Cihan University-Sulaymaniyah


Financial Mathematical

Department of:
Accounting
Stage-ONE
Prof.Dr. Obaid Mahmmood Mohsin

## Academic Year: 2023/2024



# [DOCUMENT TITLE] 

[Document subtitle]

## 2023-2024

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## MODULE DESCRIPTION FORM



## Relation with other Modules

| Prerequisite module |  | Semester |  |
| :--- | :--- | :--- | :--- |
| Co-requisites module |  | Semester |  |

Cihan University Sulaimaniya?
College of Administration \& Finance
Department: ...Accounting.......
Discipline:
Stage: $1^{\text {st }}$ stage

| Total Contact Hours: | 25 |
| :--- | :--- |
| Total Self Study Hours: | $\mathbf{5 6}$ |
| Total No. Hours: | $\mathbf{8 1}$ |
| ECTS: | $\mathbf{3}$ |


| No. of Weeks | Contact Hours |  |  |  |  | Self-Study |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Theoretical | Practical | Lab | Project | Visit | Quiz | Reading | Assignment | Report | Midterm Exam. | Final Exam. |
| $1^{\text {st }}$ Week (Registration) | - | - | - | - | - | - | - | - | - | - | - |
| $2^{\text {nd }}$ Week | 1 |  |  |  |  |  | 1 |  |  |  |  |
| $3{ }^{\text {rd }}$ Week | 1 | 1 |  |  |  |  | 1 |  |  |  |  |
| $4^{\text {th }}$ Week | 1 | 1 |  |  |  | 2 | 1 |  |  |  |  |
| $5^{\text {th }}$ Week | 1 | 1 |  |  |  |  |  |  |  |  |  |
| $6^{\text {th }}$ Week | 1 | 1 |  |  |  |  | 1 | 2 | 2 |  |  |
| $7{ }^{\text {th }}$ Week | 1 | 1 |  |  |  | 2 |  |  |  |  |  |
| $8{ }^{\text {th }}$ Week | 0 | 2 |  |  |  |  | 1 |  |  | 12 | 25 |
| 9 ${ }^{\text {th }}$ Week | 1 | 1 |  |  |  |  |  |  |  |  |  |
| $10^{\text {th }}$ Week | 0 | 2 |  |  |  |  | 1 |  |  |  |  |
| $11^{\text {th }}$ Week | 1 | 1 |  |  |  |  |  |  |  |  |  |
| $12^{\text {th }}$ Week | 0 | 2 |  |  |  |  |  | 2 |  |  |  |
| $13^{\text {th }}$ Week | 1 | 1 |  |  |  | 2 |  |  |  |  |  |
| $14^{\text {th }}$ Week | 1 | 1 |  |  |  |  | 1 |  |  |  |  |
| 15th Week (Final Exam. |  |  |  |  |  |  |  |  |  |  |  |
| 16th Week (Final Exam.) |  |  |  |  |  |  |  |  |  |  |  |
| TOTAL | 10 | 15 |  |  |  | 6 | 7 | 4 | 2 | 12 | 25 |


| Delivery Plan (Weekly Syllabus) |  |
| :---: | :---: |
|  | Material Covered |
| Week 1 |  |
| Week 2 | The types of Interest How to find Simple interest Examples |
| Week 3 | The Amount in Simple interest: Calculate the period: Examples |
| Week 4 | Types of Simple Interest: -- Standard interest-- Commercial interest—and- Exact interest The Relationship between Commercial Interest \& Exact Interest: Examples |
| Week 5 | The difference between Commercial Interest \& Exact Interest: The Numbers Method: Examples |
| Week 6 | Problems \& solutions |
| Week 7 | Mid-Term Exam |
| Week 8 | Chapter Two <br> Commercial discount with simple interest. <br> The concept of trade discount. <br> Definition of trade discount. <br> Examples |
| Week 9 | Trade discount account. -- The normal Method: -- The Constant denominator method Current value of a commercial paper Examples |
| Week 10 | Rational (Correct) Discount Examples |
| Week 11 | Problems \& solutions |
| Week 12 | Chapter Three Compound Interest |


|  | Compound Interest <br> Effective Rate of Interest: <br> Examples |
| :--- | :--- |
| Week 13 | The actual interest on simple interest rate that is equivalent to compound interest rate is <br> called the effective rate. <br> Depreciation: --- The straight -line method - The Constant -Percentage Method <br> Examples |
| Week 14 | Problems \& solutions |
| Week 15 | Preparatory Week |
| Week 16 | Final Exam |

## Delivery Plan (Weekly Lab. Syllabus)

|  | Material Covered |
| :---: | :--- |
| Week 1 |  |
| Week 2 |  |
| Week 3 |  |
| Week 4 |  |
| Week 5 |  |
| Week 6 |  |
| Week 7 |  |
| Week8 |  |
| Week9 |  |
| Week 10 |  |
| Week 11 |  |
| Week 12 |  |
| Week 13 |  |
| Week 14 |  |
| Week 15 |  |
| Week 16 |  |


| Module Aims, Learning Outcomes and Indicative Contents |  |
| :--- | :--- |
| Module Objectives | Sophisticated mathematical models, whose solution often requires <br> computers, are important in finance. This course will give students the basic <br> numerical tools and practice solving financial problems on computers. <br> students will be expected to implement basic algorithms in a high-level <br> programming language. |
| Module Learning Outcomes | On completion of this course, students should be able to <br> 1-given any two of interest rate, present value, or future value, calculate <br> .the third based on simple interest |
| 2-Given any one of the effective interest rates, the nominal |  |
| interest m-they, or the force of interest, calculate all of the other items |  |
| 3-Write the equations of value given a set of each flow and interest |  |

## Learning and Teaching Strategies

Strategies
Whiteboard, whiteboard maker, data show, course note. Assessment scheme.

| Module Evaluation |  |  |  |  |  |
| :--- | :--- | :---: | :---: | :---: | :--- |
| Assessment Types |  | Time/Number | Weight (Marks) | Week Due | Relevant Learning <br> Outcome |
| Formative <br> assessment | Quizzes | $5 / 3$ | 15 | $4,7,13$ |  |
|  | Assignments | $2 / 5$ | 10 | 6,12 |  |
|  | Projects $/$ | $1 / 5$ | 5 | 6 |  |
|  | Report |  |  |  |  |
| Summative <br> assessment | Midterm Exam | 2 hr | 20 | 8 |  |
|  | Final Exam | 3 hr | 50 | 15 |  |
|  | Total assessment |  |  | $100 \%(100$ Marks) |  |  |


| Learning and Teaching Resources |  |  |
| :--- | :--- | :--- |
|  | Text | Available in the Library? |
| Required Texts | Financial math. By Dr. Clarence H. Richardson. Mastering <br> Financial mathematics by Alastair I ..day |  |
| Recommended <br> Texts | Lectures gave to students |  |
| Websites |  |  |


| Grading Scheme مخطط الدرجات |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Group | Grade | التقدير | Marks \% | Definition |
| Success Group(50-100) | A - Excellent | امتياز | 90-100 | Outstanding Performance |
|  | B - Very Good | جيد جدا | 80-89 | Above average with some errors |
|  | C-Good | جيد | 70-79 | Sound work with notable errors |
|  | D - Satisfactory | متوسط | 60-69 | Fair but with major shortcomings |
|  | E - Sufficient | مقبول | 50-59 | Work meets minimum criteria |
| Fail Group(0-49) | FX - Fail | راسب (قيد المعالجة) | (45-49) | More work required but credit awarded |
|  | F-Fail | راسب | (0-44) | Considerable amount of work required |
|  |  |  |  |  |
| Note: Marks Decimal places above or below 0.5 will be rounded to the higher or lower full mark (for example a mark of 54.5 will be rounded to 55 , whereas a mark of 54.4 will be rounded to 54 . The University has a policy NOT to condone "near-pass fails" so the only adjustment to marks awarded by the original marker(s) will be the automatic rounding outlined above. |  |  |  |  |

