

University of Cihan-Sulaimaniya
Engineering Faculty
Architectural Engineering Department



ENGINEERING MECHANICS

Chapter 2: Force Vectors (Static)

2nd Grade- Fall Semester 2025-2026

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Chapter Description

- **Aims**
 - To review the Parallelogram Law and Trigonometry
 - To explain the Force Vectors
 - To explain the Vectors Operations (Parlaw & Cartesian)
 - To express force and position in Cartesian Vectors
- **Expected Outcomes**
 - Able to solve the problems of force vectors in the mechanics applications by using Cartesian Coordinate System
- **References**
 - Russel C. Hibbeler. Engineering Mechanics: Statics & Dynamics, 14th Edition

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Chapter Outline

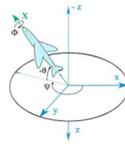
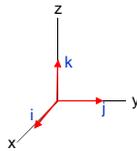
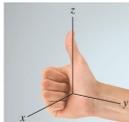
1. Scalars and Vectors – part I
2. Vectors Operations – part I
3. Vectors Addition of Forces – part I
- 4. Cartesian Vectors – part II**
5. Force and Position Vectors – part III



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2.4 Cartesian Vector

What is Cartesian Vector?



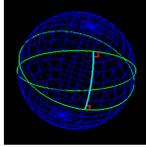
- It is a **coordinate system**
- Use to describe **position**
- Position can be defined by its **coordinate axis**
- It is a **unit vector** $u_A = A / A$
- Its magnitude is **1** and **dimensionless**
- It is denoted as **i,j,k**
- **i** is a unit vector pointing in the **x direction**
- **j** is a unit vector pointing in the **y direction**
- **k** is a unit vector pointing in the **z direction**
- **+ve** direction based on **right handed**
- It is important in **air transport**
- **Air Traffic controller** or **pilots** must know the **location** of every aircraft in the sky
- Without the coordinate system, the position or location of aircraft is **difficult** to know and may lead to **aircraft crashes**

Source: https://en.wikipedia.org/wiki/Axes_conventions

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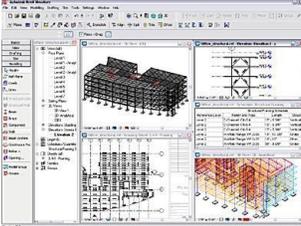
Application of Cartesian Vector






- Military Service
- Position of any body in the real world





- Location / Geographic/Latitude/longitude
- Mapping Project

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Application of Cartesian Vector

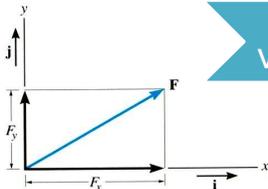
2-D
vector

→

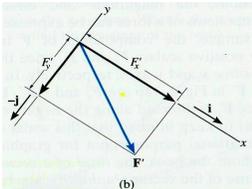
Resolve
vector into
Components

→

Addition
vector



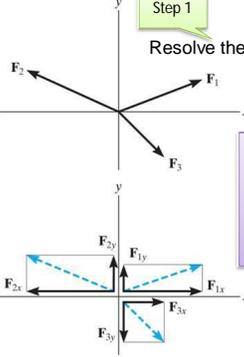
$$\mathbf{F} = F_x \mathbf{i} + F_y \mathbf{j}$$



$$\mathbf{F}' = F'_x \mathbf{i} + (-F'_y) \mathbf{j}$$

Step 1

Resolve the vectors into X and Y components

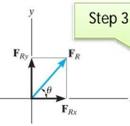


Step 2

Then add them into respective components

$$\begin{aligned} \mathbf{F}_R &= \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 \\ &= F_{1x} \mathbf{i} + F_{1y} \mathbf{j} - F_{2x} \mathbf{i} + F_{2y} \mathbf{j} + F_{3x} \mathbf{i} - F_{3y} \mathbf{j} \\ &= (F_{1x} - F_{2x} + F_{3x}) \mathbf{i} + (F_{1y} + F_{2y} - F_{3y}) \mathbf{j} \\ &= (F_{Rx}) \mathbf{i} + (F_{Ry}) \mathbf{j} \end{aligned}$$

Step 3



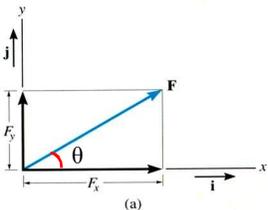
$$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2}$$

$$\theta = \tan^{-1} \left| \frac{F_{Ry}}{F_{Rx}} \right|$$

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Application of Cartesian Vector

