University of Cihan-Sulaimaniya
Engineering Faculty
Architectural Engineering Department

ENGINEERING MECHANICS

Chapter 2: Force Vectors (Static)

2nd Grade- Fall Semester 2025-2026

Instructor: Diyari B. Hussein

Chapter Description

- Aims
  - To review the Parallelogram Law and Trigonometry
  - To explain the Force Vectors
  - To explain the Vectors Operations
  - To express force and position in Cartesian Vectors
- Expected Outcomes
  - Able to solve the problems of force vectors in the mechanics applications by using Parallelogram law and Trigonometry
- References
  - Russel C. Hibbeler. Engineering Mechanics: Statics & Dynamics, 14th Edition

Chapter Outline

1. Scalars and Vectors – part I
2. Vectors Operations – part I
3. Vectors Addition of Forces – part I
4. Cartesian Vectors – part II
5. Force and Position Vectors – part III

2.1 Scalars and Vectors

What is Scalars?

A quantity that has only a magnitude

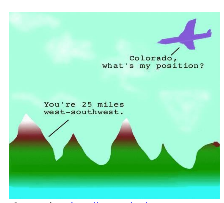
What is Vectors

A quantity that has both magnitude and direction

Source: http://www.automobiledimension.com/

Length of a mini car is **3.821** mm

↓ **quantity** ↓ **magnitude**



Source:http: http://www.shodor.org

Position of the plane is **25** miles from **west southwest**

↓ **quantity** ↓ **magnitude** ↓ **direction**

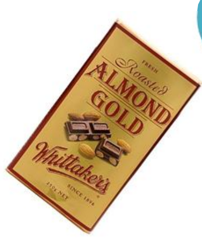
Identify scalars and vectors

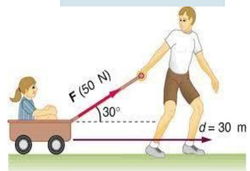
Scalars

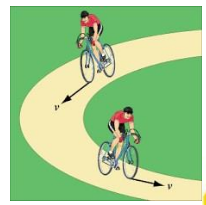
Identify example of scalars?

Vectors

Identify example of vectors?







Communitising Technology

Comparison of Scalars and Vectors

Scalars	Vectors
A quantity that has only a magnitude	A quantity that has both magnitude and direction
Mass, Length, Time, Temperature, Volume,Density	Position, Displacement, Velocity, Acceleration, Momentum,Force

Communitising Technology

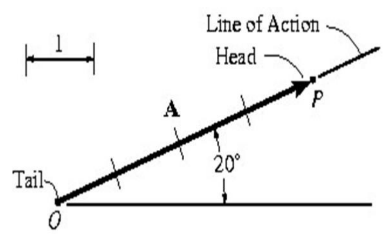
Vectors

- Represent by a letter with an arrow over it such as $\vec{A}$ or **A**
- Magnitude is designated as $|\vec{A}|$ or simply A
- Commonly, vector is presented as **A** and its magnitude (positive quantity) as A

Communitising Technology

Characteristics of Vectors

- Represented graphically as an arrow
- Length of arrow = Magnitude of Vector
- Angle between the reference axis and arrow's line of action = Direction of Vector
- Arrowhead = Sense of Vector



Communitising Technology

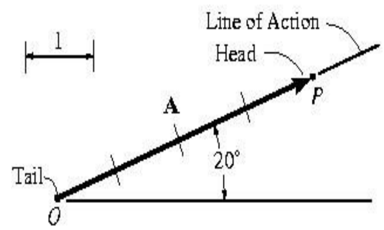
Example of Vectors

Magnitude of Vector = 4 units

Direction of Vector = 20° measured counterclockwise from the horizontal axis

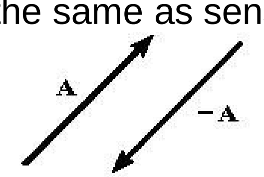
Sense of Vector = Upward and to the right

The point O is called ***tail*** of the vector and the point P is called the ***tip*** or ***head***



2.2 Vector Operations

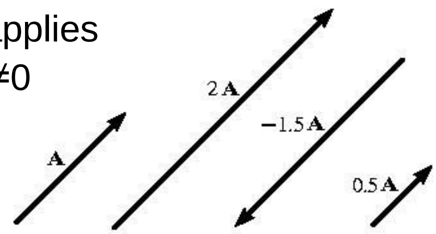
- Multiplication and Division of a Vector by a Scalar
- Product of vector **A** and scalar a = a**A**
- Magnitude = $|aA|$
- If a is positive, sense of a**A** is the same as sense of **A**
- If a is negative sense of
- a**A**, it is opposite to the
- sense of **A**