## Approved by Head of the Branch / Department

Signature

| Date | $6 / 3 / 2024$ |
| :--- | :--- |
| Name | Ass Prof. Dr.Munadhil Abd-Aljabar |

## Approved by Curriculum Development Committee and Bologna Process Committee

| Signature |  |
| :--- | :--- |
| Date |  |
| Name |  |

## Chapter One

## Simple Interest

## 1-Interest:

Interest is the core of the business and financial activities in our business life, in banks, insurance companies and others financial institutions.
Depending on some sources, the lending of "food money" was commonplace in Middle Eastern civilizations as early as 5000 BC . The argument that acquired seeds and animals could "reproduce themselves" was used to justify interest. All religions consider it charging of interest (Riba).
The first attempt to control interest rates through manipulation of the money supply was made by the Banque de France in 1847, continuous till this day and consider it as an " Investment ".
Before we enter into the main subject " Interest ", we must first know some simple things, and why there is interest.
Every one of us have income; daily or monthly, spending of this income depending on our expenses and consumptions (foods, drinks, clothes, transportations ... etc. ).

The income formula is.
$\mathrm{Y}=\mathrm{C}+\mathrm{S}$
where:
$\mathrm{Y}=$ income
$\mathrm{C}=$ consumption
$\mathrm{S}=$ saving
Some of us may have high-income with normal expenditures, so that will make some money for saving, day by day or month by month.
This will make a saving moneys increase in our house, more of us will be afraid on his moneys from (fire, steal), some of us thinking about investments to increase them and keep safety.
For these reasons we think to saving them in banks or other financial institutes:

1) To keep them safety.
2) To invest and increase them.

So that the income formula will be,
$\mathrm{Y}=\mathrm{C}+\mathrm{I}$
where:
$\mathrm{I}=$ investing.

On the other side, there are some people need moneys to building house, to buy a new car or even to do business ... etc.
So that both of these two people go to the banks, one is to save his money in it by deposit, and the other to get a loan from the bank.

Banks: are A legal intermediary that transfers the savings of individuals, businesses, and governments into loans or investments.
For example, we suppose that Mr. (Z) has surplus money (\$ 10000), he want to keep it safe and increase it, so he will take his money and deposit it in the bank ( B ); for ( 1 ) year at interest rate (10\%).

After that Mr. (K) needs a sum (\$10000) to buy a car, he will go to the bank (B) and get a loan for this sum , for ( 1 ) year at interest rate ( $12 \%$ ).

After (1) year, Mr. (K) will back to the bank (B) a sum (11200 \$) , then the Bank will back to Mr. (Z) a sum ( $11000 \$$ ), the profit of bank was (200\$)

## 2- Interest ...Definition and elements.

Now we have an imagining about the " Interest " , so we can define the Interest: " as A fee that is charged by a lender to a borrower for the right to use the borrowed funds. ", this fund we call them " Capital.
Capital: is one of the production elements.
The interest is consisting of three elements:
1- The principal (P): is a sum that will be used by others through borrowing it for specified period. May be from person to bank, or bank to person, or any other form.

2-Number of periods (n) : is the periods of using money by others; may be days, months, or years, and must be specific.

3 - Rate of interest (r): is the price of interest that will be agreed on between the two sides, credit and debit. It measures as (100) units per year, if there is no agreement for another deal.

## 3- The types of Interest

We have two types of interest:

## Simple Interest <br> and

## Compound Interest.

The simple interest is calculating each year by itself without adding any sums (one year usually),
the compound interest; calculate for first year as simple interest, and for another year adding the interest year by year ( we will explain about later).

## 4- Simple interest

Simple interest is a quick method of calculating the interest charge on a loan.
Simple interest usually calculating for one year or less than that, it consists of three elements:

1) Principal (original sum).
2) Number of periods (days, months or year).
3) Rate of interest (the price of interest; percentage).

The formula that uses to calculate the simple interest is:
I =P. n. r.
when.
I = Interest.
$\mathrm{P}=$ Principal (original sum).
$\mathrm{n}=$ number of periods.
$r=$ rate of interest (\%)
Now we will take several examples to explain how it is working,

Ex. 1: A man have a sum 1000 ID, he wants deposit it in a bank for (1) year, with rate of interest ( $10 \%$ ). Find the interest (simple interest) for him at the end of period.

Sol.
$P=1000 \mathrm{ID}, \mathrm{n}=1$ year, $\mathrm{r}=10 \%, \mathrm{l}=$ ?
I = P. n. r
$\mathrm{I}=1000$ * 1 *(10/100)
I = (1000 *1 *10 )/100
I=100 ID

Ex. 2: Mr. Anmar wants to invest a sum 2400 ID for three years, in Cihan bank with interest rate ( $13 \%$ ). Find the interest at the end of year three?

Sol.
$P=2400 \mathrm{ID}, \mathrm{n}=3$ years, $\mathrm{r}=13 \%, \mathrm{I}=$ ?
I = P. n. r
$I=2400 * 3 *(13 / 100)$
$\mathrm{I}=(2400 * 3 * 13) / 100$
I =936 ID

Ex. 3: Find interest of a sum $3060 \$$ for (6) years at (8\%)?
Sol.

$$
P=3060 \$, n=6 \text { years, } r=8 \%, I=?
$$

I = P. n. r
$I=3060 * 6 *(8 / 100)$
$\mathrm{I}=(73060 * 6 * 8) / 100$
$\mathrm{I}=1468.8$ \$
Ex. 4: Cihan bank pay interest rate (9 \%) for saving account, if a business man have 560400 ID and he want to invest it for ( 4 ) years. Find the interest?

Sol.
$P=560400 \mathrm{ID}, \mathrm{n}=4$ years, $\mathrm{r}=9 \%, \mathrm{l}=$ ?
I = P. n. r
$\mathrm{I}=560400$ * 4 * $(9 / 100)$
$\mathrm{I}=(560400 * 4 * 9) 100$
I = 201744 ID
In the case of calculating the interest by months, we must dividing the number of periods ( $n$ ) on ( 12 ) months, because it less than one year.

Here are examples,
Ex. 5: Ahmed have a sum 180000 ID, he want to deposit it in Iraqi investment bank for (8) months, at rate of interest (12 \%). Find the interest.

Sol.

$$
\begin{aligned}
& P=I D 180,000, n=8 \text { months, } r=12 \%, I=? \\
& I=P . n . r \\
& I=P *(n / 12) *(r / 100) \\
& I=180000 *(8 / 12) *(12 / 100) \\
& I=(180000 * 8 * 12) /(12 * 100) \\
& I=14400 \mathrm{ID}
\end{aligned}
$$

Ex.6: Find the interest of 102036 \$ for (11) months at rate of interest ( $11 \%$ ).

Sol.

$$
\begin{aligned}
& P=102036 \$, n=11 \text { months, } r=11 \%, I=? \\
& I=P . n . r \\
& I=P *(n / 12) *(r / 100) \\
& I=102036 *(11 / 12) *(11 / 100) \\
& I=(102036 * 11 * 11) /(12 * 100) \\
& I=10288.63 \$
\end{aligned}
$$

Ex-7: Calculate the interest for a student want to keep here money in Cihan Bank for ( 10 ) months, she have $7655.38 £$ and the interest rate in the bank is (1.5) \%.

