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SULAIMANIYA

Mathematics-1-

For Engineering

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Chapter 2
Function and graph

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OVERVIEW

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Functions are fundamental to the study of calculus. In this chapter

- we review what functions are and how they are pictured as graphs, how they are combined and transformed, and ways they can be classified

- We review the trigonometric functions, and obtain a function's graph. The real number system, Cartesian coordinates, straight lines, parabolas, and circles. We treat inverse, exponential, and logarithem

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2.1 THE BASIC FUNCTIONS AND CURVES

The standard functions and shapes are

1. Straight Lines: $y = mx + c$
2. Quadratics (parabolas): $y = ax^2 + bx + c$
3. Polynomials: $y = a_n x^n + \cdots + a_1 x + a_0$
4. Hyperbola: $y = \frac{1}{x}$
5. Exponential: $y = e^x \equiv \exp x$
6. Logarithm: $y = \ln x$
7. Sine: $y = \sin x$
8. Cosine: $y = \cos x$
9. Tangent: $y = \tan x$
10. Circles: $y^2 + x^2 = r^2$
11. Ellipses: $\left(\frac{y}{a}\right)^2 + \left(\frac{x}{b}\right)^2 = 1.$

2.2 FUNCTION PROPERTIES

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Functions are a tool for describing the real world in mathematical terms. A function can be represented by an equation, a graph, a numerical table, or a verbal description;or we can say its a rule of mapping

DEFINITION A **function** f from a set D to a set Y is a rule that assigns a *unique* (single) element $f(x) \in Y$ to each element $x \in D$.

$$y = f(x) \qquad (\text{“}y \text{ equals } f \text{ of } x\text{”}).$$

In this notation, the symbol f represents the function, the letter x is the **independent variable** representing the input value of f, and y is the **dependent variable** or output value of f at x.

A diagram showing a function as a kind of machine.

2.2 FUNCTION PROPERTIES

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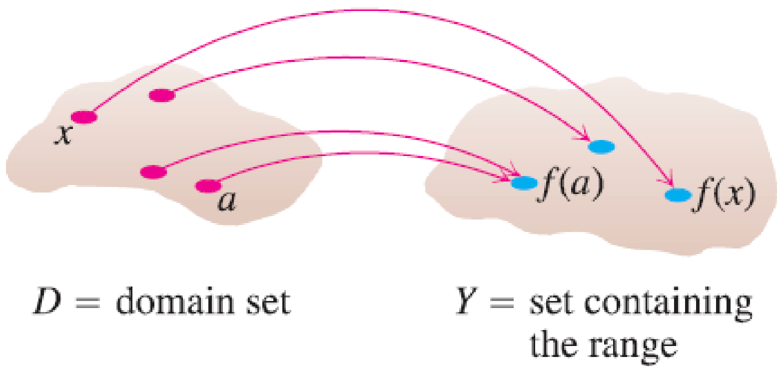
The set D of all possible input values is called the domain of the function. The set of all values of f(x) as x varies throughout D is called the range of the function.



A function is a rule for mapping one number to another. For example: $f(x) = x^2$ is a mapping from x to x^2 so that $f(3) = 3^2 = 9$.

EXAMPLES

1. If $f(x) = 3x + 1$ then $f(2) = 7$ and $f(a) = 3a + 1$.
2. If $f(z) = z^2 - 1$ then $f(1) = 0$.

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EXAMPLE

Function	Domain (x)	Range (y)
$y = x^2$	$(-\infty, \infty)$	$[0, \infty)$
$y = 1/x$	$(-\infty, 0) \cup (0, \infty)$	$(-\infty, 0) \cup (0, \infty)$
$y = \sqrt{x}$	$[0, \infty)$	$[0, \infty)$
$y = \sqrt{4 - x}$	$(-\infty, 4]$	$[0, \infty)$
$y = \sqrt{1 - x^2}$	$[-1, 1]$	$[0, 1]$

Solution The formula $y = x^2$ gives a real y-value for any real number x, so the domain is $(-\infty, \infty)$. The range of $y = x^2$ is $[0, \infty)$ because the square of any real number is nonnegative and every nonnegative number y is the square of its own square root, $y = \left(\sqrt{y}\right)^2$ for $y \geq 0$.

The formula $y = 1/x$ gives a real y-value for every x except $x = 0$. For consistency in the rules of arithmetic, *we cannot divide any number by zero*. The range of $y = 1/x$, the set of reciprocals of all nonzero real numbers, is the set of all nonzero real numbers, since $y = 1/(1/y)$. That is, for $y \neq 0$ the number $x = 1/y$ is the input assigned to the output value y.

In $y = \sqrt{4 - x}$, the quantity $4 - x$ cannot be negative. That is, $4 - x \geq 0$, or $x \leq 4$. The formula gives real y-values for all $x \leq 4$. The range of $\sqrt{4 - x}$ is $[0, \infty)$, the set of all nonnegative numbers.

The formula $y = \sqrt{1 - x^2}$ gives a real y-value for every x in the closed interval from -1 to 1. Outside this domain, $1 - x^2$ is negative and its square root is not a real number. The values of $1 - x^2$ vary from 0 to 1 on the given domain, and the square roots of these values do the same. The range of $\sqrt{1 - x^2}$ is $[0, 1]$. ■

Graphs of Functions

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If f. is a function with domain D, its graph consists of the points in the Cartesian plane whose coordinates are the input-output pairs for f. In set notation, the graph is

$$\{(x, f(x)) \mid x \in D\}.$$

The Properties of Functions

- The Domain and Range of a Function.
- The Increase and Decrease of a Function.
- The Maximum and Minimum of a Function.
- The Sign of a Function.
- The Intercepts of a Function.
- The Asymptotes of a Function.

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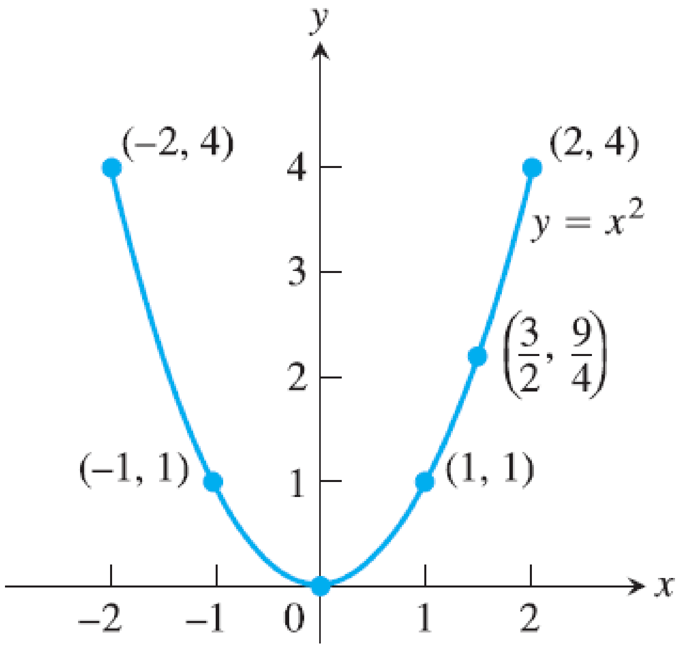
$$\{(x, f(x)) \mid x \in D\}.$$

EXAMPLE 2 Graph the function $y = x^2$ over the interval $[-2, 2]$.

x	$y = x^2$
-2	4
-1	1
0	0
1	1
$\frac{3}{2}$	$\frac{9}{4}$
2	4



Even Functions and Odd Functions: Symmetry

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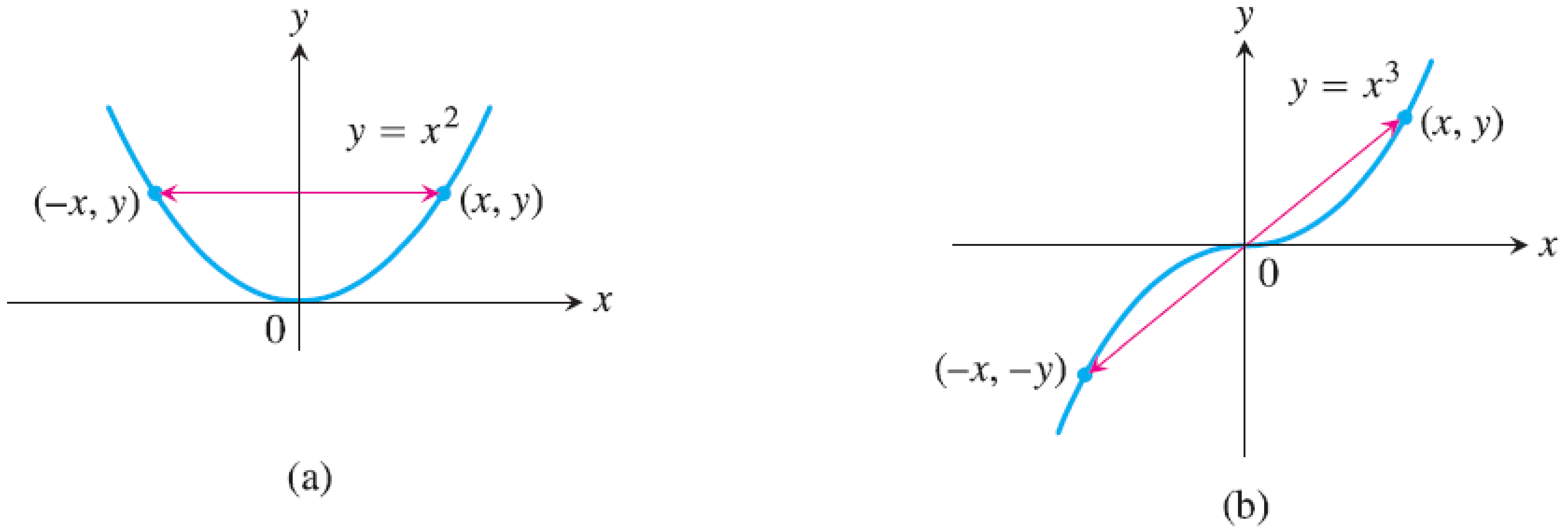
DEFINITIONS A function $y = f(x)$ is an

even function of *x* if $f(-x) = f(x)$,
odd function of *x* if $f(-x) = -f(x)$,

for every x in the function's domain.

EXAMPLES



(a) The graph of $y = x^2$ (an even function) is symmetric about the y-axis.

(b) The graph of $y = x^3$ (an odd function) is symmetric about the origin.

The **zeros** of a function, $f(x)$, are the values of x when $f(x) = 0$.

EXAMPLES

1. $f(x) = 2x + 3$ has zero $x = -\frac{3}{2}$.
2. $f(x) = x^2 + 3x + 2$ has zeros $x = -1, -2$.

The argument of a function could be the value of another function. For example if $f(x) = x^2$ and $g(x) = x + 1$ then

$$f(g(x)) = (g(x))^2 = (x+1)^2.$$

EXAMPLES

1. If $f(x) = 3x - 1$ then $f(x+1) = 3(x+1) - 1 = 3x + 2.$
2. If $f(x) = 2x + 1$ and $g(x) = \cos(x)$ then $f(g(x))$

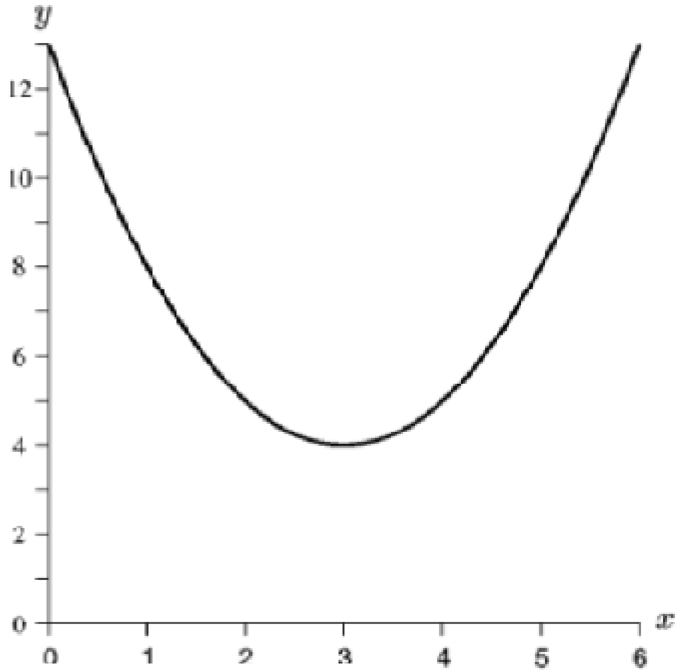
$$= 2\cos(x) + 1 \text{ and } g(f(x)) = \cos(2x+1).$$

A graph $y = f(x)$ shifted from being centred on $(0, 0)$ to being centred on (a, b) is written in the form $y - b = f(x - a)$.