Vector **A** and its negative counterpart

2.2 Vector Operations

- Multiplication and Division of a Vector by a Scalar
  - Negative of a vector is found by multiplying the vector by (-1)
  - Law of multiplication applies
  - Eg: **A**/a = (1/a) **A**, a≠0



Scalar Multiplication and Division

Communitising Technology

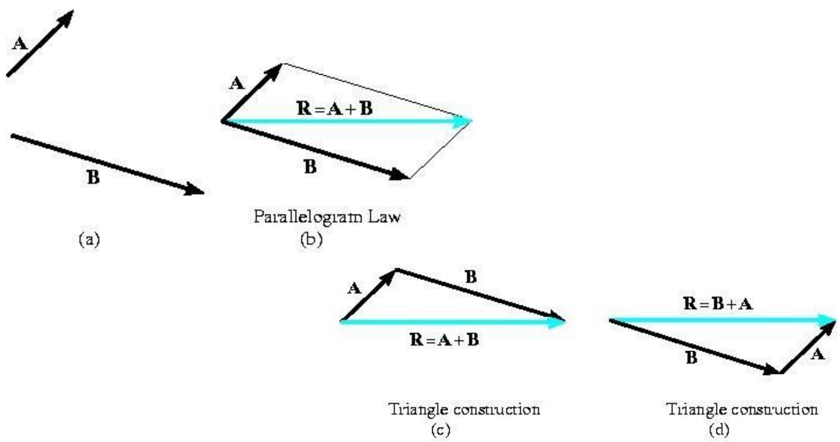
2.2 Vector Operations

- Vector Addition
  - Addition of two vectors **A** and **B** gives a resultant vector **R** by the ***parallelogram law***
  - Result **R** can be found by ***triangle construction***
  - Communicative
  - Eg: **R** = **A** + **B** = **B** + **A**

Communitising Technology

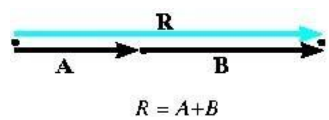
2.2 Vector Operations

- Vector Addition



2.2 Vector Operations

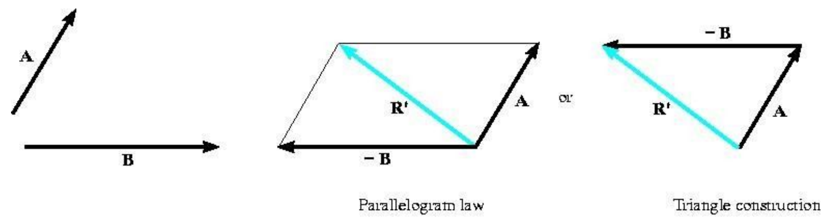
- Vector Addition
- Special case: Vectors **A** and **B** are *collinear* (both have the same line of action)



Addition of collinear vectors

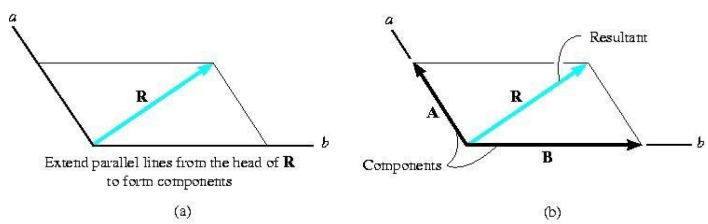
2.2 Vector Operations

- Vector Subtraction
  - Special case of addition
  - Eg: **R**' = **A** – **B** = **A** + (- **B**)
  - Rules of Vector Addition Applies



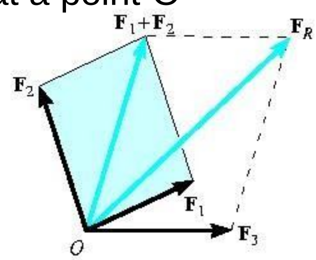
2.2 Vector Operations

- Resolution of Vector
  - Any vector can be resolved into two components by the ***parallelogram law***
  - The two components **A** and **B** are drawn such that they extend from the tail or **R** to points of intersection