Sol.

$$
\begin{aligned}
& P=7655.38 £, n=10 \text { months, } r=1.5 \%, I=? \\
& I=P . n . r \\
& I=P^{*}(n / 12) *(r / 100) \\
& I=7655.38 *(10 / 12) *(1.5 / 100) \\
& I=(7655.38 * 10 * 1.5) /(12 * 100) \\
& I=95.692 £
\end{aligned}
$$

## 5- The Amount in Simple interest:

The amount value is the principal added to it the interest of the principal that will invest at the end of period, so that the real sum anybody will keep or invest sum's own will receive at the end of period the whole sum, principal and interest together.

Amount $=$ Principal + Interest

$$
A=P+1
$$

We can also derive this formula; to find direct formula to calculate the amount.

A $=P+1$
$A=P+\left(P^{*} n^{*} r\right)$
common factor [ P ]
$A=P\left(1+\left(n^{*} r\right)\right)$
Here are examples,
Ex-8: A man deposit 30000 ID in a bank for (4) years, and the rate of interest is ( $11 \%$ ). Find the Amount that he will get at the end of year ( 4 ).

Sol.

$$
P=30,000 \mathrm{ID}, \mathrm{n}=4 \text { years, } \mathrm{r}=11 \%, \mathrm{I}=?, \mathrm{~A}=?
$$

1) At the beginning, we find the value of interest.

$$
\begin{aligned}
& I=P * n * r \\
& I=30000 * 4 *(11 / 100) \\
& I=30000(4 *(11 / 100))=30000(11 / 25)=1200 * 11 \\
& I=13200 \text { ID }
\end{aligned}
$$

2) Then we find the amount that a man will receive at the end of year (4).
$A=P+1$
$A=30000+13200$
A =43200 ID

Or
3- we can find $A$ in one step
$\mathrm{A}=\mathrm{P}\left(1+\left(\mathrm{n}^{*} \mathrm{r}\right)\right)$
$A=30000(1+(4$ * $(11 / 100))=30000(1+11 / 25)=30000(36 / 25)$
$A=1200 * 36=43200 \mathrm{ID}$

Ex-9: Mr. Ali wants to keep his sum 108788.5 \$ in a bank for (8) months, and the rate of interest of the bank is ( $15 \%$ ). Find his balance?

Sol.

$$
P=108788.5 \$, n=8 \text { months, } r=15 \%, \mathrm{l}=?, \mathrm{~A}=?
$$

Here we will use the direct way to find the amount by this formula:
$A=P .(1+(n * r))$
$A=108788.5^{*}\left[1+(8 / 12)^{\star}(15 / 100)\right]$
$A=108788.5 *[1+(8 * 15) /(12 * 100)]$
$A=108788.5$ * $[1+(120 / 1200)]$
$A=108788.5$ * [ $1+(0.1)]$
$\mathrm{A}=108788.5$ *[1.1]
$A=119667.35$ \$

Ex-10: Find the amount of sum 160585.35 ID for (15) months, at ( $13 \%$ )?
Sol.

$$
\begin{aligned}
& P=160585.35 \mathrm{ID}, \mathrm{n}=15 \text { months, } \mathrm{r}=13 \%, \mathrm{~A}=? \\
& A=P *(1+(n * r)) \\
& A=160585.35 *[1+(15 / 12) *(13 / 100)] \\
& A=160585.35 *[1+((15 / 12) *(13 / 100))] \\
& A=160585.35 *\left[1+\left(15^{*} 13\right) /(12 * 100)\right] \\
& A=160585.35 *[1+(195 / 1200)] \\
& A=160585.35 *[1+(0.1625)] \\
& A=160585.35 *[1.1625] \\
& A=186680.47 \mathrm{ID}
\end{aligned}
$$

## 6- Calculate the period:

The financial activities in banks usually do every day; deposit, withdraw or discount , etc. So, we need in this case to calculate the period by days.
In the commercial activates usually depend on standard Time of period as (360) days by one year ; it is mean (30) days for every month.
There is also Real time of period as ( 365 or 366 ) days by one year ; it is mean ( 30 or 31 or 29 or 28 ) days for every month.

The following table, can explain and simple the way of calculating:

No.
1 -
2 -
3 -
4 -
5 -
6 -
7 -
8 -
9 -
10 -
11 -
12 -

Name Short
Jan.
Feb.
Mar.
Apr.
May
June
July
Aug.
September Sept. 3030

October Oct. 31

November Nov.
30
December Dec.31

To calculate the period of days we must:
1 - Don't use the first day of deposit.
2 - Use the last day of withdraw.
3 - Calculate the days of months between deposit and withdraw.

To explain more, we will take some examples:
Ex-11: Find the period between 1st March to 20th May?
Sol.

$$
\begin{array}{llcc} 
& \text { March } \rightarrow & \text { April } \rightarrow & \text { May } \\
& \mathbf{3 1} & \mathbf{3 0} & * * \mathbf{2 0} \\
& * \mathbf{3 1 - 1} & \mathbf{3 0} & \mathbf{2 0} \\
\mathbf{3 0}+ & \mathbf{3 0} & + & \mathbf{2 0}=\mathbf{8 0} \text { days }
\end{array}
$$

> * Subtract days of all months from the day of deposit.
> ** the last month, we just use the date till to withdraw

money; not all the days of month

Ex -12: Calculate the number of days between 22nd May to 14th Sep.

Sol.

| May $\rightarrow$ | June $\rightarrow$ | July $\rightarrow$ | Aug. $\rightarrow$ | Sep. |
| :---: | :---: | :---: | :---: | :---: |
| 31 | 30 | 31 | 31 | 14 |
| $31-22$ | 30 | 31 | 31 | 14 |
| $9+30+31+31+14=115$ days |  |  |  |  |

## 7- Types of Simple Interest:

There are three types of simple interest:
1 ) Standard interest : this type of interest calculate by dividing the number of periods ( $n$ ) on ( 360 ) days, which represents every month by ( 30 ) days.

$$
S(I)=P^{*}(n / 360) *(r / 100)
$$

when,
S = standard interest

2- Commercial interest: this type of interest is also calculated by dividing the number of periods ( n ) on ( 360 ) days, which represent every month by ( 30 ) days.

Com.(I) $=P^{*}(n / 360) *(r / 100)$
when,

Com. (I) = commercial interest

3- Exact interest : this type of interest is calculated by divide the number of periods ( $n$ ) on ( 365 or 366 ) days, which represent the days of whole year.
Ex. (I) $=P$ *(n/360 or366) *(r/100)
when,
EX. (I) = Exact interest

We notice that the standard and commercial interest are the same in the way of calculation, so for that we depend on commercial interest in our financial activities.

Ex-13: Mr. Kamal want to invest his sum 36000 ID for (100) days in Trade Bank of Iraq, the rate of interest is ( $12 \%$ ). Find commercial interest.

Sol.
$P=36000 \mathrm{ID}, \mathrm{n}=100$ days, $\mathrm{r}=12 \%$, Com. $(\mathrm{I})=$ ?

Com. (I) $=\mathrm{P} *(\mathrm{n} / 360) *(\mathrm{r} / 100)$
Com. (I) $=36000$ * (100/360) *(12/100)
Com. (I) $=(36000$ * 100 * 12 )/( 360 *100)
Com.(I) = 1200 ID

Ex-14: Cihan bank pay interest rate ( $9 \%$ ) for investing sum , a man have 146800 ID want to invest it for ( 187 ) days. Calculate the commercial interest he will gain.
Sol.
$\mathrm{P}=146800 \mathrm{ID}, \mathrm{n}=187$ days, $\mathrm{r}=9 \%$, Com. $(\mathrm{I})=$ ?
Com (I) $=\mathrm{P} *(\mathrm{n} / 360) *(\mathrm{r} / 100)$
Com. (I) $=146800$ * $(187 / 360)$ *(9/100)
Com ,(I) = (146800 *187 *9 )/ (360 * 100)
Com . (I) $=6862.9$ ID

Ex-15: A merchant wants to invest his sum 100,640 \$ for 300 days, and Cihan bank pay ( $14 \%$ ). Find commercial interest?