EXAMPLES

1. A circle with centre (1,2) has form $(x-1)^2 + (y-2)^2 = r^2$.

2. A parabola $y = x^2$ with turning point (0,0) if shifted to having turning point (3,4) has equation $(y-4) = (x-3)^2$.



A function is **even** if $f(-x) = f(x)$ and **odd** if $f(-x) = -f(x)$.

EXAMPLES

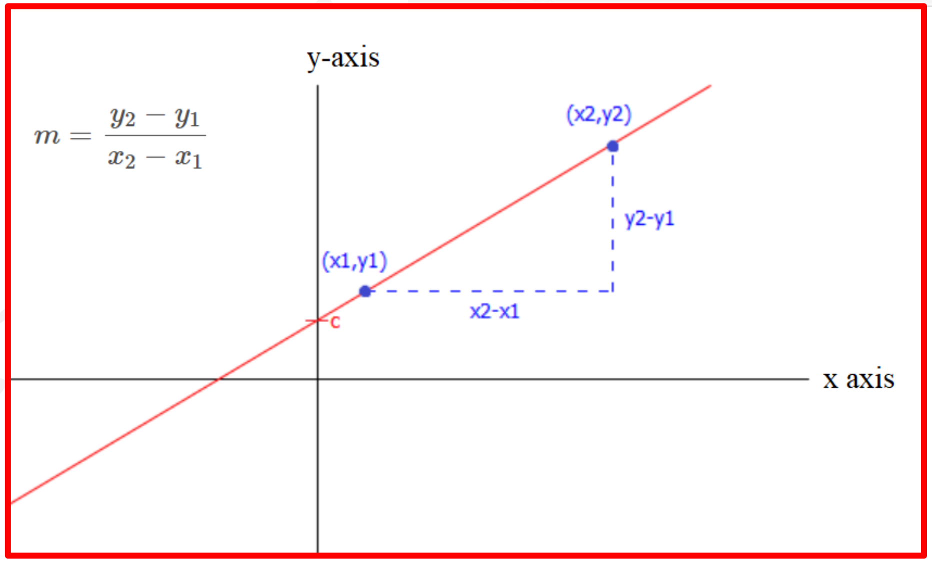
1. $y = f(x) = x^3$ is odd since $f(-x) = (-x)^3 = -x^3 = -f(x)$.

2. $y = f(x) = x^4$ is even since $f(-x) = (-x)^4 = x^4 = f(x)$.

2.3 STRAIGHT LINES

The *equation of a straight line* is **y=mx+c**

where m is the *gradient* and c is the height at which the line crosses the y-axis, also known as the y-*intercept*.



Example
Find the equation of the straight line through the points (−5,7) and (1,3).

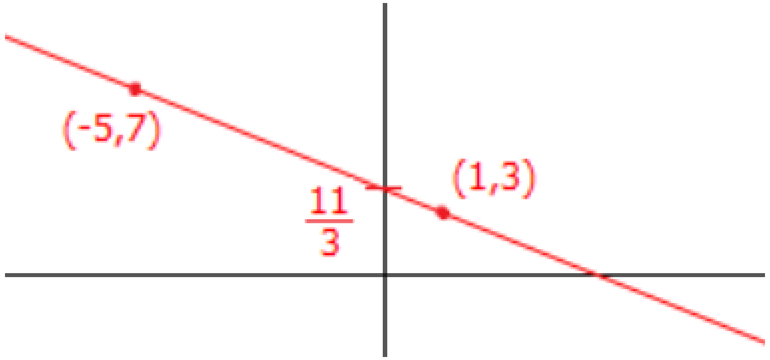
Solution
First, find the gradient by substituting the coordinates x1=−5, y1=7, x2=1and y2=3 into the formula for the gradient

$$m = \frac{y_2 - y_1}{x_2 - x_1} \Rightarrow = \frac{3-7}{1-(-5)} = -\frac{2}{3}$$

the formula

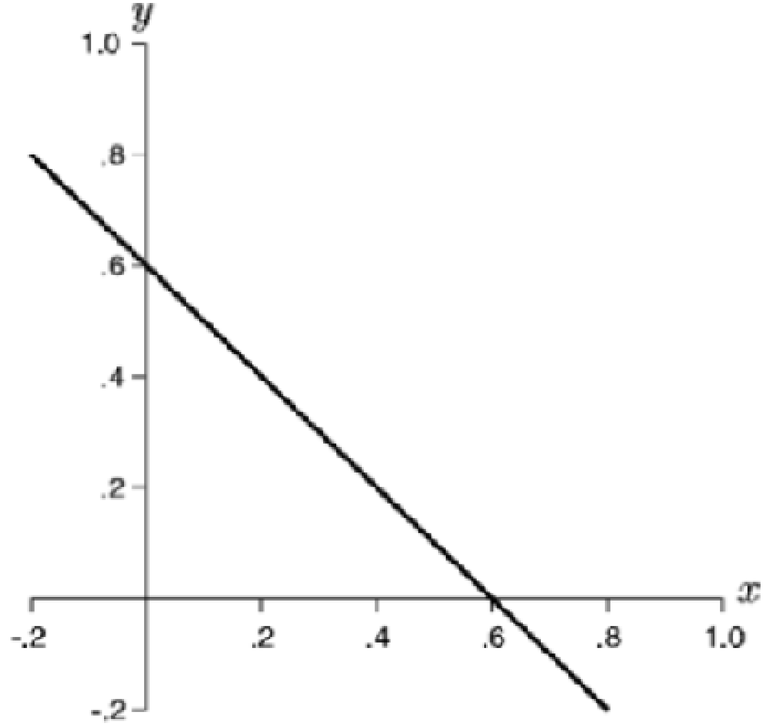
$$y - y_1 = m(x - x_1)$$
$$y - 7 = -\frac{2}{3}(x - (-5))$$

$$y - 7 = -\frac{2}{3}x - \frac{10}{3}$$
$$y = -\frac{2}{3}x + \frac{11}{3}$$



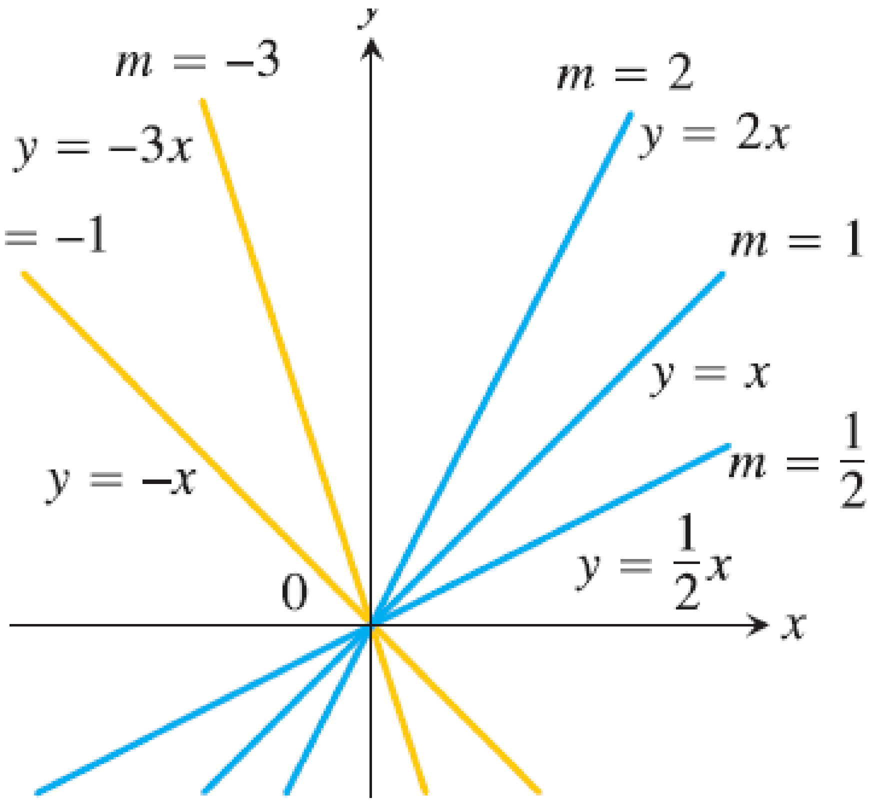
EXAMPLES

1. Part of the straight line $y = 0.6 - x$ is drawn in the following diagram:



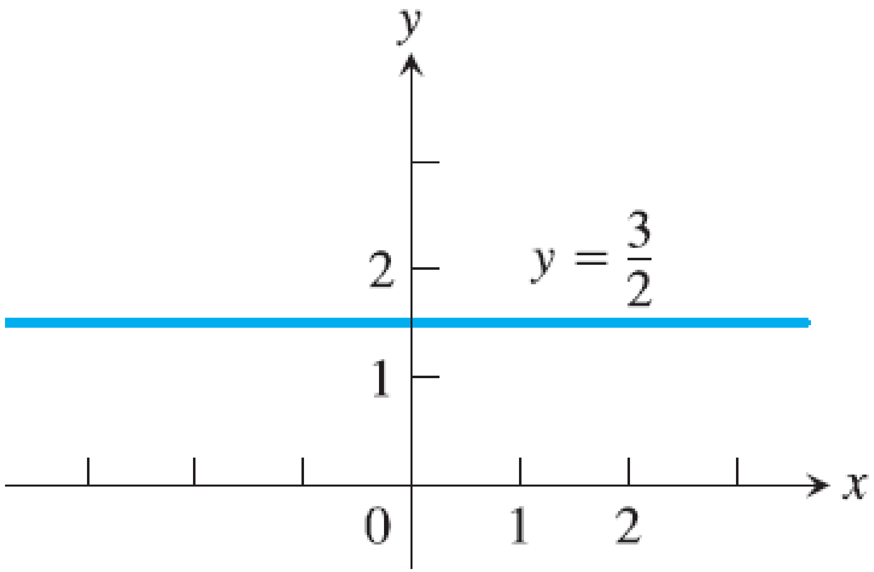
2. The line $y = 2x + 1$ cuts the x axis when $y = 0$ giving $x = -\frac{1}{2}$ as the zero.

EXAMPLES



(a)

(a) Lines through the origin with slope m.



(b)

(b) A constant function with slope $m = 0$.

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The domain of a function can be given in different ways: sets of numbers, intervals, and brackets.

EXAMPLES

1. $y = x^2 + 4$ has domain of all real numbers.
2. $y = 1/(x-1)$ has domain $x \neq 1$. That is, all real numbers *except* $x = 1$ can be used in this function. If $x = 1$ then the function is undefined because of division by zero.

Sometimes the domain is defined as part of the function such as $y = x^2$ for $0 < x < 3$ so that the domain is restricted to be in the interval zero to three.