How to resolve into components Vector?



$$\mathbf{F} = F_x \mathbf{i} + F_y \mathbf{j}$$

$$|F_x| = F_x = F \cos \theta$$

$$|F_y| = F_y = F \sin \theta$$

The magnitude of \mathbf{F}

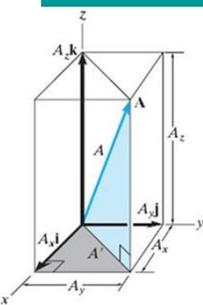
$$F = \sqrt{F_x^2 + F_y^2}$$

The direction of \mathbf{F}

$$\theta = \tan^{-1} \left| \frac{F_y}{F_x} \right|$$

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Application of Cartesian Vector



3-D vector

Resolve vector into components

Addition vector

The vector \mathbf{A} can be resolved as

$$\mathbf{A} = A_x \mathbf{i} + A_y \mathbf{j} + A_z \mathbf{k}$$

Given

$$\mathbf{A} = A_x \mathbf{i} + A_y \mathbf{j} + A_z \mathbf{k}$$

$$\mathbf{B} = B_x \mathbf{i} + B_y \mathbf{j} + B_z \mathbf{k}$$

If $\mathbf{A} + \mathbf{B} = \mathbf{C}$

Sum of the vectors \mathbf{A} and \mathbf{B} can obtained vector \mathbf{C}

$$\mathbf{F}_R = \Sigma \mathbf{F} = \Sigma F_x \mathbf{i} + \Sigma F_y \mathbf{j} + \Sigma F_z \mathbf{k}$$

$$\begin{aligned} \mathbf{C} &= (A_x \mathbf{i} + A_y \mathbf{j} + A_z \mathbf{k}) + (B_x \mathbf{i} + B_y \mathbf{j} + B_z \mathbf{k}) \\ &= (A_x + B_x) \mathbf{i} + (A_y + B_y) \mathbf{j} + (A_z + B_z) \mathbf{k} \\ &= C_x \mathbf{i} + C_y \mathbf{j} + C_z \mathbf{k} \end{aligned}$$

Once the vectors are present in Cartesian term, it easy to add or subtract

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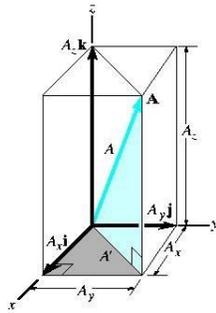
Application of Cartesian Vector

How to determine magnitude and direction angle in 3-D vector?

It should be noted that in 3-D vector information is given as:

- **Magnitude** and the **coordinate direction angles**, or
- **Magnitude** and **projection angles**

- **Magnitude of a Cartesian Vector \mathbf{A} in the x-y plane is A'**



- From the colored triangle,

$$A = \sqrt{A'^2 + A_z^2}$$

- From the shaded triangle,

$$A' = \sqrt{A_x^2 + A_y^2}$$

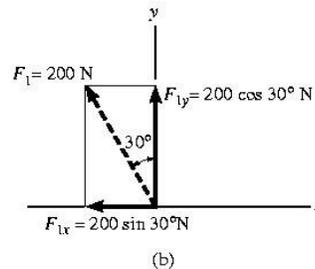
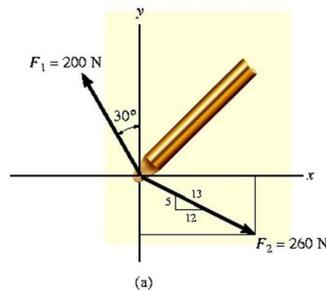
- **The magnitude of the position vector \mathbf{A}**
-Combining the equations gives magnitude of \mathbf{A}

$$A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

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Example 2.4

Determine the X and Y components of F_1 and F_2 in Cartesian vectors



$$F_{1x} = -200 \sin 30^\circ \text{ N} = -100 \text{ N} = 100 \text{ N} \leftarrow$$

$$F_{1y} = 200 \cos 30^\circ \text{ N} = 173 \text{ N} = 173 \text{ N} \uparrow$$