Sol.
$P=100640 \mathrm{ID}, \mathrm{n}=300$ days, $\mathrm{r}=14 \%$, Com. $(\mathrm{I})=$ ?
Com (I) $=\mathrm{P} *(\mathrm{n} / 360)$ *(r/100)
Com. (I) $=100640$ * (300/360) *(14/100)
Com. $(I)=(10040 * 300 * 14) /(360 * 100)$
Com. (I) $=(422688000 / 36000)$

Com.(I) $=11741.33$ ID

Ex-16: Find the commercial interest of sum 90830 ID, if you know that Cihan bank pay ( $15 \%$ ) , for 256 days?

Sol.
$\mathrm{P}=90,830 \mathrm{ID}, \mathrm{n}=256$ days, $\mathrm{r}=15 \%$, Com. $(\mathrm{I})=$ ?
Com. (I) $=\mathrm{P} *(\mathrm{n} / 360)$ * $(\mathrm{r} / 100)$
Com. (I) $=90830 *(256 / 360) *(15 / 100)$
Com. (I) $=348787200 / 36000$
Com. (I) $=9688.53$ ID

Ex-17: A man want to invest a sum 7432 ID for 169 Days at (10\% ). Find the Exact interest.
Sol.
P = ID 7,432 , $\mathrm{n}=169$ days , $\mathrm{r}=10 \%$, Ex. (I) $=$ ?
Ex.(I) $=P *(n / 360) *(r / 100)$
Ex. (I) $=7432$ * $(169 / 365)$ * (10/100)
Ex. (I) $=(7432$ * 169 *10)/36500
Ex. (I) $=12560080 / 36500$
Ex.(I) =344.11 ID

Ex-18: Cihan bank pay (14\%) as rate of interest, if you think to invest a sum 288500 ID for ( 135 ) days; how much will be the Exact interest you will receive?
Sol.
$P=288,500 \mathrm{ID}, \mathrm{n}=135$ days, $\mathrm{r}=14 \%$, Ex. (I) $=$ ?
Ex. (I) $=\mathrm{P}$ *( $\mathrm{n} / 365$ ) *(r/100)
Ex. (I) $=288500 *(135 / 365) *(14 / 100)$
Ex. (I) $=(288500 * 135 * 14) /(36500)$
Ex. (I) $=545265000 / 36500$
Ex. (I) $=14938.76$ ID

Ex-19: A man want to invest his money in Trade Bank of Iraq (TBI) for 201 days, and the rate of interest is ( $9 \%$ ) for 86340 ID he want to invest. Calculate exact interest?

Sol.
$\mathrm{P}=86340 \mathrm{ID}, \mathrm{n}=201$ days, $\mathrm{r}=9 \%$, Ex. ( I ) = ?
$E x(I)=P *(n / 365)^{*}(r / 100)$
$E x(I)=86340$ * $(201 / 365)$ * $(9 / 100)$
$E x(I)=(86340 * 201 * 9) / 36500$
$E x(I)=156189060 / 36500$
$E x(I)=4279.15 \mathrm{ID}$

Ex-20: How much will be the Exact interest if a merchant want to invest his sum $165218 \$$ for 364 days at ( $10 \%$ ) ?
Sol.
$\mathrm{P}=165218 \mathrm{ID}, \mathrm{n}=364$ days , $\mathrm{r}=10 \%$, Ex. ( I ) = ?
$E x(I)=P *(n / 365) *(r / 100)$
$E x(I)=165218$ *(364/365) *(10/100)
$E x(I)=(165218 * 364 * 10) / 36500$
Ex (I) $=601393520 / 36500$
$E x(I)=16476.53 \mathrm{ID}$

## 8- The Relationship between Commercial Interest \& Exact Interest:

We can find any of (Commercial or Exact) Interest in terms of Other, if we look at the equation of each of them:
$\operatorname{Com}(\mathrm{I})=\mathrm{P} *(\mathrm{n} / 360) *(\mathrm{r} / 100)$

Ex. (I) $=P$ * $(n / 365) *(r / 100)$

The difference is just in the period, the
Com. (I) = 360 and Ex. (I) = 365, if we divided the equations.

Com.(I) /Ex.(I) $=(P *(n / 360) * r / 100)) /(P *(n / 365) *(r / 100))$
Com.(I) $/ E x$.(I) $=365 / 360$
Com (I) $/$ Ex.(I) $=73 / 72$

Either,
Com.(I) $=$ Ex. (I) *73/72
Or
Ex.(I) = Com.(I) *72/73

## 9- The difference between Commercial Interest \& Exact Interest:

The difference between the commercial interest and exact interest can be calculate by using the formula:

Diff $=$ Com.(I) -Ex.(I)
one can cross-multiply to get either;

Ex. $(\mathrm{I})=72$ *Diff
Or
Com.(I) $=73$ *Diff
here are examples,
Ex-21: How much will be the Commercial Interest, if the Exact interest is ID 7200 ?
Sol.

Com. (I) = Ex. (I) * 73/72

Com.(I) $=7200$ *73/72

Com.(I) $=(7200 * 73$ )/72

Com.(I) $=7300$ ID

Ex-22: Find the Exact Interest, when the commercial interest is \$ 3420? Sol.
Ex. (I) = Com. (I) *72/73

Ex. $(\mathrm{I})=3420 * 72 / 73$

Ex. (I) $=(3420 * 72) / 73$

Ex. (I) = 3373.15 ID

Ex-23: A difference between commercial interest and exact interest is 1035 ID, find the Com. (I) \& Ex. (I)?
Sol.

Com.(I) $=73$ *Diff

Com.(I) $=73$ *1035

Com.(I) =75555 ID

Ex. (I) = 72 *Diff

Ex. (I) $=72 * 1035$

Ex. (I) = 74520 ID

Ex-24: Find the commercial interest and exact interest,
if you know that the difference between them is 4197 ID?
Sol.

Com.(I) $=73$ *Diff
Com. $(I)=73$ * 4197

Com.(I) = 306381 ID
Ex.(I) $=72$ *Diff
Ex. (I) $=72$ * 4197
$E x .(I)=302184 I D$

## 10- The Numbers Method:

The banks usually have many deals with many corporates and individuals every day or week or month; deposit, investment or withdraw.
In this case they make current accounts with the same rate of interest for corporate or individual, to facilities and accuracy the calculation of the bank activates.
The numbers method used for several amounts with different periods at same rate of interest, also it's called (Table method).
To calculate the sums by this way, we must make some steps:

1 - Make a table:

| Principal | Number of <br> periods <br> Days or <br> months | No. |
| :--- | :--- | :--- |
| xxx | Days or <br> months | xxx |
| Xxx | P $=$ xxx | $-------------~$ <br> --- |
|  | No. $=\mathrm{xxx}$ |  |

2 - Calculate the summation of principals.
3 - Multiple every principal by it period; to find the No.
4 - Calculate the summation of No.