3. What is the domain of the function f(x)=12x−1=1? The function is plotted as follows.

The domain of this function is formed by all the real numbers because the values that the variable x can have are all the values between negative and positive infinity. In mathematical language, the domain can be written in the following ways:

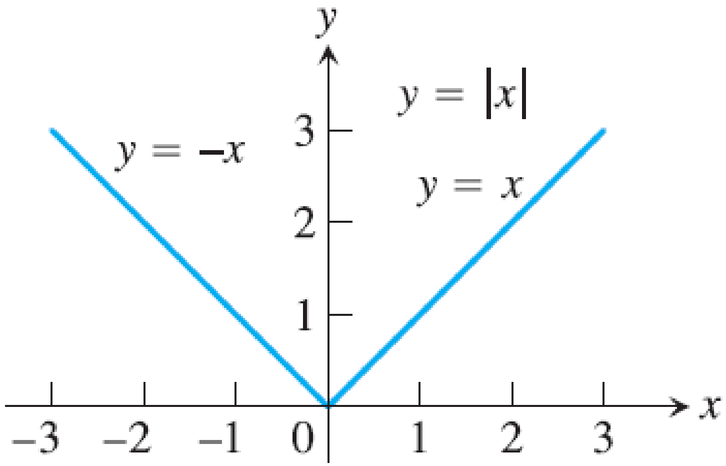
$\text{dom}(f) = \mathbb{R}$ or $\text{dom}(f) =]-\infty, +\infty[$ or $\text{dom}(f) = \{x \in \mathbb{R}\}$

Instead of writing the interval]−∞,+∞[, we simply write R.

EXAMPLE

absolute value function

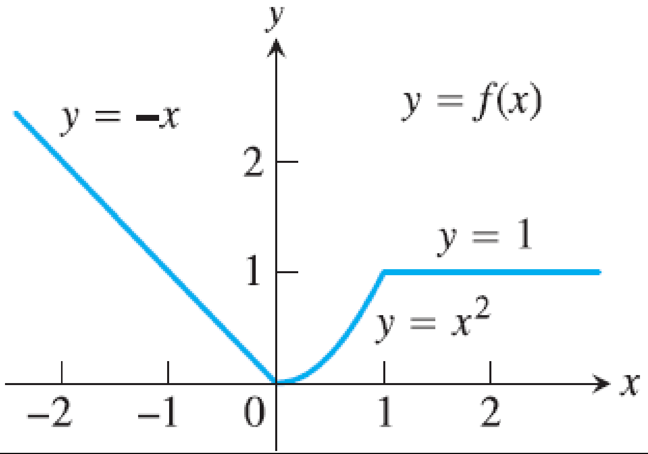
$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0, \end{cases}$$



The absolute value function has domain $(-\infty, \infty)$ and range $[0, \infty)$.

EXAMPLE The function

$$f(x) = \begin{cases} -x, & x < 0 \\ x^2, & 0 \leq x \leq 1 \\ 1, & x > 1 \end{cases}$$



EXAMPLE QUESTIONS

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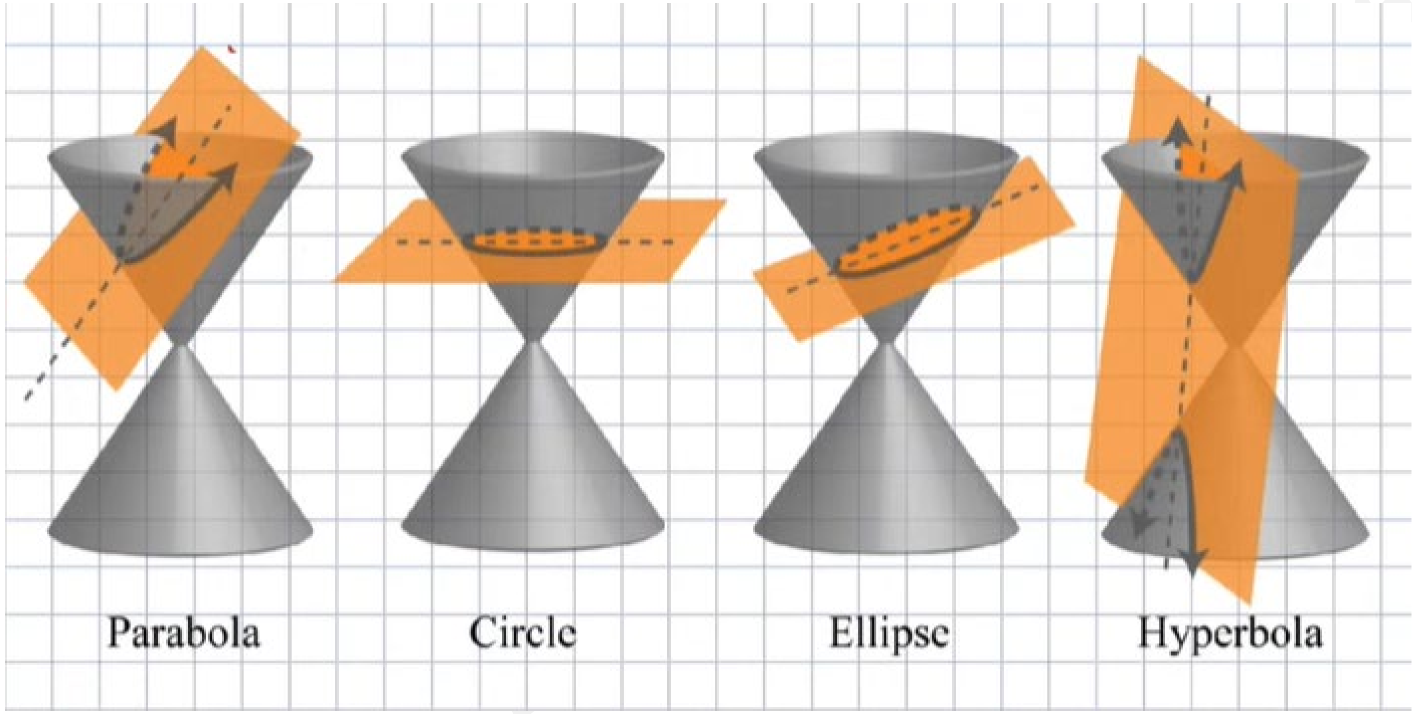
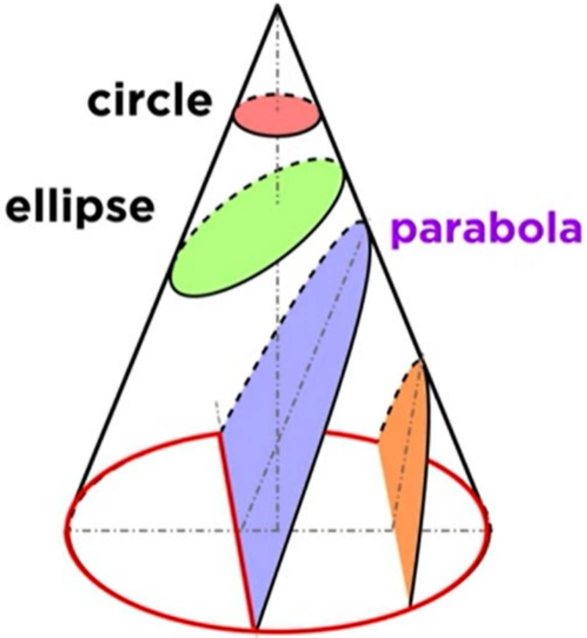
HW???

1. Draw the line $y = -2x + 1$ for $x \in [0, 1]$.
2. Where is the zero of the line $y = x - 1$?
3. Where does the line $2y + x - 1 = 0$ cross the y axis? What is the slope of the line?
4. Draw $3y - x + 3 = 0$ for $x \in [0, 4]$.

Imagine the contraction of Parabola, hyperbola, circle and Ellipse





2.4 QUADRATICS

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A quadratic (parabola) has the general form $y = ax^2 + bx + c$ and can have either no real zeros, one real zero or two real zeros.

If the quadratic has two real zeros, c_1, c_2 then it can also be written as

$$y = a(x - c_1)(x - c_2).$$

n is called the **degree** of the polynomial. Polynomials of degree 2, usually written as $p(x) = ax^2 + bx + c$, are called **quadratic functions.**

What is Parabola Graph?

A parabola is a U-shaped curve that is drawn for a quadratic function, f(x) = ax2 + bx + c. The graph of the parabola is downward (or opens down), when the value of a is less than 0, a < 0. The graph of parabola is upward (or opens up) when the value of a is more than 0, a > 0. Hence, the direction of parabola is determined by sign of coefficient 'a'

Vertex

The vertex of parabola will represent the maximum and minimum point of parabola

Axis of Symmetry

The axis of symmetry of parabola always passes through its vertex and is parallel to y-axis

y-intercept

The point at which the parabola graph passes through the y-axis is called y-intercept. The parabola of quadratic function passes through an only a single point at the y-axis,

x-intercepts

The points at which the parabola graph passes through the x-axis, are called x-intercepts, which expresses the roots of quadratic function.

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The standard form of parabola equation is expressed as follows:

$$f(x) = y = ax^2 + bx + c$$

The orientation of the parabola graph is determined using the "a" value.

- If the value of a is greater than 0 ($a>0$), then the parabola graph is oriented towards the upward direction.
- If the value of a is less than 0 ($a<0$), then the parabola graph opens downwards.

The axis of symmetry from the standard form of the parabola equation is given as $x = -b/2a$.

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Solved Examples

Graphing Parabola

Example 1:

Draw a graph for the equation **$y = 2x^2 + x + 1$.**

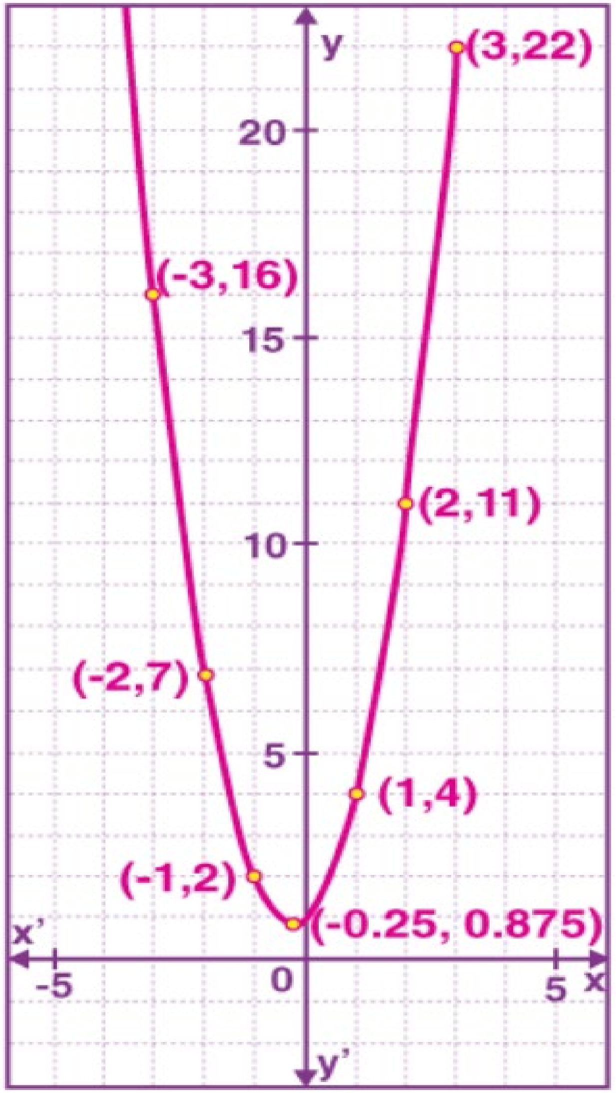
Solution:

The given equation is $y = 2x^2 + x + 1$.
Here, $a = 2$, $b = 1$ and $c = 1$.
It needs to find the vertex now
$x = -b/(2a)$
$x = -1/(2(2))$
$x = -0.25$

Now putting $x = -0.25$ in the equa
$y = 2x^2 + x + 1$
$y = 2(-0.25)^2 + (-0.25) + 1$.
$y = 2(0.0625) - 0.25 + 1$
$y = 0.125 - 0.25 + 1$
$y = 0.875$

Then we obtain

x	1	2	3	-1	-2	-3
y	4	11	22	2	7	16



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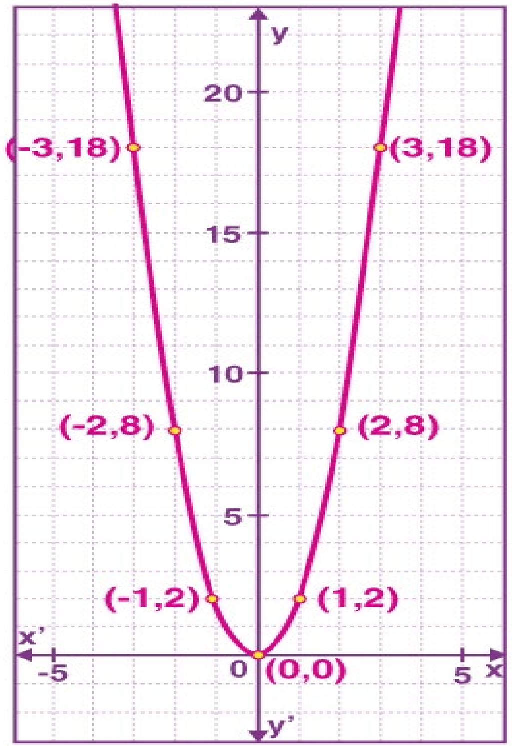
Example :

Draw a graph for the equation $y = 2x^2$.