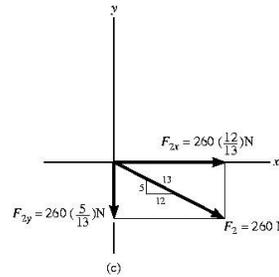
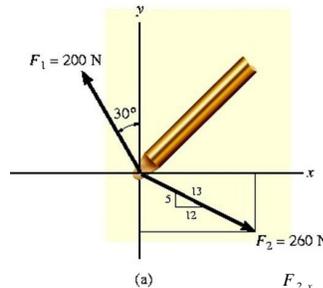
Cartesian Vector Notation:

$$\mathbf{F}_1 = \{-100\mathbf{i} + 173\mathbf{j}\} \text{ N}$$

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Example 2.4

Determine the X and Y components of F_1 and F_2 in Cartesian vectors



$$\frac{F_{2x}}{260\text{ N}} = \frac{12}{13}$$

$$F_{2x} = 260\text{ N} \left(\frac{12}{13}\right) = 240\text{ N} \rightarrow F_{2y} = 260\text{ N} \left(\frac{5}{13}\right) = 100\text{ N} \downarrow$$

Cartesian Vector Notation:

$$\mathbf{F}_2 = \{240\mathbf{i} - 100\mathbf{j}\}\text{ N}$$

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Example 2.5

Determine the magnitude and direction of the resultant force

Step 1

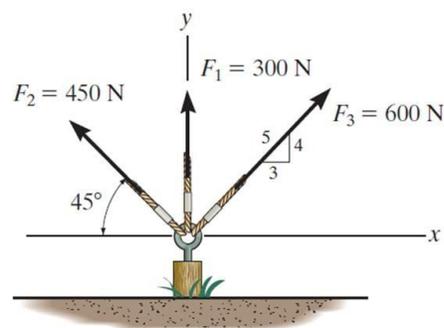
Resolve the forces into components x and y

Step 2

Then add the respective components to get the resultant forces

Step 3

Calculate the magnitude and direction from the resultant forces

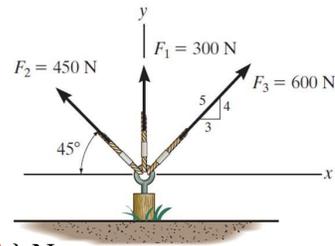


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Solution Example 2.5

Step 1

Resolve the forces into components x and y



$$\mathbf{F}_1 = \{ 0 \mathbf{i} + 300 \mathbf{j} \} \text{ N}$$

$$\begin{aligned} \mathbf{F}_2 &= \{ -450 \cos(45^\circ) \mathbf{i} + 450 \sin(45^\circ) \mathbf{j} \} \text{ N} \\ &= \{ -318.2 \mathbf{i} + 318.2 \mathbf{j} \} \text{ N} \end{aligned}$$

$$\begin{aligned} \mathbf{F}_3 &= \left\{ \left(\frac{3}{5}\right) 600 \mathbf{i} + \left(\frac{4}{5}\right) 600 \mathbf{j} \right\} \text{ N} \\ &= \{ 360 \mathbf{i} + 480 \mathbf{j} \} \text{ N} \end{aligned}$$

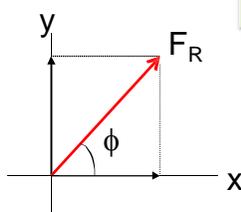
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Solution Example 2.5

Step 2

- Summing up all the \mathbf{i} and \mathbf{j} components respectively:

$$\begin{aligned} \mathbf{F}_R &= \{ (0 - 318.2 + 360) \mathbf{i} + (300 + 318.2 + 480) \mathbf{j} \} \text{ N} \\ &= \{ 41.80 \mathbf{i} + 1098 \mathbf{j} \} \text{ N} \end{aligned}$$



Step 3

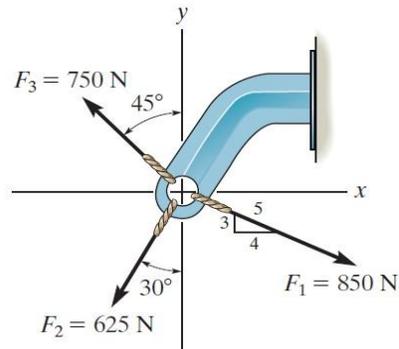
- Magnitude and direction:

$$\begin{aligned} F_R &= ((41.80)^2 + (1098)^2)^{1/2} = \underline{1099 \text{ N}} \\ \phi &= \tan^{-1}(1098/41.80) = \underline{87.8^\circ} \end{aligned}$$