5 - Apply the suitable formula.

$$
\sum I=\sum \text { No. *(1/12 or } 360 \text { ) *(r/100) }
$$

For more explain, let's take some example :
Ex-25: Find the interest in sums at 14\%
1500 ID for 30 days
2500 ID for 50 days
4000 ID for 70 days

Sol.

| P | N | No. |
| :--- | :--- | :--- |
| 1500 | 30 | 45000 |
| 2500 | 50 | 125000 |
| 4000 | 70 | 280000 |
| $\Sigma \mathrm{P}=8000$ | $--------------\quad \Sigma$ No. $=450000$ |  |

$\Sigma I=\sum$ No. * $(1 / 360){ }^{*}(r / 100)$
$\Sigma I=450000$ * $(1 / 360)$ * $(14 / 100)$
$\Sigma I=(450000 * 1 * 14) /(360 * 100)$
$\Sigma I=6300000 / 36000$
$\sum I=175 I D$

Ex-26: A merchant want to invest sums at 9\%
10500 ID for 5 months
13000 ID for 7 months
16250 ID for 8 months
19400 ID for 11 months

Find the total amount?

Sol.

| P | N | No. |
| :--- | :--- | :--- |
| 10500 | 5 | 52500 |
| 13000 | 7 | 91000 |
| 16250 | 8 | 130000 |
| 19400 | 11 | 213400 |
| $\Sigma \mathrm{P}=59150$ | $--------------\quad \Sigma$ No. $=486900$ |  |

$\sum I=\sum$ No. *(1/12) * (r/100)
$\sum I=486900$ *(1/12) * (9/100)
$\sum \mathrm{I}=(486900 * 1 * 9) /(12 * 100)$
$\Sigma I=4382100 / 1200$
$\Sigma I=3651.75 \mathrm{ID}$
$A=\sum P+\sum 1$
$A=59150+3651.75$
$A=62801.75 \mathrm{ID}$

Ex-27: Baghdad company want to be investing their sums in Cihan bank:
120000 ID on 1st May 2016
136000 ID on 12th June 2016
145000 ID on 17th July 2016
All of sums investing end on 21st September 2016 at 13\%, find the commercial interest?

1 st May $\rightarrow$ 21st Sep. $=143$ days
12th June $\rightarrow$ 21st Sep. $=101$ days

17th July $\rightarrow 21$ st Sep. $=66$ days

| P | $\mathrm{N} /$ days |
| :--- | :--- |
| 120000 | 143 |
| 136000 | 101 |
| 145000 | 66 |
| $\Sigma \mathrm{P}=401000$ | ------------ |

$$
\begin{aligned}
& \Sigma \text { Com. }(I)=\Sigma N o *(1 / 360) *(r / 100) \\
& \Sigma I=40466000 *(1 / 360) *(13 / 100) \\
& \Sigma I=(40466000 * 1 * 13) /(360 * 100) \\
& \Sigma I=526058000 / 36000 \\
& \Sigma I=14612.72 I D
\end{aligned}
$$

No.
17160000
13736000
9570000
$\Sigma$ No. $=40466000$

## Problems

1- Salar invested 10000 \$ in a saving bank account that earned $2 \%$ simple interest.

Find the interest earned if the amount was kept in the bank for 4 years.

2- $\quad$ Ryan borrowed $15000 \$$ from a bank to buy a car at $10 \%$ simple interest. If he paid $9000 \$$ as interest while clearing the loan.

Find the time for which the loan was given.

3- In how much time will the simple interest on $3500 \$$ at the rate of $9 \%$, be the same as simple interest on $4000 \$$ at $10.5 \%$, for 4 years?

4- Ian deposit 3000\$ in State bank of India for 3 years which earn him an interest of 8\%.

What is the amount he gets after 1 year, 2 years, and 3 years?

5- Kamal deposited 10000, for 4 years at a rate of $6 \%$.
Find the interest and amount Kamal got.

6- $\quad$ Tim invested $1500 \$$ at the rate of $6 \%$, for 7 years and 3 months.
Find the amount he got back.

7- Ahmed requests a 2-years loan of $6500 \$$ from Cihan bank. The bank approves the loan at an annual rate of $14 \%$ :
(i) What is the simple interest of the loan?
(ii) What is the maturity value of the loan?

8- A student borrows $600 \$$ to buy a camera. The loan is over two years, and the simple interest rate is $6 \%$ per year. How much interest does the student pay? What is the total amount of money repaid?

9- An investor lent $12000 \$$ to a business associate for 6 months at an interest rate of $8 \%$ per year. Calculate the interest the investor earns and how much in total will be repaid?

10- Find the exact and ordinary simple interest on a 60-day loan of 1950 \$ at 13.5 \%

11- A loan made a loan $100 \$$ to be repaid with $120 \$$ at the end of one month. What was the annual interest rate?

12- How long will it take 5000 \$ to earn $60 \$$ interest at $6 \%$ ?

13- How long will take 1000 \$?
(i) To earn $100 \$$ at $15 \%$ simple interest?
(ii) To calculate to 1270 \$ at $13.5 \%$ simple interest?

14- Find the accumulated value (using both ordinary \& exact simple interest)
(i) $500 \$$ at $11 \%$ over 60 days.
(ii) $1000 \$$ at $15 \%$ over 150 days.

15- What principal will accumulate to:
(i) $5100 \$$ in 6 months at $9 \%$ simple interest?
(ii) $580 \$$ in 120 days at $18 \%$ exact interest?

16 - A deposit of $1500 \$$ is made into a fund-on March (18). The fund earns simple interest at $5 \%$ on August (5), the interest rate changes to $4.5 \%$, how much is in the fund on October (23)?

17- A sum of $42000 \$$ is invested from May (18) ,2006 to April (8) ,2007, at $4.5 \%$ simple interest. What amount must he repay in 7 months?

## Chapter Two

## Commercial discount with simple interest.

## 1- The concept of trade discount.

The debtor may need cash, so he resorts to collecting and discounting the commercial papers he owns, and then presents them, to the bank before its due date, the bank makes a discount by subtracting the interest amount resulting from the period, the interval between the discount date and the maturity date of the commercial paper is the nominal value of the commercial paper. The remaining value is provided to its holder, which is the current value of the paper.

## 2- Definition of trade discount.

It is the bank's interest in the process of converting the value of a commercial paper into cash before its maturity date. It is a process that allows the ownership of a commercial paper to be exchanged, thus making the bank the beneficiary of the nominal value (the amount of the paper). The bank keeps the amount of the nominal value and provides the holder of the paper with an amount (the current value), before its maturity date.

## 3-Trade discount account.

The trade discount is calculated based on the nominal value of the commercial paper, at a specific rate of the bank's tip is called the discount rate, and the discount period ( n ), which is the period between the maturity date of the paper and the discount date. There are two ways to calculate the trade discount.

## The normal Method:

$\mathrm{EC}=\left(\mathrm{C} * \mathrm{r}^{*} \mathrm{n}\right) / 36000$
Where
$\mathrm{C}=$ The face value of the paper
$\mathrm{r}=$ Discount rate
$\mathrm{n}=$ Discount period.

## The Constant denominator method:

EC=N/D
Where
$\mathrm{N}=\mathrm{C} * \mathrm{n} * \mathrm{D}=36000 / \mathrm{r}$

Ex-1: On 13/4/ 2020, I submitted a commercial paper for discount, with a nominal value of 20,000 , due on $30 / 6 / 2020$, at the bank, to which the discount rate of $5 \%$ was applied. Calculate the discount amount.

Sol:
$\mathrm{n}=(30-13)+31+30=78$ days
EC= (C *r*n)/36000
EC= $(20000 * 5 * 78) / 36000$
EC =216.66 ID

EX-2: What is the nominal value of a commercial bond that was discounted 66 days before its maturity date at a discount rate of $6 \%$, and the discount amount was 550 ID.

