F = ks$$

- where **s** is determined from the difference in spring's deformed length **l** and its undeformed length **l$_o$**

$$s = l - l_o$$

$s = l - l_o$

- If **s** is positive, **F** "pull" onto the spring
- If **s** is negative, **F** "push" onto the spring

l = 0.2 m

l = 0.6 m

l$_0$ = 0.4 m

s = – 0.2 m

k = 500 N/m

(s = 0)

s = 0.2 m

– **F**

F

+ s

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Cables and Pulley

- Cables (or cords) are assumed to have negligible weight and they cannot stretch
- A cable only support tension or pulling force
- Tension always acts in the direction of the cable
- Tension force in a continuous cable must have a constant magnitude for equilibrium
- For any angle θ, the cable is subjected to a constant tension **T** throughout its length

θ

T

T

Cable is in tension

With a frictionless pulley and cable
$T_1 = T_2$.

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3.3 Coplanar Systems 2-D
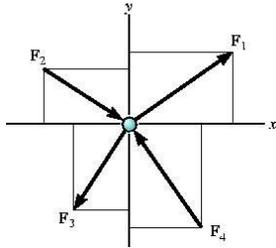
- A particle is subjected to coplanar forces in the x-y plane
- Resolve into **i** and **j** components for equilibrium
$$\sum \mathbf{F}_x = 0$$
$$\sum \mathbf{F}_y = 0$$
- Scalar equations of equilibrium require that the algebraic sum of the x and y components to equal o (zero)

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Scalar Notation

- Sense of direction = an algebraic sign that corresponds to the arrowhead direction of the component along each axis
- For unknown magnitude, assume arrowhead sense of the force
- Since magnitude of the force is always positive, if the scalar is negative, the force is acting in the opposite direction

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Step to draw FBD

Step 1: Sketch outline shape

Step 2:Show all the forces that act on body and indicate the active (set the body in motion) or reactive forces (tend to resist the motion)

Step 4: Apply EE and calculate the unknown forces(can be write in letters)

Step 3: Labeled the known forces (magnitude and direction)

B

$30°$

D

A

C 40 kg

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Select the correct FBD of Particle A

$30°$ A $40°$

100 lb

A)   A   100 lb

B) F_1 F_2 $30°$ $40°$ A

C)   F $30°$ A   100 lb

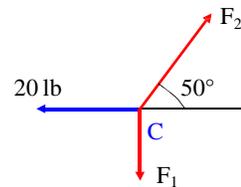D) F_1 F_2 $30°$ $40°$ A   100 lb

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FBD

Using this FBD of Point C, the sum of forces in the x-direction ($\Sigma \ F_x$) is__. Use a sign convention of $+ \rightarrow$.
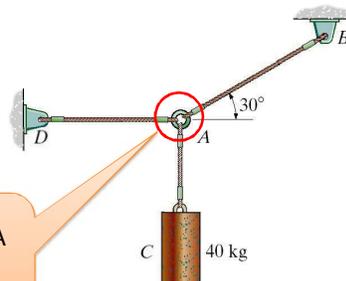
A) $F_2 \sin 50° - 20 = 0$

B) $F_2 \cos 50° - 20 = 0$

C) $F_2 \sin 50° - F_1 = 0$

D) $F_2 \cos 50° + 20 = 0$

20 lb 50° F_2 C F_1

Example 3.1

This is an example of a 2-D or coplanar force system. If the whole assembly is in equilibrium, then particle A is also in equilibrium. Determine the tensions in the cables for a given weight of cylinder = 40kg

B 30° D A

Step 1: FBD @A
(Sketch outline shape)

C 40 kg

Solution Example 3.1

FBD at A

Step 4: Apply EE and calculate the unknown forces(can be write in letters)

Step 2:Show all the forces that act on body and indicate the active (set the body in motion) or reactive forces (tend to resist the motion)

F_B

30°

F_D

A

x

y

$F_C = 40 \times 9.81 = 392.4\,N$

Step 3: Labeled the known forces (magnitude and direction)

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Solution Example 3.1

Step 4: Apply EE and calculate the unknown forces(can be write in letters)

FBD at A

Since particle A is in equilibrium, the net force at A is zero.

So $F_B + F_C + F_D = 0$

or $\Sigma F = 0$

F_B

30°

F_D

A

x

y

$F_C = 392.4\,N$

In general, for a particle in equilibrium,

$\Sigma F = 0$ or $\Sigma F_x\, i + \Sigma F_y\, j = 0 = 0\, i + 0\, j$ (a vector equation)

Or, written in a scalar form,
$\Sigma F_x = 0$ and $\Sigma F_y = 0$

- Two scalar equations of equilibrium (E-of-E)
- Used to solve for up to two unknowns
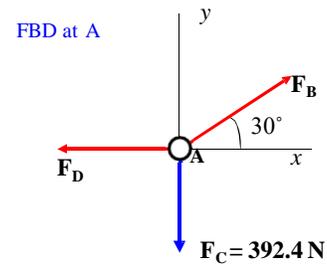
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Solution Example 3.1