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Example 2.6

Determine the magnitude and direction of the resultant force



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Solution Example 2.6

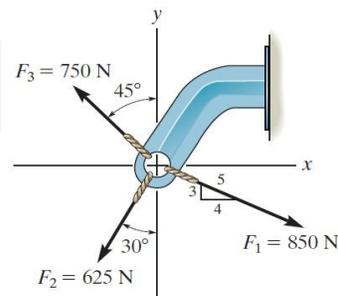
Step 1

Resolve the forces into components

$$\begin{aligned} \mathbf{F}_3 &= \{-750 \sin(45^\circ) \mathbf{i} + 750 \cos(45^\circ) \mathbf{j}\} \text{ N} \\ &= \{-530.3 \mathbf{i} + 530.3 \mathbf{j}\} \text{ N} \end{aligned}$$

$$\begin{aligned} \mathbf{F}_2 &= \{-625 \sin(30^\circ) \mathbf{i} - 625 \cos(30^\circ) \mathbf{j}\} \text{ N} \\ &= \{-312.5 \mathbf{i} - 541.3 \mathbf{j}\} \text{ N} \end{aligned}$$

$$\begin{aligned} \mathbf{F}_1 &= \{850(4/5) \mathbf{i} - 850(3/5) \mathbf{j}\} \text{ N} \\ &= \{680 \mathbf{i} - 510 \mathbf{j}\} \text{ N} \end{aligned}$$



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Solution Example 2.6

Step 2

- Summing all the i and j components, respectively:

$$\begin{aligned} \mathbf{F}_R &= \{ (680 - 312.5 - 530.3) \mathbf{i} + (-510 - 541.3 + 530.3) \mathbf{j} \} \text{N} \\ &= \{ -162.8 \mathbf{i} - 520.9 \mathbf{j} \} \text{N} \end{aligned}$$

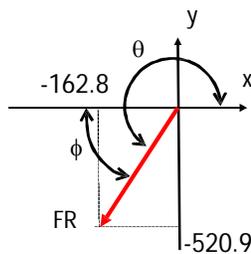
Step 3

- Magnitude and direction (angle):

$$F_R = ((-162.8)^2 + (-520.9)^2)^{1/2} = \underline{546 \text{ N}}$$

$$\phi = \tan^{-1}(520.9 / 162.8) = \underline{72.6^\circ}$$

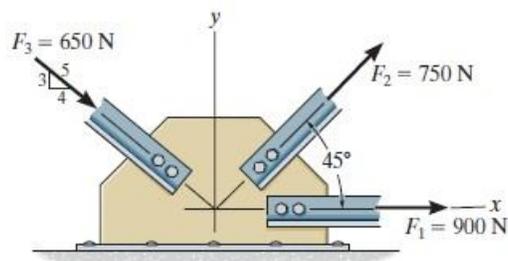
$$\text{From the positive x-axis, } \theta = 253^\circ$$



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Example 2.7

Determine the magnitude of the resultant force acting on the plate and its direction, measured counterclockwise from the positive x axis.



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Solution Example 2.7

Rectangular Components: By referring to Fig. a, the x and y components of F_1 , F_2 , and F_3 can be written as

$$\begin{aligned}(F_1)_x &= 900 \text{ N} & (F_1)_y &= 0 \\ (F_2)_x &= 750 \cos 45^\circ = 530.33 \text{ N} & (F_2)_y &= 750 \sin 45^\circ = 530.33 \text{ N} \\ (F_3)_x &= 650 \left(\frac{4}{5}\right) = 520 \text{ N} & (F_3)_y &= 650 \left(\frac{3}{5}\right) = 390 \text{ N}\end{aligned}$$

Resultant Force: Summing the force components algebraically along the x and y axes, we have

$$\begin{aligned}\rightarrow \Sigma(F_R)_x &= \Sigma F_x; & (F_R)_x &= 900 + 530.33 + 520 = 1950.33 \text{ N} \rightarrow \\ \uparrow \Sigma(F_R)_y &= \Sigma F_y; & (F_R)_y &= 530.33 - 390 = 140.33 \text{ N} \uparrow\end{aligned}$$