2.3 Vector Addition of Forces

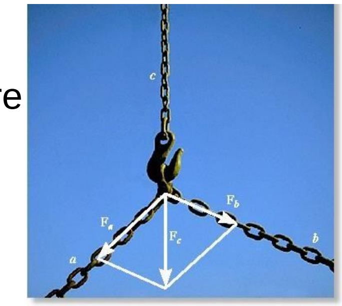
- When two or more forces are added, successive applications of the ***parallelogram law*** is carried out to find the resultant
- Eg: Forces $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ acts at a point O
- First, find resultant of
- $\mathbf{F}_1 + \mathbf{F}_2$
- Resultant,
- $\mathbf{F}_R = (\mathbf{F}_1 + \mathbf{F}_2) + \mathbf{F}_3$



Communitising Technology

Example of Vector Addition of Forces

- $\mathbf{F}_a$ and $\mathbf{F}_b$ are forces exerting on the hook.
- Resultant, $\mathbf{F}_c$ can be found using the *parallelogram law*
- Lines parallel to a and b
- from the heads of $\mathbf{F}_a$ and $\mathbf{F}_b$ are
- drawn to form a parallelogram
- Similarly, given $\mathbf{F}_c$, $\mathbf{F}_a$ and $\mathbf{F}_b$
- can be found



Communitising Technology

Steps to Solve the Vectors Operations

- Parallelogram Law
- Make a sketch using the *parallelogram law*
- Two components forces add to form the resultant force
- Resultant force is shown by the diagonal of the parallelogram
- The components is shown by the sides of the parallelogram

Communitising Technology

Steps to Solve the Vectors Operations

- Parallelogram Law
- To resolve a force into components along two axes directed from the tail of the force
- Start at the head, constructing lines parallel to the axes
- Label all the known and unknown force magnitudes and angles
- Identify the two unknown components

Communitising Technology

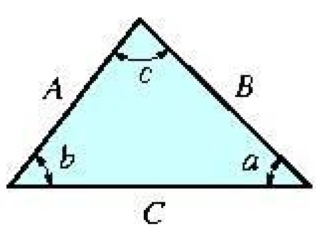
Steps to Solve the Vectors Operations

- Trigonometry
- Redraw half portion of the parallelogram
- Magnitude of the resultant force can be determined by the ***law of cosines***
- Direction if the resultant force can be determined by the ***law of sines***

Communitising Technology

Steps to Solve the Vectors Operations

- Trigonometry
- Magnitude of the two components can be determined by the ***law of sines***



Sine law:

$$\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}$$

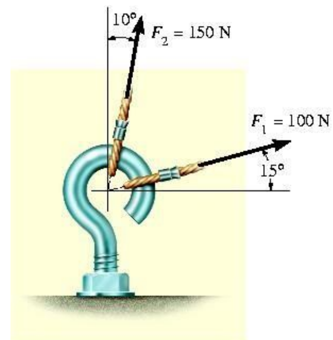
Cosine law:

$$C = \sqrt{A^2 + B^2 - 2AB\cos c}$$

Communitising Technology

Example 2.1

The screw eye is subjected to two forces $\mathbf{F}_1$ and $\mathbf{F}_2$. Determine the magnitude and direction of the resultant force.



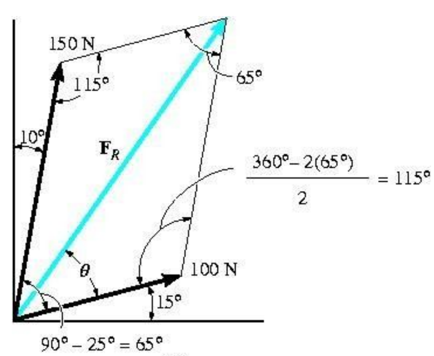
(a)

Communitising Technology

Solution Example 2.1

From Parallelogram Law

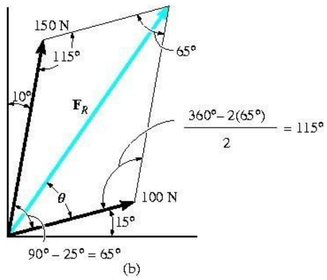
Unknown: magnitude of $\mathbf{F}_R$ and angle θ



(b)