Sol:
EC = (C *r *n )/36000
$550=(C * 6 * 66) / 36000$
C=(EC*36000)/ (r*n)
$C=(550 * 36000) /(6 * 66)$
$\mathrm{C}=50000 \mathrm{ID}$

EX-3: In22/3/2020. An institution sold a commodity for the amount of 58,500 ID, and the debt was recognized after postponing it with a commercial paper due for payment in25/5/2020, but the holder of the paper needed money, so he approached the bank to negotiate the paper before its maturity date, so the amount of 520 ID was deducted from him at a rate of $10 \%$.

Required is to find the discount date.
Sol:
$E C=(C * r * n) / 36000$
$\mathrm{n}=(\mathrm{EC} * 36000) /(\mathrm{C} * \mathrm{r})=(520 * 36000) /(58500 * 10)=32$ days
The deducted date is 23/4/2020.
EX-4: On 3/7/2020 a paper with a nominal value of 48,000 was negotiated with a bank, and its maturity date was 30/11/2020. The bank received interest from the transaction in the amount of 1,400 . Calculate the discount rate.

Sol:
EC $=(C * r * n) / 36000$
1400= (48000 *r * 150)/ 36000
$1400=(4 * r * 50)$
$r=1400 / 200=7 \%$

## 4-Current value of a commercial paper

It is the value of the amount that the owner of the commercial paper gets when deducting it, and we symbolize it with the symbol V.

There are two methods to calculate the current value.

## (i) The normal Method:

$$
\begin{aligned}
V & =C-E C=C-(C * r * n) / 36000 \\
& =C\left(1-\left(r^{*} n / 36000\right)\right.
\end{aligned}
$$

## (ii) The Constant denominator method:

$\mathrm{V}=\mathrm{C}-((\mathrm{C} * \mathrm{n}) / \mathrm{D})=\mathrm{C}((\mathrm{D}-\mathrm{n}) / \mathrm{D})$

EX-5: Calculate the current value of a commercial paper with a nominal value of 20,000 ID, presented for discount at a bank 60 days before its maturity date at a rate of $6 \%$.

Sol:
$V=C(1-((r * n) / 36000)=20000(1-((6 * 60) / 36000)=19800$ ID

EX-6: A merchant presented a commercial paper for discount at a bank 4 months before its maturity. If you know that the bank applies a discount rate of $6 \%$,and the value that the merchant gets after the discount process is 83,300 . Calculate the face value of the paper.

Sol:
$\mathrm{V}=\mathrm{C}-((\mathrm{C} * \mathrm{r} * \mathrm{n}) / 1200)$
$83300=C(1-(r * n) / 1200))$
$83300 n=C(1-(6 * 4) / 1200))$
$83300=C(49 / 50)$
C=85000 ID
EX-7: A commercial paper with a nominal value of 60,000 , due for payment on 12/30/2017, presented for discount at a bank n days before its maturity date, at a discount rate of $6 \%$. If you know that its holder may obtain a value of 59,400 , specify the discount date.

Sol:

$$
\begin{aligned}
& V=C-((C * r * n) / 36000) \\
& 59400=60000(1-(6 * n) / 36000) \\
& 59400=10(6000-n)
\end{aligned}
$$

$59400=60000-10 n$
$10 n=60000-59400=600$
$\mathrm{n}=60$ days

## 5-Rational (Correct) Discount

It is the discount that is calculated on the basis of real data, specifically on the basis of the current rational value of the commercial paper, which we symbolize with the symbol ( $\mathrm{V}^{\wedge}$ ) and not on the basis of the nominal value, which we symbolize with the symbol (ER). The rational discount is calculated.

$$
E R=\left(V^{\wedge} *_{r}{ }^{*} n\right) / 36000
$$

Where ER = the rational (correct) discount.
$\mathrm{V}^{\wedge}=$ The current rational value of the commercial paper .

$$
\begin{aligned}
& V^{\wedge}=C-E R=C-\left(\left(V^{\wedge} *_{r} * n\right) / 36000\right) \\
& V^{\wedge}+\left(\left(V^{\wedge} *_{r} *_{n}\right) / 36000\right)=C
\end{aligned}
$$

$V^{\wedge}(1+(r * n) / 36000)=C$
$\mathrm{V}^{\wedge}=\mathrm{C} /\left(1+\left(\mathrm{r}^{*} \mathrm{n}\right) / 36000\right)=(36000 * \mathrm{C}) /(36000+(r * n))$

## For the discount correct,

calculate the following formula:
$E R=C-V^{\wedge}=C-((36000 * C) /(36000+(r * n))$
$E R=(C * r * n) /(36000+(r * n))$

## EX-8

Commercial paper with a nominal value of 70,000 ID, negotiated 42 days before the maturity date, at a discount rate of $7.5 \%$.

Find:

> Calculate the trade discount. (EC) The correct discount (ER) current value (V) Rational present value ( $V^{\wedge}$ )

Sol:
the trade discounts.
$\mathrm{EC}=(\mathrm{C} * \mathrm{r} * \mathrm{n}) /(36000)=(70000 * 7.5 * 42) / 36000=612.5 \mathrm{ID}$
The correct discount
$E R=(C * r n) /(36000+(r * n))=(70000 * 7.5 * 42) /(36000+(7.5 * 42))$
$=607.18 \mathrm{ID}$
current value
$V=C-E C=70000-612.5=69387.5 \mathrm{ID}$

Rational present value
$\mathrm{V}^{\wedge}=\mathrm{C}-\mathrm{ER}=70000-607.18=69392.82 \mathrm{ID}$

## EX-9

Commercial paper with a nominal value of 90,000 ID, negotiated 4 months before the maturity date, at a discount rate of $8.5 \%$.

Find:
(i) -Calculate the trade discount.
(ii)-The correct discount
(iii)-current value
(iv)-Rational present value

Sol:
(i) Trade discount EC

$$
\begin{aligned}
& \mathrm{EC}=(\mathrm{C} * r * n) / 1200 \\
& \mathrm{EC}=(90000 * 8.5 * 4) / 1200 \\
& \mathrm{EC}=2550 \mathrm{ID}
\end{aligned}
$$

(ii) The correct discount ER

$$
\begin{aligned}
& E R=(C * r * n) /(1200+(r * n)) \\
& E R=(90000 * 8.5 * 4)(1200+(8.5 * 4)) \\
& E R=3060000 / 1234 \\
& E R=2479.74 \mathrm{ID}
\end{aligned}
$$

(iii) Current value V
$V=C-E C=90000-2550=87450 \mathrm{ID}$
(iv) Rational present value $\mathrm{V}^{\wedge}$

$$
V^{\wedge}=C-E R=90000-2479.74=87520.26 \text { ID }
$$

## Problems

1-If you know that the difference between the commercial discount and the rational discount is 45.454 dinars, and their ratio is $12 / 11$ required
(i) -Calculate both commercial and rational discount
(ii)- Calculating the nominal value of the debt.

2-A commercial paper with a nominal value of 34,500, which matures after 45 days at an estimated rate of $8 \%$.

Find the discount value.
3 -A commercial paper with a value of 15,000 , maturity date 20/6/2020, presented for commercial discount on $5 / 6 / 2020$, at a discount rate of $8 \%$ Required:
(i)-Calculate the number of days of discount.
(ii)-Calculate the discount amount.

4- What is the nominal value of a commercial bond that was discounted 70 days before its maturity date at a discount rate of $8 \%$, and the discount amount was 750 ID.

5- Calculate the current value of a commercial paper with a nominal value of 40,000 ID, presented for discount at a bank 30 days before its maturity date at a rate of $7 \%$.

6- Commercial paper with a nominal value of 70,000 ID, negotiated 3 months before the maturity date, at a discount rate of $7.5 \%$.

Find:
(i) -Calculate the trade discount.
(ii)-The correct discount
(iii)-current value

> (iv)-Rational present value

7- Commercial paper with a nominal value of 100,000 ID, negotiated 45 days before the maturity date, at a discount rate of $8 \%$.