The magnitude of the resultant force F_R is

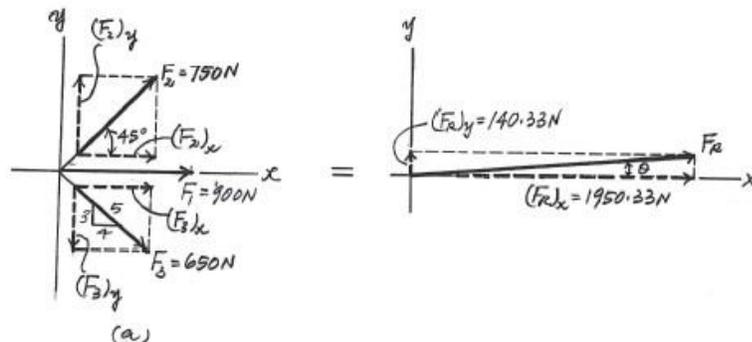
$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{1950.33^2 + 140.33^2} = 1955 \text{ N} = 1.96 \text{ kN} \text{ Ans.}$$

The direction angle θ of F_R , measured clockwise from the positive x axis, is

$$\theta = \tan^{-1} \left[\frac{(F_R)_y}{(F_R)_x} \right] = \tan^{-1} \left(\frac{140.33}{1950.33} \right) = 4.12^\circ \text{ Ans.}$$

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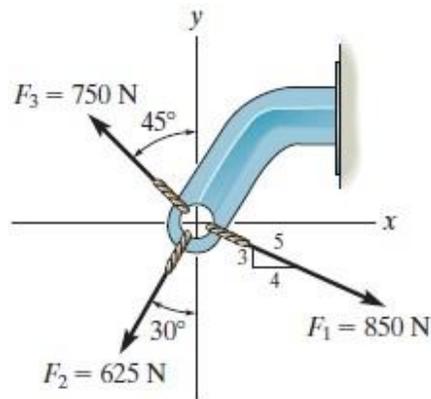
Solution Example 2.7



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Example 2.8

Determine the magnitude of the resultant force and its direction measured counterclockwise from the positive x axis.



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Solution Example 2.8

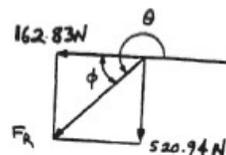
$$\rightarrow F_{Rx} = \Sigma F_x; \quad F_{Rx} = \frac{4}{5}(850) - 625 \sin 30^\circ - 750 \sin 45^\circ = -162.83 \text{ N}$$

$$+\uparrow F_{Ry} = \Sigma F_y; \quad F_{Ry} = -\frac{3}{5}(850) - 625 \cos 30^\circ + 750 \cos 45^\circ = -520.94 \text{ N}$$

$$F_R = \sqrt{(-162.83)^2 + (-520.94)^2} = 546 \text{ N} \quad \text{Ans.}$$

$$\phi = \tan^{-1}\left(\frac{520.94}{162.83}\right) = 72.64^\circ$$

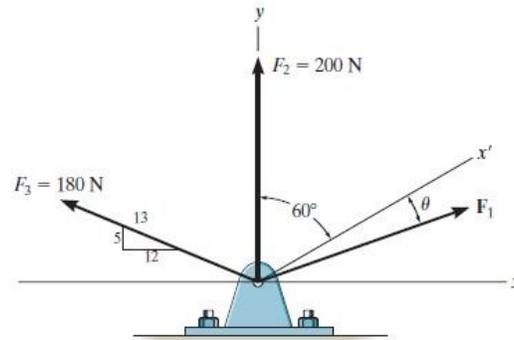
$$\theta = 180^\circ + 72.64^\circ = 253^\circ \quad \text{Ans.}$$



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Example 2.9

Three forces act on the bracket. Determine the magnitude and direction θ of F_1 so that the resultant force is directed along the positive x' axis and has a magnitude of 800 N.



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Solution Example 2.9

$$\rightarrow F_{Rx} = \Sigma F_x; \quad 800 \sin 60^\circ = F_1 \sin(60^\circ + \theta) - \frac{12}{13}(180)$$

$$+\uparrow F_{Ry} = \Sigma F_y; \quad 800 \cos 60^\circ = F_1 \cos(60^\circ + \theta) + 200 + \frac{5}{13}(180)$$

$$60^\circ + \theta = 81.34^\circ$$

$$\theta = 21.3^\circ$$

$$F_1 = 869 \text{ N}$$

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Conclusion of The Chapter 2 part II

- Conclusions
 - The Cartesian vector have been identified and determined in the mechanics
 - The 2-D and 3-D vector has been represent in a Cartesian coordinate system
 - The magnitude and direction of 2-D of resultant forces have been determined in a Cartesian coordinate system



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Thank you

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