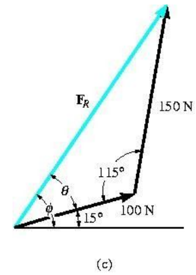
Communitising Technology

Solution Example 2.1



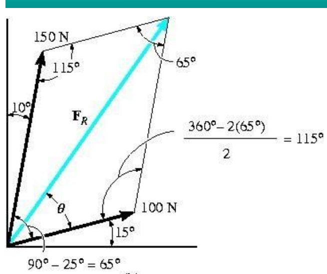
Trigonometry

Law of Cosines



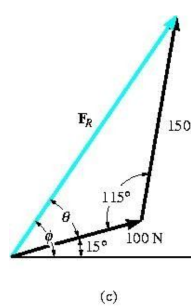
$$
\begin{aligned}
F_R &= \sqrt{(100N)^2 + (150N)^2 - 2(100N)(150N)\cos 115^\circ} \\
&= \sqrt{10000 + 22500 - 30000(-0.4226)} \\
&= 212.6N \\
&= 213N
\end{aligned}
$$

Solution Example 2.1



Trigonometry

Law of Sines

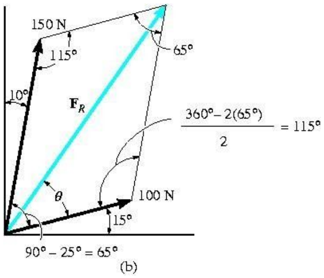


$$\frac{150N}{\sin\theta} = \frac{212.6N}{\sin 115^\circ}$$

$$\sin\theta = \frac{150N}{212.6N}(0.9063)$$

$$\theta = 39.8^\circ$$

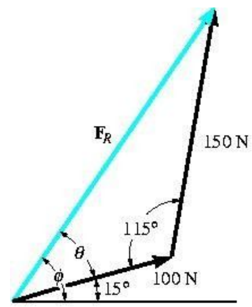
Solution Example 2.1



Trigonometry

Direction Φ of $\mathbf{F}_R$ measured from the horizontal

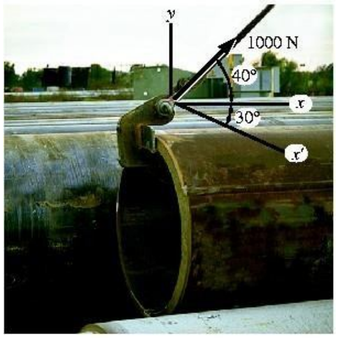
$$\phi = 39.8^\circ + 15^\circ$$
$$= 54.8^\circ \angle^{\phi}$$



Communitising Technology

Example 2.2

Resolve the 1000 N (≈ 100kg) force acting on the pipe into the components in the
(a) x and y directions,
(b) x' and y directions.



(a)