Solution:

The given equation is $y = 2x^2$.
Here a = 2, b = 0 and c = 0.
It needs to find the vertex now
$x = -b/(2a)$
$x = 0$
Now putting x = 0 in the equation $y = 2x^2$.
$y = 2x^2$
$y = 2(0)^2$
$y = 0$
Now putting in different values for x and calculate the corresponding values for y.

- When $x = 1 \Rightarrow y = 2x^2 \Rightarrow y = 2(1)^2 \Rightarrow y = 2$
- When $x = 2 \Rightarrow y = 2x^2 \Rightarrow y = 2(2)^2 \Rightarrow y = 8$
- When $x = 3 \Rightarrow y = 2x^2 \Rightarrow y = 2(3)^2 \Rightarrow y = 18$
- When $x = -1 \Rightarrow y = 2x^2 \Rightarrow y = 2(-1)^2 \Rightarrow y = 2$
- When $x = -2 \Rightarrow y = 2x^2 \Rightarrow y = 2(-2)^2 \Rightarrow y = 8$
- When $x = -3 \Rightarrow y = 2x^2 \Rightarrow y = 2(-3)^2 \Rightarrow y = 18$



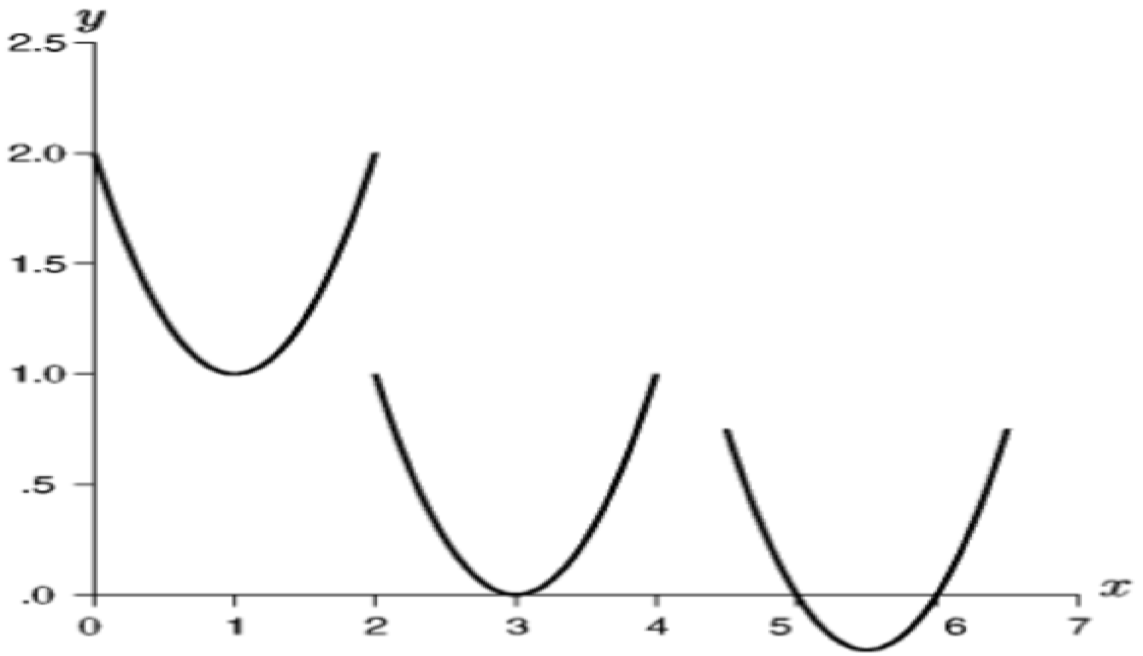
EXAMPLES

Sections of the three quadratic functions

$y = (x-1)^2 + 1, \quad y = (x-3)^2,$

$y = (x-5)(x-6)$

are drawn in the following diagram:



Quadratics

5. Draw the quadratic $y = x^2 - 2x + 1$ for $x \in [0, 2]$.

6. Where are the zeros of the curve $y = (x-3)(x-4)$?

HW??

2.5 POLYNOMIALS

A polynomial has the general form

$$y = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

where $a_i, i = 0 \ldots n$, are real numbers, and has the following properties.

1. The polynomial has **degree** n if its highest power is x^n.
2. A polynomial of degree n has n zeros (some of which may be complex).
3. The **constant** term in the above polynomial is a_0.
4. The **leading order** term in the above polynomial is $a_n x^n$ since this is the term that dominates as $x \to \infty$.

Types of Polynomial Functions

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There are various types of polynomial functions based on the degree of the polynomial. The most common types are:

- **Constant Polynomial Function: $P(x) = a = ax^0$**
- **Zero Polynomial Function: $P(x) = 0$; where all a_i's are zero, $i = 0, 1, 2, 3, \ldots, n$.**
- **Linear Polynomial Function: $P(x) = ax + b$**
- **Quadratic Polynomial Function: $P(x) = ax^2+bx+c$**
- **Cubic Polynomial Function: ax^3+bx^2+cx+d**
- **Quartic Polynomial** Function: $ax^4+bx^3+cx^2+dx+e$

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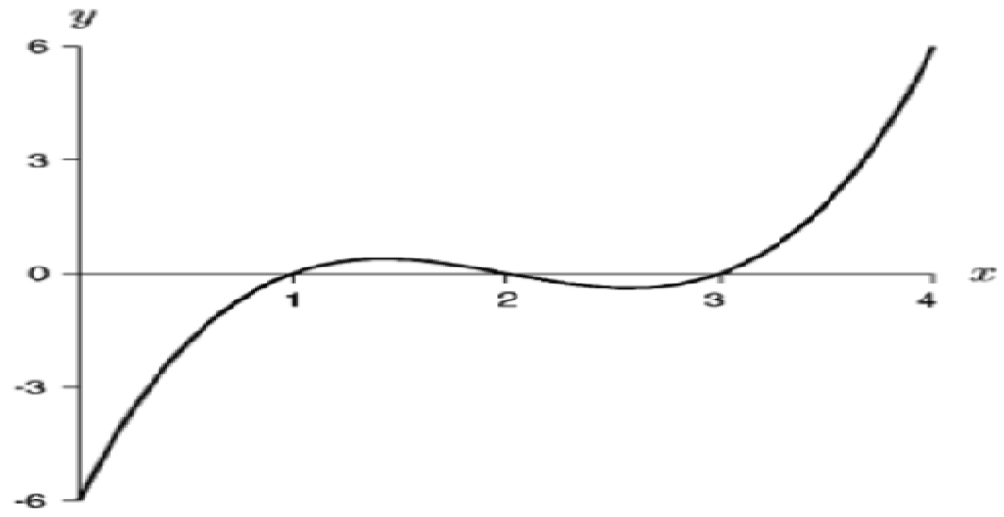
EXAMPLES

1. $y = 2x^3 + 4x^2 + 1$ has degree 3, constant term 1 and leading order term $2x^3$.
2. $y = x^2 + 5x + 6$ has two zeros $x = -3$ and $x = -2$.
3. The third degree polynomial $y = (x-1)(x-2)(x-3)$
   $= x^3 - 6x^2 + 11x - 6$ is plotted below for $x \in [0, 4]$:



2.6 HYPERBOLA

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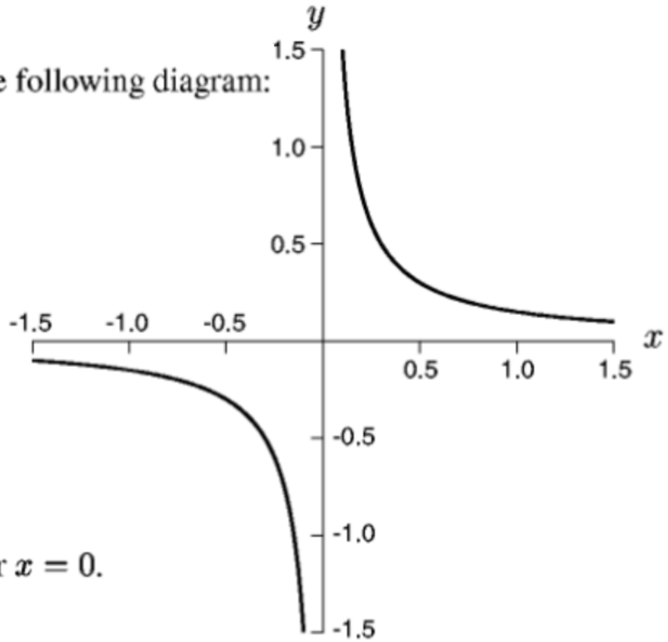
A hyperbola centred on the origin is usually written in the form $y = \frac{k}{x}$

although other orientations of hyperbolas can be written as

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad \text{or} \quad \frac{y^2}{a^2} - \frac{x^2}{b^2} = 1.$$

EXAMPLES

The hyperbola $y = \frac{0.15}{x}$ is drawn in the following diagram:



The hyperbola above is not defined for $x = 0$.

The general form of hyperbola

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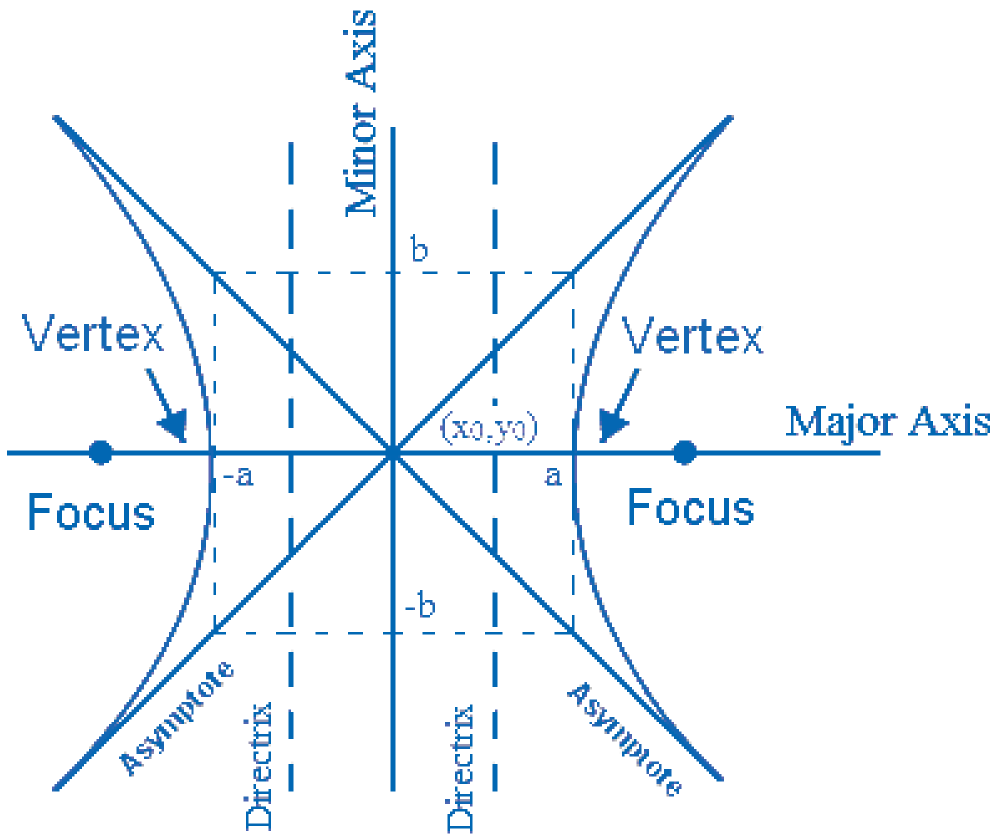

The equation for hyperbola is,

$$\frac{(x-x_0)^2}{a^2} - \frac{(y-y_0)^2}{b^2} = 1$$

Where, x_0, y_0 are the center points.