Write the scalar E-of-E:

$+ \rightarrow \Sigma F_x = F_B \cos 30° - F_D = 0$

$+ \uparrow \Sigma F_y = F_B \sin 30° - 392.4 \text{ N} = 0$

Solving the second equation, $F_B = 785 \text{ N} \rightarrow$

From the first equation, $F_D = 680 \text{ N} \leftarrow$

FBD at A

F_B

30°

F_D

$F_C = 392.4 \text{ N}$

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Example 3.2

This is an example of a 2-D or coplanar force system. If the whole assembly is in equilibrium, then particle E is also in equilibrium. Determine the tensions in the cables DE,EA and EB for a given weight of cylinder = 40kg

B

30°

D E

Step 1: Draw FBD @ point E

Step 2

C 40 kg

A
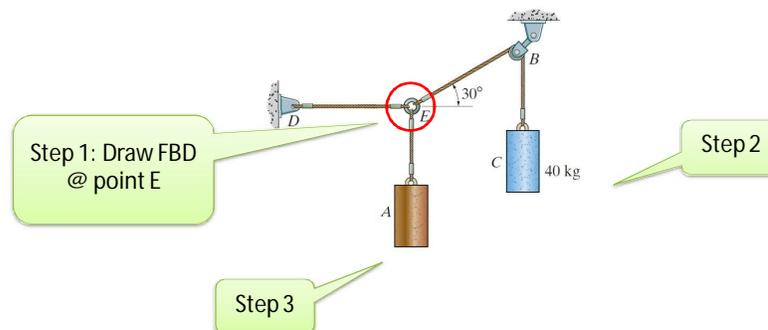
Step 3

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Solution Example 3.2

FBD at point E

Step 2

y

$\mathbf{T_{EB}} = 40*9.81 \text{ N}$

$\mathbf{T_{ED}}$

30°

E x

$\mathbf{T_{EA}}$

Step 3

Step 4

Applying the scalar E-of-E at E,

$+ \rightarrow \Sigma F_x = -T_{ED} + (40*9.81) \cos 30° = 0$

$+ \uparrow \Sigma F_y = (40*9.81) \sin 30° - T_{EA} = 0$

Solving the above equations,

$\underline{T_{ED} = 340 \text{ N}} \leftarrow$ and $\underline{T_{EA} = 196 \text{ N}} \downarrow$

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Example 3.3

This is an example of a 2-D or coplanar force system. If the whole assembly is in equilibrium, then particle C and D are in equilibrium. Determine the force in each cables for a given weight of lamp = 20kg
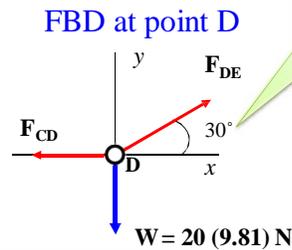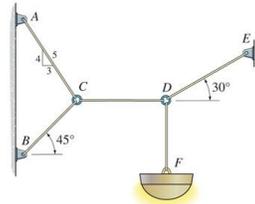
How many unknown forces?

Step 2: Draw FBD @ point C to solve F_{CB} & F_{CA}

Step 1: Draw FBD @ point D to solve F_{CD} & F_{DE}

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Solution Example 3.3

FBD at point D

Step 1: Draw FBD @ point D to solve F_{CD} & F_{DE}

F_{DE}

F_{CD}

30°

D

y

x

$W = 20\ (9.81)\ N$

Applying the scalar E-of-E at D,

$+\uparrow \Sigma F_y = F_{DE} \sin 30° - 20\ (9.81) = 0$

$+\rightarrow \Sigma F_x = F_{DE} \cos 30° - F_{CD} = 0$

Solving the above equations,

$F_{DE} = 392\ N$ and $F_{CD} = 340\ N$

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Solution Example 3.3

FBD at point C

Step 2: Draw FBD @ point C to solve F_{CB} & F_{CA}

F_{AC}

4 | 5
3

$F_{CD} = 340\ N$

C

F_{BC}

45°

y

x
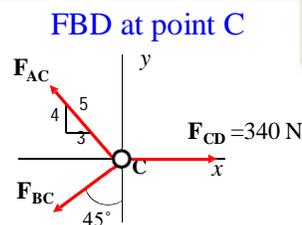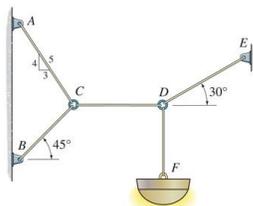
Applying the scalar E-of-E at C,

$+\rightarrow \Sigma F_x = 340 - F_{BC} \sin 45° - F_{AC}\ (3/5) = 0$

$+\uparrow \Sigma F_y = F_{AC}\ (4/5) - F_{BC} \cos 45° = 0$

Solving the above equations,

$F_{BC} = 275\ N$ and $F_{AC} = 243\ N$

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Conclusion of The Chapter 3

- Conclusions

  - The Equilibrium and FBD have been identified
  - The Equilibrium Equation have been implemented to solve a particle problems in Coplanar Forces Systems

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Thank you

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