Find:
Calculate the trade discount. (EC)
The correct discount (ER)
current value ( $V$ )
Rational present value ( $V^{\wedge}$ )

## Chapter Three

## Compound Interest

## 1- Compound Interest

Compounding means that interest calculated not only on the initial investment but also on the interest of previous period.

In the compound interest the money growth faster than investment in simple interest.

The compound interest may be compounded either:
1-annually (once a year)
2-half- year (semiannually) (twice a year)
3-quartery year (4 times a year)
4-monthly (12 times a year)
5-weekly (52 times a year)
6- daily (365 times a year)
When computing compound interest , the first thing to do is find what is called the period interest rate ( PIR)

The
PIR = yearly interest rate or $r=R / n$
Number of interest per year
To find the PIR when interest is compounded yearly : divide the yearly interest rate by 1 .

Half yearly: $==\quad=\quad=2$.
Quarterly : = = = = 4 .
Monthly: = = = = 12 .
weekly : = = = = 52 .
daily $:===\quad==365$.

Ex-1: Find the period interest rate (PIR) if the annual interest rate is $8 \%$ and the interest is compounded quarterly.

Sol:
If the interest is compounded quarterly , then the number of interest
payments per year is 4 , Bence
$P I R=(8 \%) / 4=0.08 / 4=0.02=2 \%$

The next example shows the difference between simple \& compound interest

Ex-2: Find the amount of the
(i) Simple interest
(ii) The compound interests

On a 2000 \$ investment as an annual rate of 4\% for 3 years .
Sol:
For simple interest, use the formula
$I=P * r * n$, when $P=2000 \$, r=4 \%, n=3$
$I=2000 * 0.04 * 3=240 \$$
For compound interest, the calculations need to be done three times, one for each year .

For year 1
I =P *r *n = 2000 \$ *0.04 * $1=80$ \$
During the second year, the $80 \$$ interest also generates interest, and so the principal is
$2000 \$+80 \$=2080 \$$

For year 2
$I=P * r * n=2080 \$ * 0.04 * 1=83.20 \$$
For year 3 :
The principal is $2080 \$+83.20 \$=2163.20 \$$
Hence:
$I=2163.20 \$ * 0.04 * 1=86.53 \$$
Hence the total compound interest is
$80 \$+83.20 \$+86.53 \$=249.73 \$$
The total a mount of the investment is
$2000 \$+249.73$ \$ = 2249.73 \$

If interest is compounded half yearly, the same problem would require six calculations. if interest is compounded quarterly, the problem would require 12 calculation and so on .

The following formula can be used :
$\mathrm{FV}=\mathrm{S}=\mathrm{P}(1+\mathrm{r})^{\mathrm{n}}$
Where:
$\mathrm{FV}(\mathrm{S})=$ Future value (a mount)
P = Principal
$r=$ period interest rate , r=PIR
$\mathrm{n}=$ number of periods per year times the number of years
Ex-3:
Find the future value and compound interest on 2000 \$ investment at a rate $4 \%$ compounded yearly for 3 years .

Sol:

To this case , the period interest rate is $4 \%$ since the interest is compounded yearly and $n=1 * 3=3$

$$
\begin{aligned}
\mathrm{FV}=\mathrm{P}(1+r)^{\mathrm{n}} & =2000 \$(1+0.04)^{3} \\
& =2249.73 \$
\end{aligned}
$$

The amount of the compound interest is
$I=F V-P=2249.73 \$-2000 \$=249.73 \$$

## EX-4

Find the future value and compound interest on a 6000 \$ investment at $10 \%$ compounded half-yearly for 6 years .

Sol:
The period investment rate is ( $10 \% / 2$ ) $=0.05$ or $5 \%$
The number of periods is $2 * 6=12$

$$
\begin{aligned}
\mathrm{FV} & =P(1+r)^{n} \\
& =6000 \$(1+0.05)^{12} \\
& =10775.14 \$
\end{aligned}
$$

The amount of the compound interest is

$$
10775.14 \$-6000 \$=4775.14 \$
$$

## Ex-5:

Find the future value ad compound interest on a 500 \$ investment at 6\% compounded daily for 5 years .

Sol:
The PIR $=(6 \%) / 365$.the number of periods is $365 * 5=1825$
$F V=P(1+r)^{n}$

$$
\begin{aligned}
= & 500 \$(1+(0.06 / 365))^{1825} \\
& =674.91 \$
\end{aligned}
$$

The compounded interest is 674.91 \$ $-500 \$=174.91$ \$

## EX-6

A person invested $\$ 75000$ for a period of 3 years at an annual compound interest rate of $7 \%$. Calculate the interest value for each year.

Sol:
$\mathrm{I}_{1}=\mathrm{P}^{*} \mathrm{n} * \mathrm{r}=75000 * 1 *(7 / 100)=5250 \mathrm{ID}$
$\mathrm{I}_{2}=(75000+5250)(7 / 100)=80250 *(7 / 100)=5617.5 \mathrm{ID}$
$\mathrm{I}_{3}=(80250+5617.5) *(7 / 100)=85867.5 *(7 / 100)=6010.725$

## EX-7

A man invested $\$ 12,000$ in a bank that offers compound interest at a rate of $4 \%$ every three months. What is the value obtained after two years?

Sol:
$A=P(1+r)^{n}=P(1+0.04)^{8}=12000 *(1+0.04)^{8}$
$A=12000 * 1.368569=16422.82 \mathrm{ID}$

## Note: Numbers are taken up to six digits after the comma.

## EX-8

A person invested a certain amount for 4 years at an annual compound interest rate of $8 \%$, and he obtained an amount of $1,360,489$ at the end of the investment period. Calculate the amount invested.

Sol:
$A=P(1+r)^{n}$
$\mathrm{P}=\mathrm{A}(1+\mathrm{r})^{-\mathrm{n}}=1360.489(1+0.08)^{-4}$
$(1.0 .8)^{-4}=0.735029$
$P=1360.489$ * $0.735029 \quad=1000 \mathrm{ID}$

## EX-9

An amount of 72,800 ID was invested for a period of 6 years with compound interest, to produce a value obtained at the end of the investment of 125491.9848 ID. Calculate the investment rate.

Sol:
$A=P(1+r)^{n}$
$(1+r)^{n}=(A / P)$
$(1+r)^{6}=(125491.984 / 72800)$
$(1+r)^{6}=1.723791$
$r=9.5 \%$

## EX-10

A person invested an amount of 2,000 ID at a compound interest rate of 6\% for a certain period. At the end of the period, he obtained an amount of 2,676,452
ID. Calculate the investment period.
Sol:
$A=P(1+r)^{n}$
$(1+r)^{n}=A / P=(2676.452) / 2000$
$(1+r)^{n}=1.338226$
$n=5$ years

EX-11

A person deposited an amount of 98,200 ID in a commercial bank at an annual interest rate of $8.5 \%$ for a period of 7 years

Required:
1- The interest earned at the end of the first year of deposit
2-Interest earned at the end of the fifth year only
3- The amount obtained at the end of the period
Sol:
1- $\quad \mathrm{l}_{1}=\mathrm{P} * \mathrm{r} * \mathrm{n}=98200 * 1 *(8.5 / 100)=8347 \mathrm{ID}$
2- $\quad \mathrm{I}_{5}=\mathrm{A} 4 * r=P(1+r)^{4} * r=98200(1+0.085)^{4 *} 0.085$
$=98200(1.085)^{4 *} 0.085=11567.76257$ ID

3- $\quad A n=P(1+r)^{n}$
$A 7=98200(1+0.085)^{7}=98200(1.085)^{7}=173827.9687 \mathrm{ID}$

The following steps will help you calculate the formula FV $=P(1+r)^{n}$
1- $\quad$ Enter the value for $P$
2- Press *
3- Press (
4- Enter 1
5- Press +
6- Enter PIR
7- Press )
8- $\quad$ Press the exponent key ( $\mathrm{x}, \wedge$ ory x $\qquad$

9- Find the value of the number of period.