Question: The equation of the hyperbola is given as:

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$$\frac{(x-4)^2}{9^2} - \frac{(y-2)^2}{7^2}$$

Find the following: Vertex, Asymptote, Major Axis, Minor Axis and Directrix?

Solution:

Given,

$x_0 = 4$
$y_0 = 2$
$a = 9$
$b = 7$

⇨ The vertex point: (a, y_0) and $(-a, y_0)$ are $(9, 2)$ and $(-9, 2)$ and

Major Axis

a = 9

Minor Axis

b = 7

Asymptote

$$y = \tfrac{7}{9}(x-4)+2$$
$$y = -\tfrac{7}{9}(x-4)+2$$

Directrix

$$x = \frac{\pm 9^2}{\sqrt{9^2+7^2}} = \pm\frac{81}{\sqrt{81+49}} = 7.1$$

Asymptotes.

$$y = y_0 + \frac{b}{a}x - \frac{b}{a}x_0$$
$$y = y_0 - \frac{b}{a}x + \frac{b}{a}x_0$$

Directrix of a hyperbola $x = \dfrac{\pm a^2}{\sqrt{a^2+b^2}}$

(a, y_0) and $(-a, y_0)$ **VERTEX**

Focus (foci)

$$\left(x_0 + \sqrt{a^2+b^2}, y_0\right)$$
$$\left(x_0 - \sqrt{a^2+b^2}, y_0\right)$$

2.7 EXPONENTIAL AND LOGARITHM FUNCTIONS

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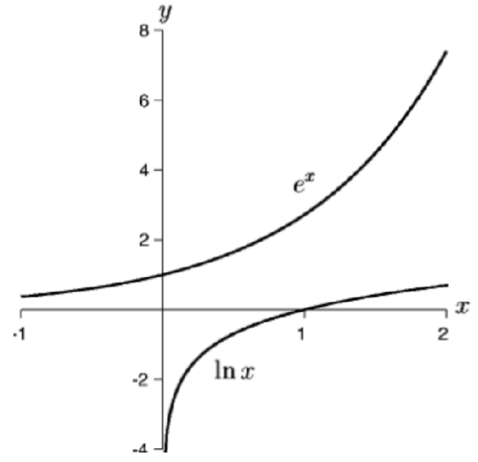
Exponential Functions Functions of the form $f(x) = a^x$, where the base $a > 0$ is a positive constant and $a \neq 1$, are called **exponential functions**.

The exponential function is $y = e^x \equiv \exp x$

with its inverse the logarithm function $y = \ln x$.

The general properties of the exponential are listed in the next chapter on transcendental functions.

The exponential function $y = e^x$ (upper curve) and logarithm function $y = \ln x$ (lower curve) are drawn in the following diagram:



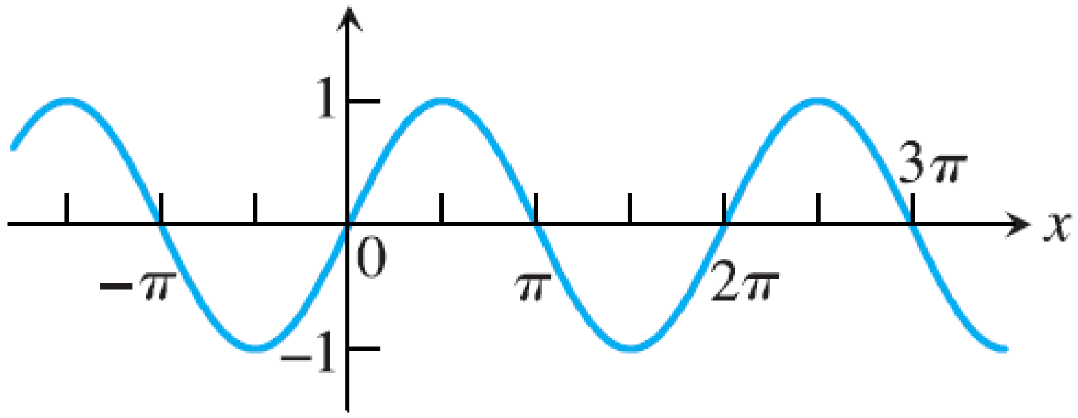
Logarithmic Functions These are the functions $f(x) = \log_a x$, where the base $a \neq 1$ is a positive constant.

Note:

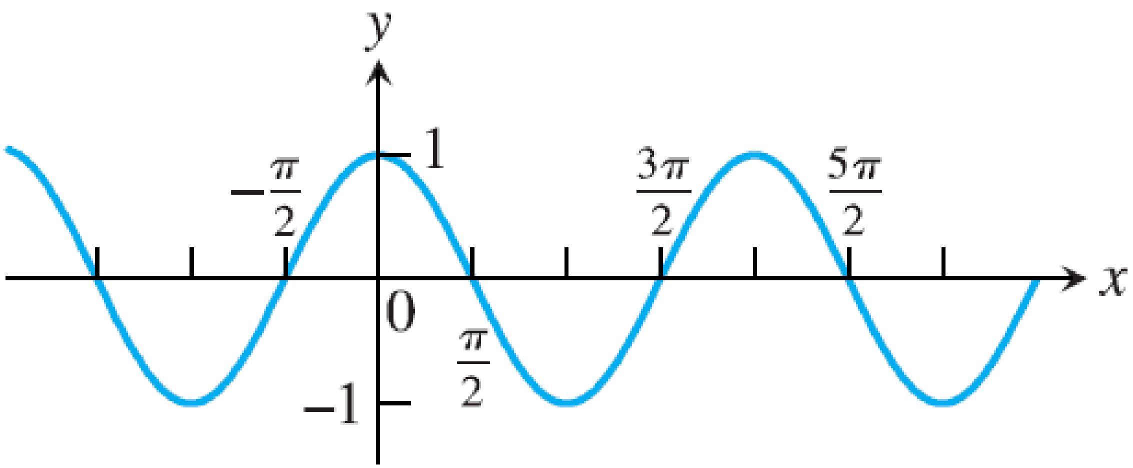
The logarithm function is not defined for $x \leq 0$.

2.8 TRIGONOMETRIC FUNCTIONS

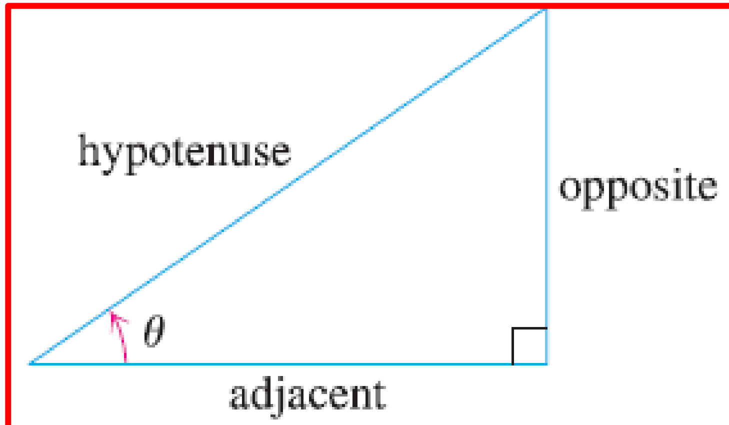
The graphs of the sine and cosine functions are shown in Figure



(a) $f(x) = \sin x$



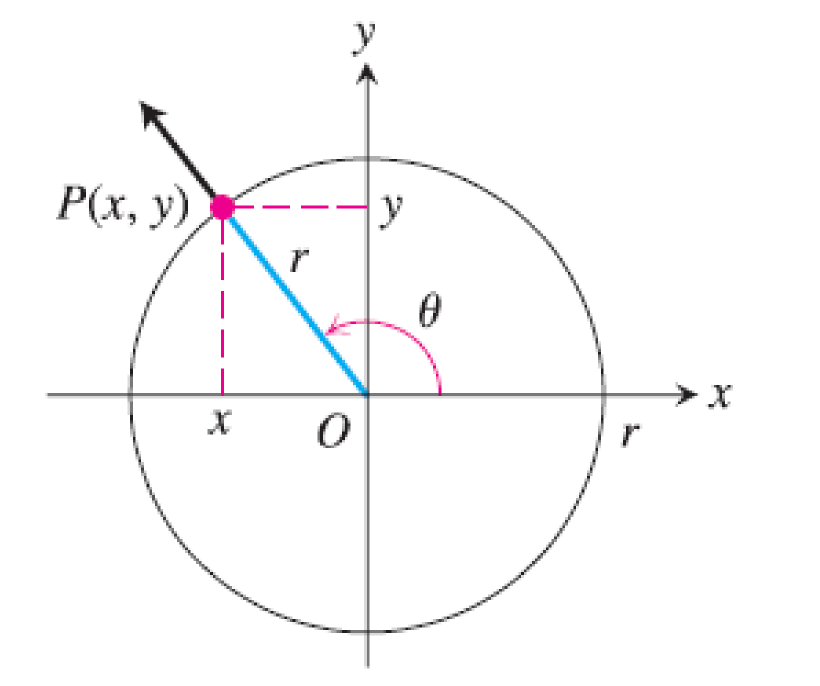
(b) $f(x) = \cos x$



$$\sin\theta = \frac{\text{opp}}{\text{hyp}} \qquad \csc\theta = \frac{\text{hyp}}{\text{opp}}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} \qquad \sec\theta = \frac{\text{hyp}}{\text{adj}}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} \qquad \cot\theta = \frac{\text{adj}}{\text{opp}}$$



sine: $\sin\theta = \frac{y}{r}$ **cosecant:** $\csc\theta = \frac{r}{y}$

cosine: $\cos\theta = \frac{x}{r}$ **secant:** $\sec\theta = \frac{r}{x}$

tangent: $\tan\theta = \frac{y}{x}$ **cotangent:** $\cot\theta = \frac{x}{y}$

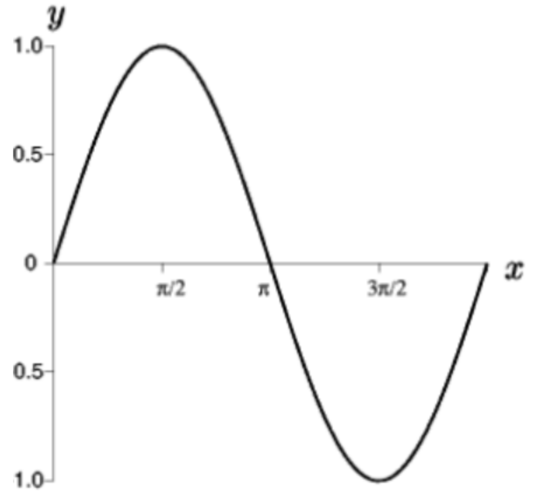
$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