Communitising Technology

Solution Example 2.2

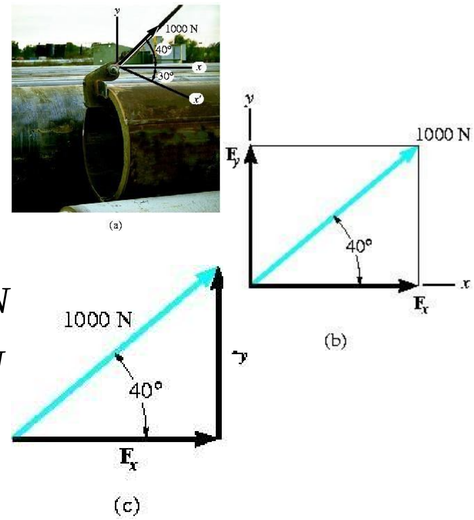
(a) Parallelogram Law

From the vector diagram,

$$F = F_x + F_y$$

$$F_x = 1000\cos 40^\circ = 766N$$

$$F_y = 1000\sin 40^\circ = 643N$$

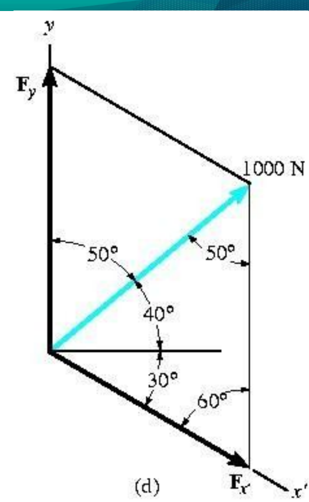


Communitising Technology

Solution Example 2.2

(b) Parallelogram Law

$$F = F_x + F_{y'}$$



Communitising Technology

Solution Example 2.2

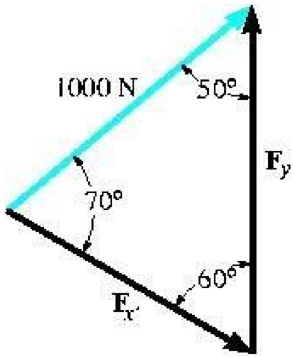
(b) Law of Sines

$$\frac{F_{x'}}{\sin 50^\circ} = \frac{1000N}{\sin 60^\circ}$$

$$F_{x'} = 1000N\left(\frac{\sin 50^\circ}{\sin 60^\circ}\right) = 884.6N$$

$$\frac{F_y}{\sin 70^\circ} = \frac{1000N}{\sin 60^\circ}$$

$$F_y = 1000N\left(\frac{\sin 70^\circ}{\sin 60^\circ}\right) = 1085N$$

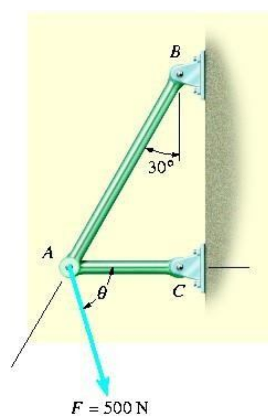


(e)

Communitising Technology

Example 2.3

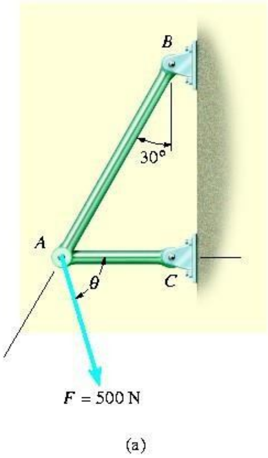
The force **F** acting on the frame has a magnitude of 500N and is to be resolved into two components acting along the members AB and AC. Determine the angle θ, measured below the horizontal, so that components $\mathbf{F}_{AC}$ is directed from A towards C and has a magnitude of 400N.



(a)

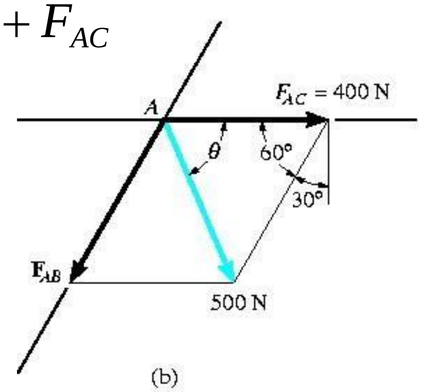
Communitising Technology

Solution Example 2.3



Parallelogram Law

$$500\,N = F_{AB} + F_{AC}$$



Communitising Technology

Solution Example 2.3

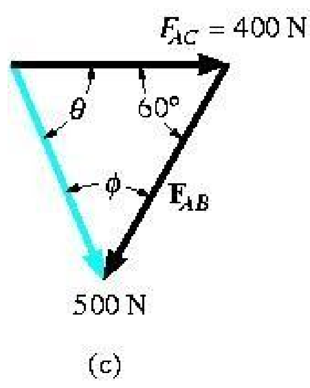
Law of Sines

$$\frac{400\,N}{\sin\phi} = \frac{500\,N}{\sin 60^\circ}$$

$$\sin\phi = \left(\frac{400\,N}{500\,N}\right)\sin 60^\circ$$

$$\sin\phi = 0.6928$$

$$\phi = 43.9^\circ$$



Communitising Technology

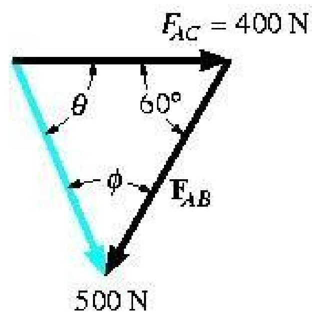
Solution Example 2.3

Hence,

$$\theta = 180^\circ - 60^\circ - 43.9^\circ = 76.1^\circ \angle^{\theta}$$

By Law of Cosines or Law of Sines

Hence, show that $\mathbf{F}_{AB}$ has a magnitude of 561N



(c)

Conclusion of The Chapter 2 part I

- Conclusions
  - The scalars and vectors have been identified and implemented in the mechanics
  - The vector operations have been identified and implemented in the mechanics

Thank you

Communitising Technology