## 2-Effective Rate of Interest:

When the interest is compounded more than once a year . it is sometimes necessary to determine an equivalent simple interest rate.

For example, suppose a person invested $800 \$$ in a saving account for 1 year at a rate of $8 \%$ compounded quarterly .

The future value would be
$\mathrm{FV}=\mathrm{P}(1+r)^{\mathrm{n}}=800 \$(1+(0.08 / 4)) 1 * 4=865.95 \$$
The total interest is 865.95 \$ - 800 \$ = 65.95 \$
Now the comparable simple interest rate is
$R=1 /(P * n)=65.95 \$ /(800 \$ * 1)=0.0824$ or $8.24 \%$
In other words, if a person invested 800 \$ in saving account paying $8.24 \%$ simple interest for 1 year , he or she would receive $65.95 \$$ in interest , which is the same amount as the person investing $800 \$$ for year at $8 \%$ compounded quarterly .

## 3-The actual interest on simple interest rate that is equivalent to compound interest rate is called the effective rate.

For the loan, this rate is called the annual percentage rate APR.
The following formula can be used for calculating the effective rate
$\mathrm{E}=(1+r)^{\mathrm{n}}-1$ where:
$E=$ effective rate
$r=$ period interest rate $R / n$ \& $n=$ the number of periods .

## Ex-12:

Find the effective rate of interest that is equivalent to a $5 \%$ rate compounded

Semiannually (half yearly)
Sol:
Since the interest is compounded half yearly
$R=R / n=0.05 / 2=0.025$
$\mathrm{E}=(1+\mathrm{r})^{\mathrm{n}}-1=(1+0.025)^{2}-1=0.0506$ or $5.06 \%$

## 4-Depreciation:

Businesses most often need to purchase equipment, forint buildings, and other necessary items in order to operate their items are called assets.

The worth of an item most often decreases over the year. This decrease in worth is due to what is collect depreciation.

## 5-The straight -line method:

## EX-13:

A factory machine was purchased for $15000 \$$ and has an estimated lifetime of 5 years. if the scrap value at the end of 5 years is estimated of $3000 \$$, find the amount of the depreciation for each year .

Sol:
Find the depreciation amount (DA)
DA =original principle - scarp value $=15000 \$-3000 \$=12000 \$$
Divide the depreciation amount by the estimated lifetime.
12000\$ $/ 5=2400 \$$ per yar
In other words, the value of the machine depreciates $2400 \$$ per year.

## EX-14:

Find the book value (the value of an item often depreciation is calculated) at the end of 4 years of a digital (video camera costing $2400 \$$ with an expected lifetime of 6 years. the scrap value at the end of lifetime is $3000 \$$.

Sol:
$D A=2400 \$-3000 \$=2100 \$$
2100 \$ /6 =3500 \$
The amount of depreciation at the end of 4 years is
3500 \$ *4 = 14000 \$
The book value of the camera at the end of 4 years is :
24000 \$ - 14000 \$ = 10000 \$

## 6-The Constant -Percentage Method:

## Ex-15:

A car costing 24000 \$ depreciates $25 \%$ of its value each year. find the book value at the end of 5 years.

Sol:

$$
\begin{aligned}
B & =P(1-r)^{n} \\
B & =24000 \$(1-0.25)^{5} \\
& =24000 \$(0.75)^{5} \\
& =5695.31 \$
\end{aligned}
$$

## EX-16:

Determine the rate of depreciation for the machine costing $40000 \$$ which is estimated to have a useful lifetime of 5 years and scrap value of $5000 \$$.

Sol:
$B=P(1-r)^{n}$
$5000=40000(1-r)^{5}$
$5000 / 40000=(1-r)^{5}$
$(1-r)^{5}=0.125$
$(1-r)=(0.125)^{1 / 5}=(0.125)^{0.2}$
$1-r=0.65975$
$r=1-0.65975=0.340246=0.34=34 \%$

## Problems

## Compound Interest

1- Find the future value and compound interest on an investment account of $700 \$$, compounded quarterly at $10 \%$ for 3 years .
2- Mr. Zaki deposited 4000 \$ in a saving account paying $2 \%$ interest compounded daily, find the future value of the money and the compound interest he earned at the end of 5 years.
And find the effective rates:
(i) $-6 \%$ compounded quarterly.

$$
\begin{aligned}
& \text { (ii) }-9 \%=
\end{aligned} \quad \text { semiannually. }
$$

3- The owner of Dana pharmacy plans to remodel his store in 3 years. he estimates that she will need $15000 \$$ for the project. how much should he set aside at 4\% interest compounded quarterly to have $15000 \$$ in 3 years.
4- Mrs Young wants to set aside some money for her daughter's college education. She estimates that in 15 years, her 3 -years old daughter's college costs will be about $60000 \$$. She can obtain an interest rate of 6.5 \% compounded semiannually. How much money should invest now.

5- Compute the accumulated value of an annuity of $100 \$$ invested at the end of each month for one year at an annual rate of $6 \%$.
6- Suppose that Ahmed deposits $1000 \$$ every 3 months in a saving account which pays $8 \%$ annual interest compounded quarterly. How much will Ahmed have in the bank at the end of 5 years?
7- XYZ Savings bank pays interest at the rate of $4 \%$ annually compounded quarterly. How much money will Suzan have in the bank at the end of 5 years if she deposits $250 \$$ at the end of each quarter?
8 - Find the yearly depreciation of a loan tractor costing $1800 \$$. It has an expected lifetime of 5 years and a scrap value of $200 \$$. using the straight -line depreciation method.
9- How long it will take until your 50000 \$ saving grow to 100000 \$ at $3 \%$ ?

10-Suppose that Ahmed deposits 2000 \$ every 6 months in a saving account which pays $12 \%$ annual interest compounded half-yearly. How much will Ahmed have in the bank at the end of 8 years. Assuming that Ahmed deposits his money at the beginning of each 6-month period.

11-Ahmed purchases a life insurance policy which has an annual premium of $1500 \$$ due at the beginning of the year. if he elects to pay his premium in quarterly installments, how much should he pay at the beginning of each quarter if the interest rate is $10 \%$ compounded quarterly.

12-Mr. Smith would like to receive 4000 \$ each quarter for 10 years after he retires. How much money (to nearest dollar) does he have to save in a money market fund which pays at the rate of $8 \%$ compounded quarterly.

13-When Suzan's husband died, she became the beneficiary of a 100000 \$ life insurance policy. Instead of taking the money in a lump sum, she elects to receive a monthly stipend over a period of 20 years. If the insurance company pays interest at the rate of $6 \%$ compounded monthly, what will her monthly income be?

14- Mark must choose between receiving $50000 \$$ five years from now or taking a lump sum payment of $30000 \$$ today. He assumes he can earn $10 \%$ compounded annually if he invests $30000 \$$. What is the present value of the $50000 \$$ future payment?

15-You can afford monthly payments of $250 \$$ into an ordinary account that pays $3.5 \%$ compounded monthly. How long will it be until the account contains 10000 \$ (round to the next-higher month if not exact).

16- A file cabinet costs $700 \$$. If it has an expected lifetime of 10 years, find the yearly depreciation using the straight-line method. The scrap value is 100 \$.

17- Mark buys an Audi for 200000 \$. If the car depreciates by $10 \%$ per year, what is the value of the car in 10 years?