$$\sec\theta = \frac{1}{\cos\theta} \qquad \csc\theta = \frac{1}{\sin\theta}$$

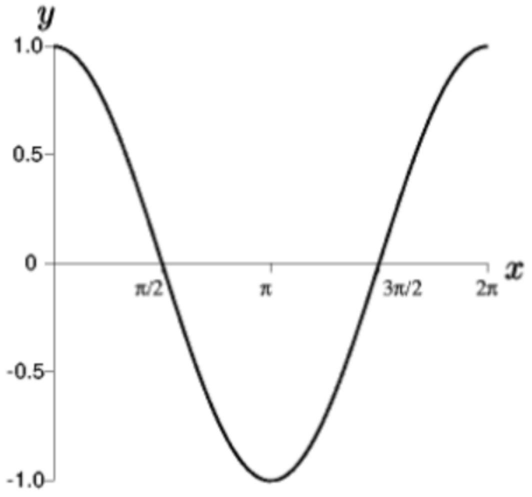
Examples

1. The function $y = \sin 2x$ will have a period of π.

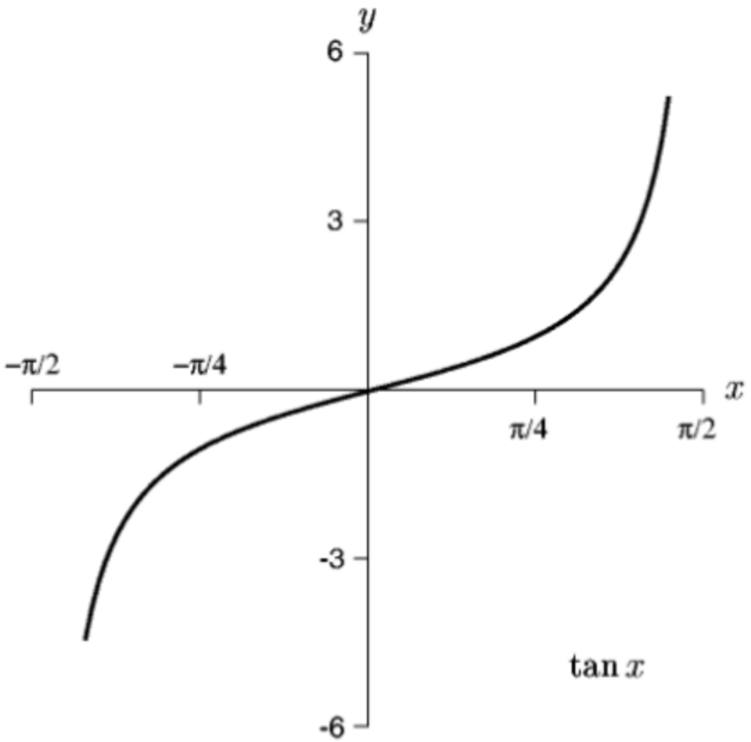
2. The functions $\sin x$ and $\cos x$ are plotted below for the first period $x \in [0, 2\pi]$, while $\tan x = \sin x / \cos x$ is plotted for $x \in [-\pi/2, \pi/2]$.



$\sin x$



$\cos x$



$\tan x$

2.9 CIRCLES

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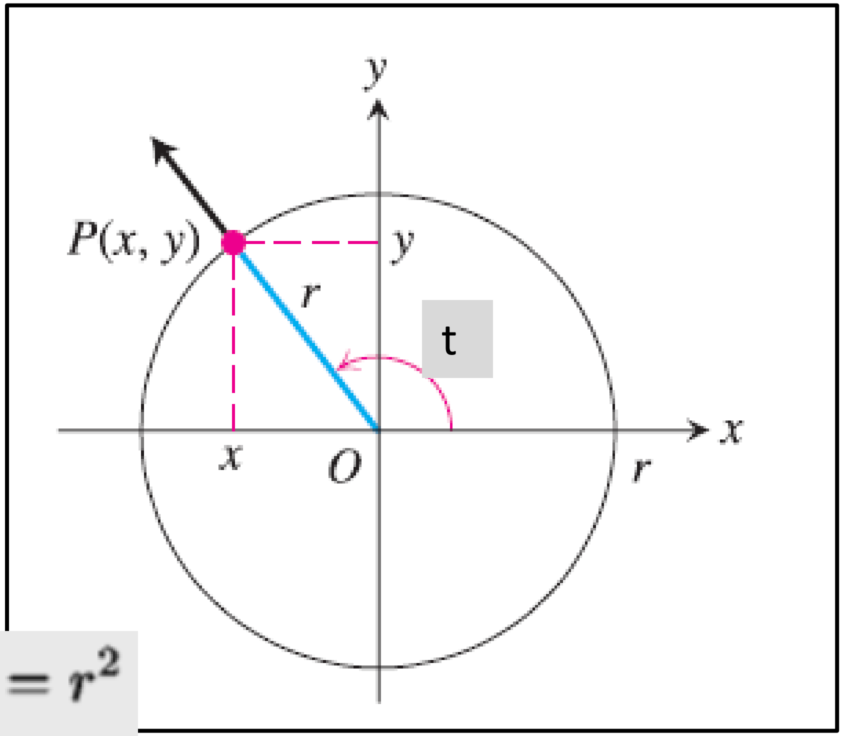
The standard **equation of a circle** is given by:

$(x-h)^2 + (y-k)^2 = r^2$

Where (h,k) is the coordinates of center of the circle and r is the radius

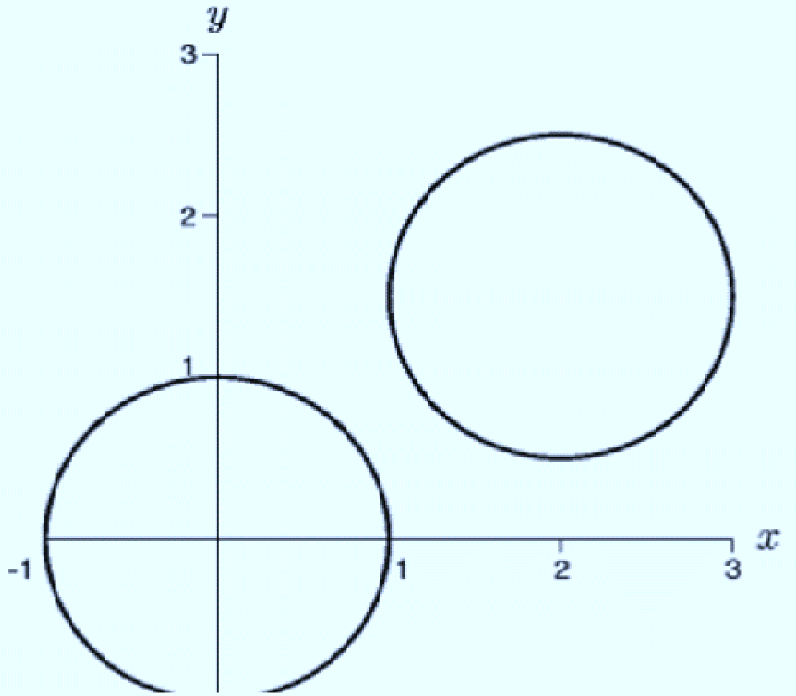


A **circle** centred on the origin has the general equation $x^2 + y^2 = r^2$ where r is the radius. This is often written in parametric form

$$x(t) = r\cos t, \quad y(t) = r\sin t, \quad t \in [0, 2\pi].$$

EXAMPLES

1. The circles $x^2 + y^2 = 1$ and $(x - 2)^2 + (y - 1.5)^2 = 1$ are drawn in the following diagram:



note

General form of Equation of a Circle

The general equation of any type of circle is represented by:

$x^2 + y^2 + 2gx + 2fy + c = 0$, for all values of g, f and c.

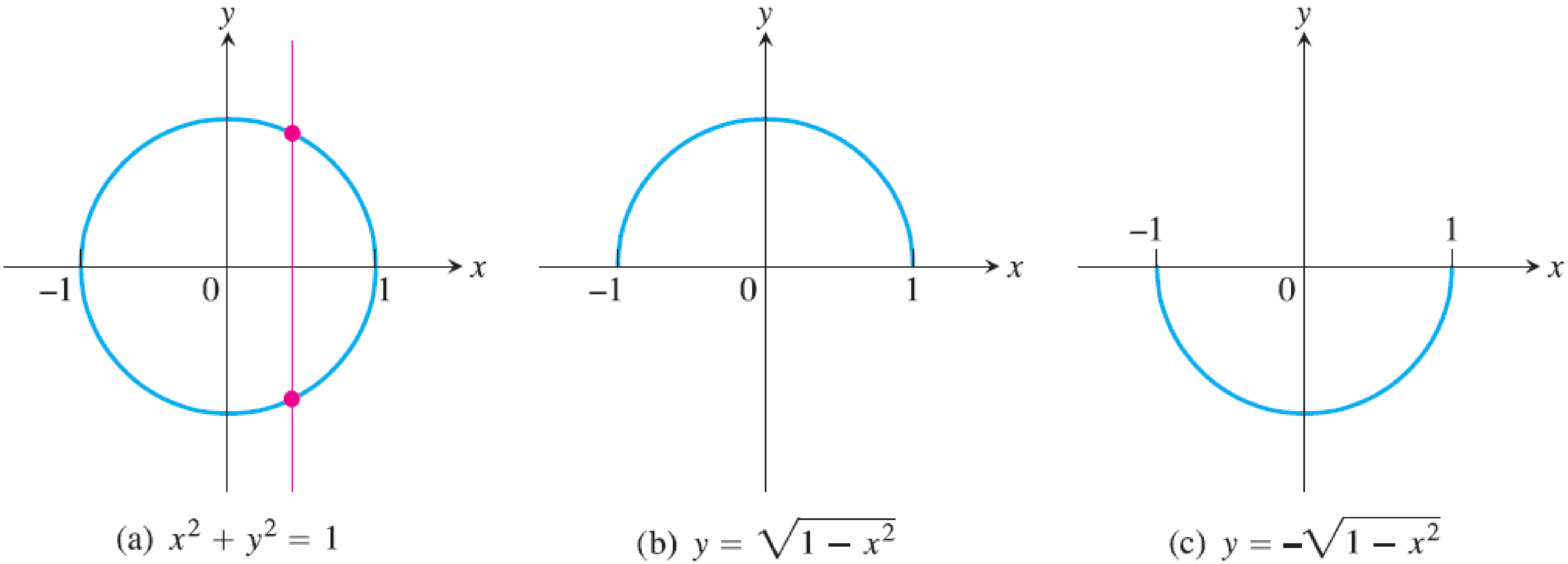


FIGURE (a) The circle is not the graph of a function; it fails the vertical line test. (b) The upper semicircle is the graph of a function $f(x) = \sqrt{1 - x^2}$. (c) The lower semicircle is the graph of a function $g(x) = -\sqrt{1 - x^2}$.

EXAMPLES

2. The curve $x^2 + 2x + y^2 + 4y = -4$ can be written as $(x+1)^2 + (y+2)^2 = 1$, which is a circle centred on $(-1, -2)$ with radius 1.

3. The curve represented by $x(t) = 2\cos t + 1$, $y(t) = 2\sin t - 3$, $t \in [0, 2\pi)$ is the circle radius 2 centred on $(1, -3)$.

Example 2:
Find the equation of the circle whose center is (3,5) and the radius is 4 units.
Solution:
Here, the center of the circle is not an origin. Therefore, the general equation of the circle is,
$(x-3)^2 + (y-5)^2 = 4^2$
$x^2 - 6x + 9 + y^2 - 10y + 25 = 16$
$x^2 + y^2 - 6x - 10y + 18 = 0$

Example 3:
Equation of a circle is $x^2+y^2-12x-16y+19=0$.
Find the center and radius of the circle.
????????

2.10 ELLIPSES

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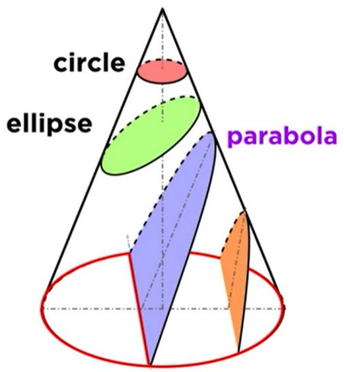
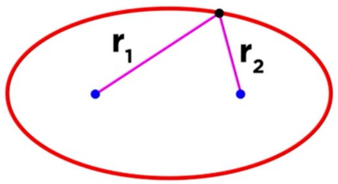
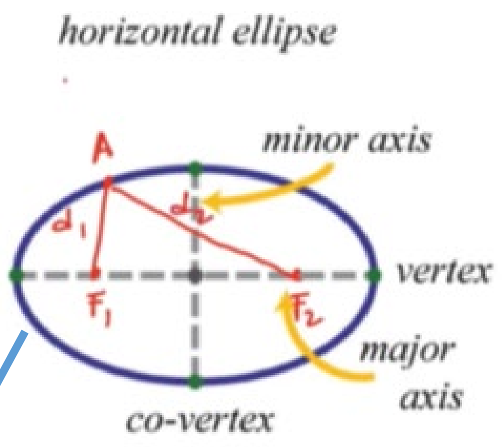
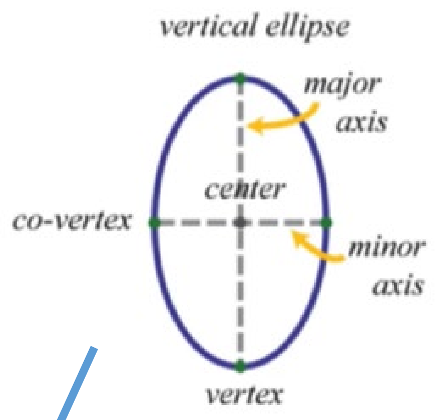


Defining the Ellipse



the set of all points surrounding two foci such that $r_1 + r_2 =$ constant





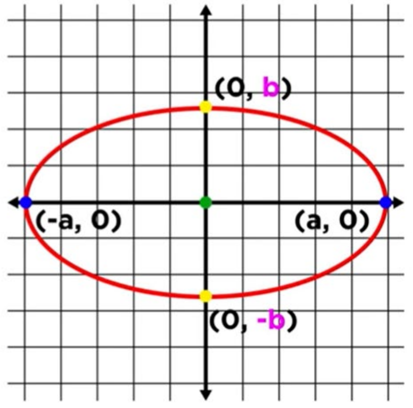
Graphing Ellipses in Standard Form

standard form of an ellipse:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

length of major axis = 2a
length of minor axis = 2b
(a = distance from center to vertex)

vertices are at:
(-a, 0) and (a, 0)



Graphing Ellipses in Standard Form

History of Astronomy Part 4: Kepler's Laws and Beyond

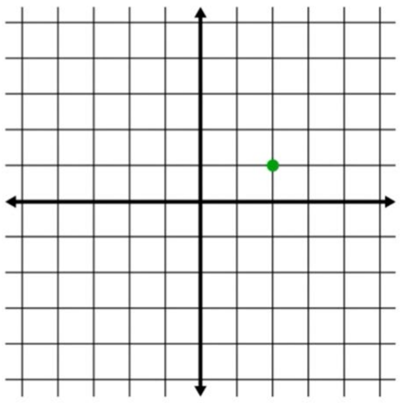
standard form of an ellipse:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{(x-2)^2}{4} + \frac{(y-1)^2}{3} = 1$$

h = horizontal shift
k = vertical shift

center: (2, 1)

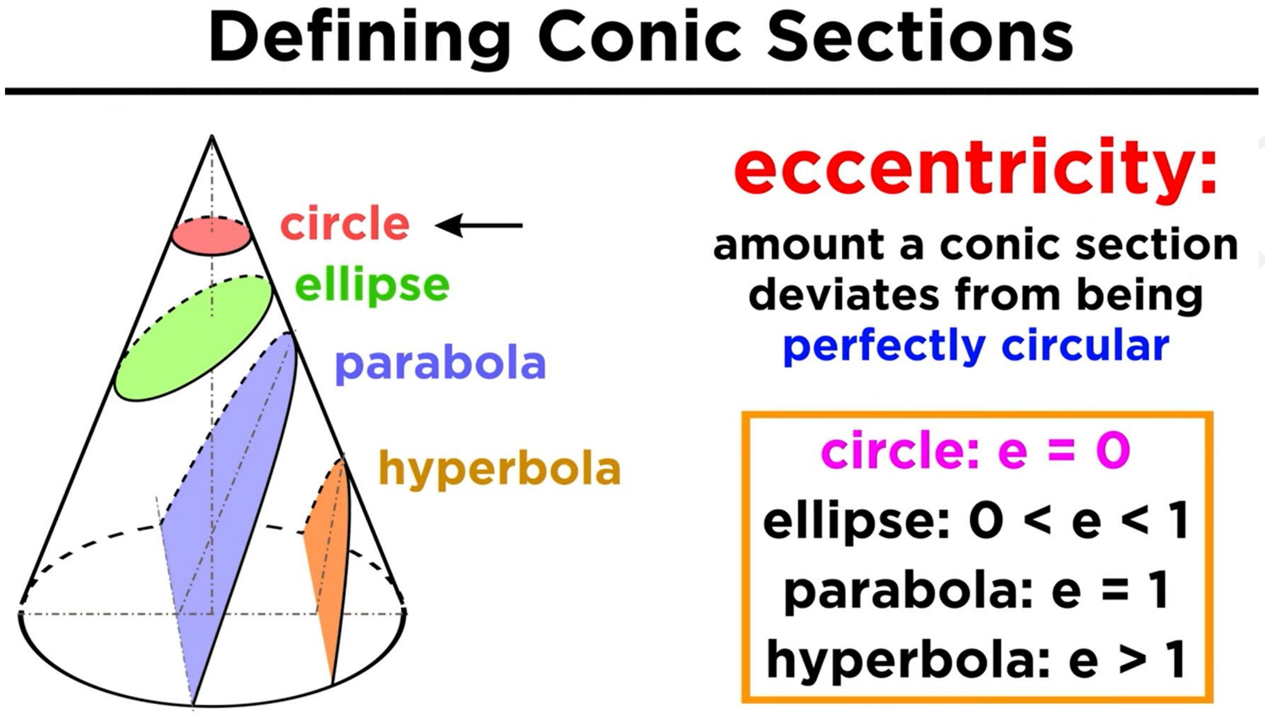


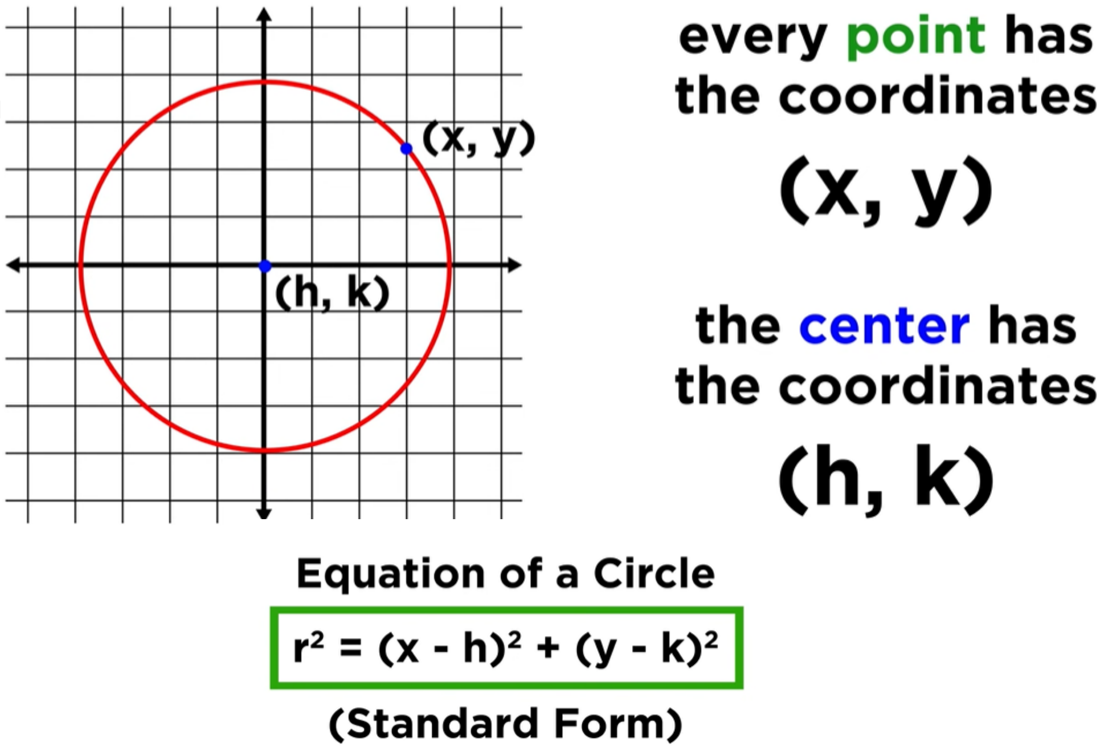
Ellipse

An ellipse is the set of points in a plane such that the sum of the distances from two fixed points in that plane stays constant. The two points are each called a focus.

Center: (0, 0)	Ellipse with foci on the x - *axis*	Ellipse with foci on the y -*axis*
Equation	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ where $c=\sqrt{a^2-b^2}$ and $a>b$	$\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$ where $c=\sqrt{a^2-b^2}$ and $a>b$
Vertices	(±a, 0)	(0, ±a)
Foci	(±c, 0)	(0, ±c)
Major Axis	Equation: y = 0 Location: On the x - axis Length: 2*a* Endpoints: (±a, 0)	Equation: x = 0 Location: On the y - axis Length: 2*a* Endpoints: (0, ±a)
Minor Axis	Equation: x = 0 Location: On the y - axis Length: 2*b* Endpoints: (0, ±b)	Equation: y = 0 Location: On the x - axis Length: 2*b* Endpoints: (±b, 0)
x - intercepts	±a	±b
y - intercepts	±b	±a
Directrices	$x=\pm\frac{a^2}{c}$	$y=\pm\frac{a^2}{c}$
Latus Rectum	Equation: $x=\pm c$ Direction: vertical Length: $\frac{2b^2}{a}$ Endpoints: $\left(-c,\ \pm\frac{b^2}{a}\right)$ & $\left(c,\ \pm\frac{b^2}{a}\right)$	Equation: $y=\pm c$ Direction: horizontal Length: $\frac{2b^2}{a}$ Endpoints: $\left(\pm\frac{b^2}{a},-c\right)$ & $\left(\pm\frac{b^2}{a},c\right)$
Permissible Values	$-a\le x\le a$ $-b\le y\le b$	$-b\le x\le b$ $-a\le y\le a$

How one construct different shapes from con





Parabola

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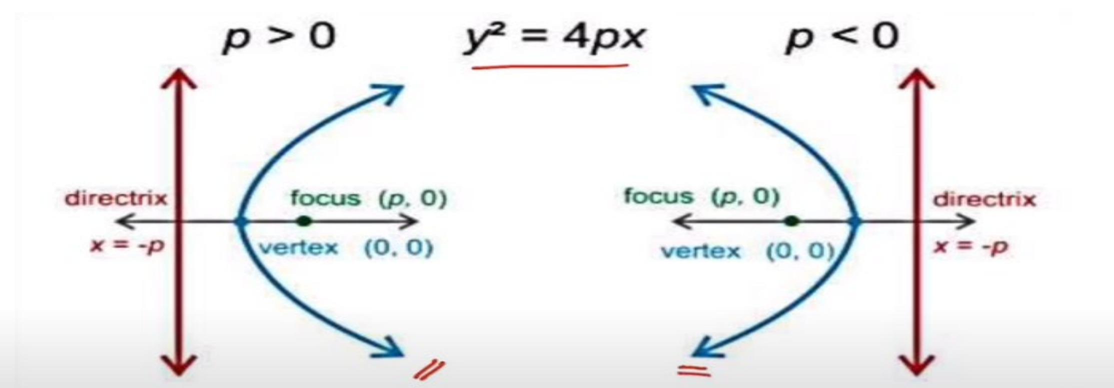
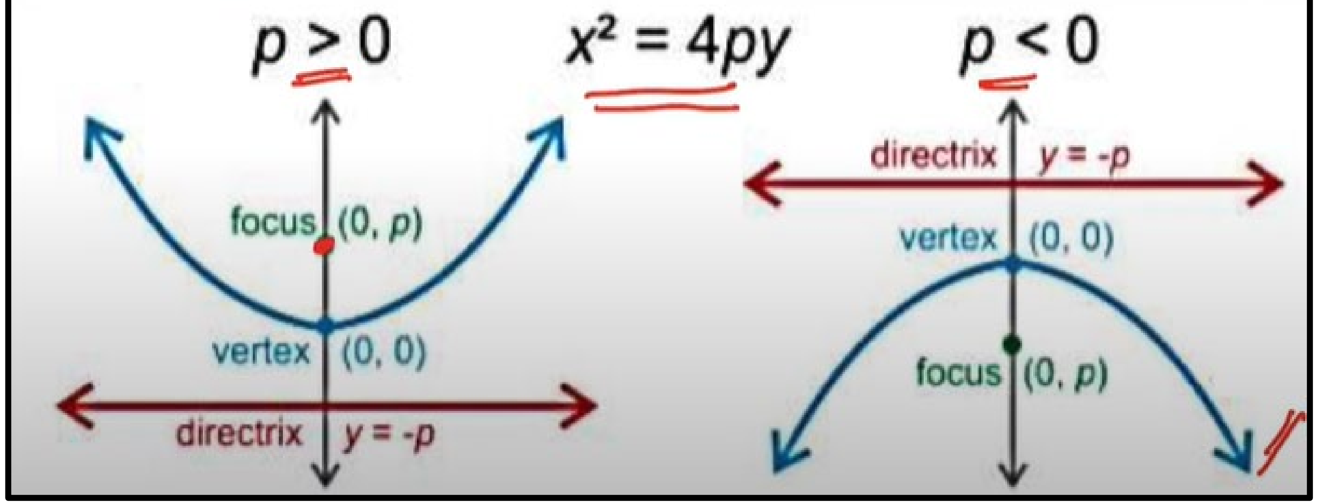
Equation of Parabola in standard form: Vertex at the origin (O, O)





The characteristics of parabolas with vertex at (0,0) are summarized below:

Vertex (0,0)	Parabola with horizontal axis of symmetry	Parabola with vertical axis of symmetry
Equation	$y^2 = 4px$	$x^2 = 4py$
Focus	$(p, 0)$	$(0, p)$
Opening of Parabola	If **p<0**, parabola opens to the **left**. If **p>0**, parabola opens to the **right**.	If **p<0**, parabola opens **downward**. If **p>0**, parabola opens **upward**.
Latus Rectum	**Equation:** $x = \pm p$ **Direction:** vertical **Length:** 4p **Endpoints:** $(p, \pm 2p)$	**Equation:** $y = \pm p$ **Direction:** horizontal **Length:** 4p **Endpoints:** $(\pm 2p, p)$
Axis of Symmetry	**Equation:** $y = k; k = 0$ **Direction:** horizontal	**Equation:** $x = h; h = 0$ **Direction:** vertical
Directrix	**Equation:** $x = -p$ **Direction:** vertical	**Equation:** $y = -p$ **Direction:** horizontal

EXAMPLES

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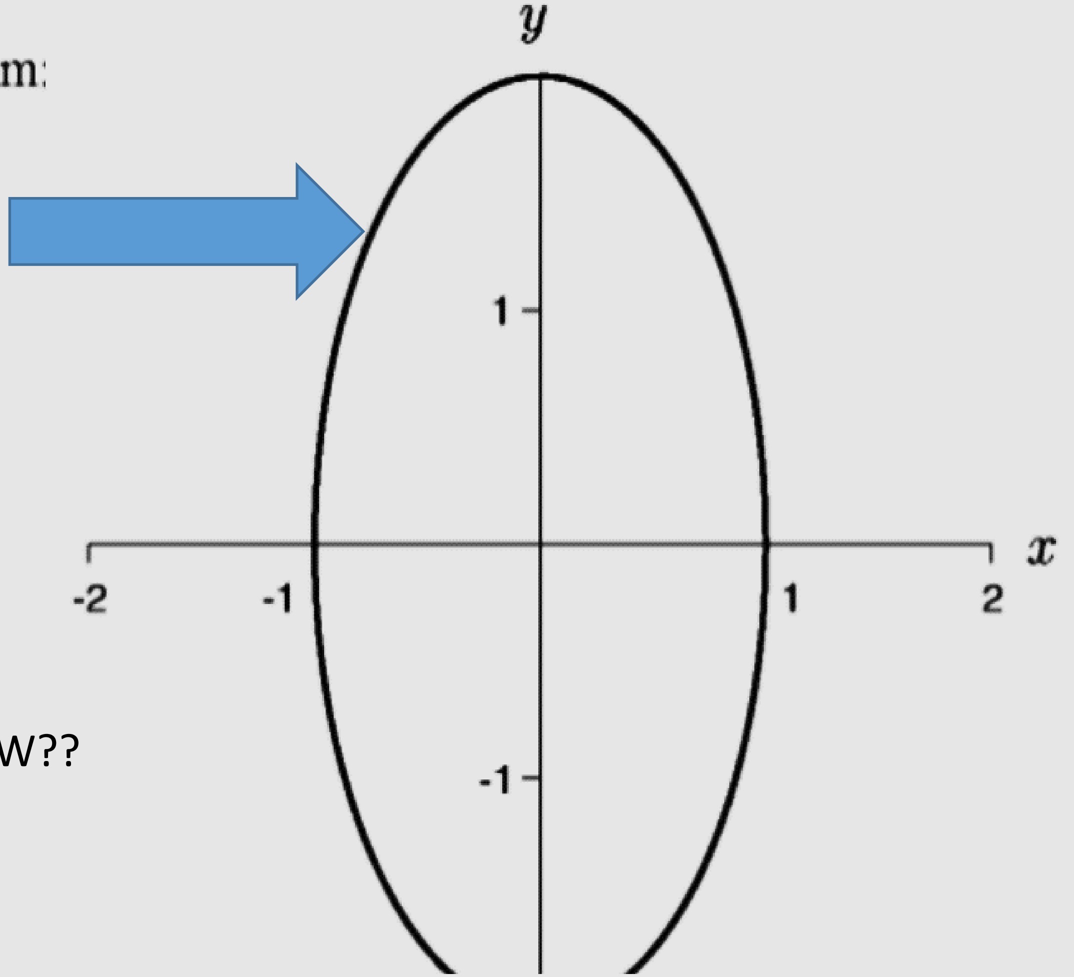
1. The ellipse $\left(\frac{y}{2}\right)^2 + x^2 = 1$ is drawn in the following diagram:



2. The curve $(x-2)^2 + 16y^2 = 1$ is an ellipse centred on $(2, 0)$ with major axis of length 2 in the x direction and minor axis of length $\frac{1}{2}$.

HW??

EXAMPLE QUESTIONS

HW???

General

30. What type of curve has equation $y^2 + (x-1)^2 - 2 = 0$?

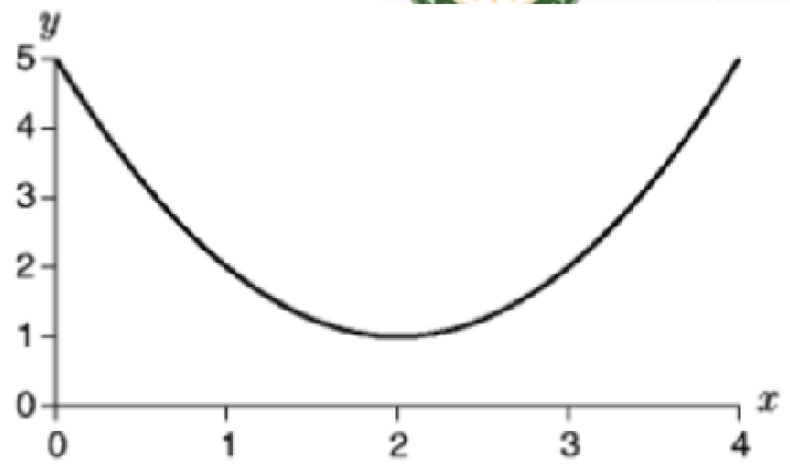
31. What type of curve has equation $2y^2 + (x-1)^2 - 2 = 0$?

32. What type of curve has equation $2y + (x-1)^2 - 2 = 0$?

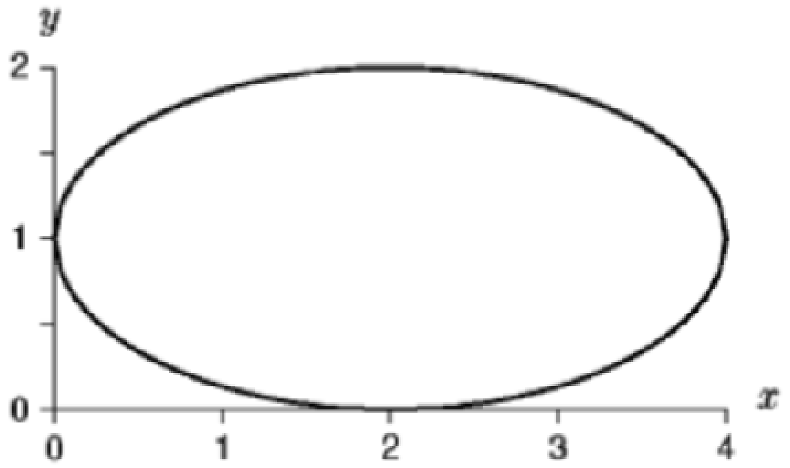
33. What type of curve has equation $2y + (x-1) = 0$?

34. What type of curve has equation $\dfrac{2}{y-1} + (x-1) = 0$?

35. What is the equation of the quadratic below:



36. What is the equation of the shape below:



2.11 EXAMPLE QUESTIONS

1. If $f(x) = x^3 + 1$ what is $f(2)$?
2. If $f(x) = x^3 + 1$ what is $f(g)$?
3. If $f(x) = x^3 + 1$ and $g(x) = (x - 1)$ what is $f(g(x))$?
4. If $f(x) = x^3 + 1$ and $g(x) = (x - 1)$ what is $f(g(b))$?
5. If $f(x) = x^2$ and $g(z) = \sin z$ find $f(g(a))$ and $g(f(x))$.
6. If $f(x) = x^2 + 1$ find $f(f(x))$.
7. If $f(x) = (x - 1)^2$ and $g(x) = x^2 - 1$ find $f(g(x))$ and $g(f(x))$.
8. If $f(x) = \frac{1}{x} + 1$ find the inverse $f^{-1}(x)$.
9. If $f(x) = \frac{1}{x + 1}$ find the inverse $f^{-1}(x)$.
10. If $f(x) = \frac{1}{x^2} + 1$ find the inverse $f^{-1}(x)$.

Answes

1. 9 2. $g^3 + 1$ 3. $(x - 1)^3 + 1$ 4. $(b - 1)^3 + 1$ 5. $f(g(a)) = \sin^2(a)$, $g(f(x)) = \sin x^2$
6. $f(f(x)) = (x^2 + 1)^2 + 1$ 7. $f(g(x)) = (x^2 - 2)^2$, $g(f(x)) = (x - 1)^4$. 8. $f^{-1}(x) = \frac{1}{x - 1}$
9. $f^{-1}(x) = \frac{1}{x} - 1$ 10. $f^{-1}(x) = \frac{1}{\sqrt{x - 1}}$

EXAMPLE QUESTIONS

HW???

Sines and cosines

17. Draw the curve $y = 2\sin 3x$ from $x = 0$ to $x = \pi$.
18. Draw the curve $y = \cos\frac{x}{2}$ from $x = 0$ to $x = 4\pi$.
19. Draw the curve $y = \cos 2x + 1$ from $x = 0$ to $x = 2\pi$.
20. What is the period of $y = \sin(x + 1)$?
21. What is the period of $y = \cos 3x$?
22. What is the period of $y = \sin(3x + 1)$?

EXAMPLE QUESTIONS

HW???

Circles and ellipses

23. Draw the circle $y^2 + (x-2)^2 = 4$.
24. Draw the ellipse $y^2 + 2x^2 = 1$.
25. Draw the ellipse $4y^2 + (x-1)^2 = 1$.
26. Where does the ellipse $(x-1)^2 + 2y^2 = 1$ cut the x axis?
27. What is the equation for an ellipse centred on (0,0) with x axis twice as long as the y axis?
28. What is the equation for a circle centred on $(1,2)$ with radius 2?
29. What is the equation for a circle centred on $(a,2)$ with radius 3?

EXAMPLES

2. The curve $x^2 + 2x + y^2 + 4y = -4$ can be written as $(x+1)^2 + (y+2)^2 = 1$, which is a circle centred on $(-1, -2)$ with radius 1.

3. The curve represented by $x(t) = 2\cos t + 1$, $y(t) = 2\sin t - 3$, $t \in [0, 2\pi)$ is the circle radius 2 centred on $(1, -3)$.

Prof. dr. Salah Raza Saeed

THANKS FOR YOUR ATTENTION