Cihan University-Sulaymaniyah

Operation Research

Department of :

Business Administration &Accounting

Stage-TWO

Prof.Dr. Obaid Mahmmood Mohsin

Academic Year: 2023/2024

Department of … *Business Administration &Accounting*

University of …Cihan University

Subject: Operation Research

Course Book – Year 2,

Lecturer's name –Prof .Dr. Obaid Mahmmood Mohsin

Academic Year: 2023/2024

Course Book

1. Course name	**Operation Research**
2. Lecturer in charge	Prof.Dr.Obaid Mahmood Mohsin
3. Department/ College	Business Administration & Accounting
4. Contact	e-mail:obed.muhsin@sulicihan.edu.krd Tel:
5. Time (in hours) per week	2 hours per week
6. Office hours	3 hours morning
7. Course code	BAD32208
8. Teacher's academic profile	e.g Webpage, Blog, Moodle… or few paragraphs about not less than 100 words
9. Keywords	

10- Description

Concepts covered in this course Network Analysis, Critical Path Method (C.P.M.) (PERT) Computations, Decision Theory, Decision Tree, Game Theory, Inventory Models.

11- **Objective**

The main objective of this course is to give student a good theoretical and practical knowledge of operations methods. The student will take courses from a variety of technique that focus extensively on statistical methodology, mathematical modeling, and computer implementation issues.
The student will be able to solve and interpret correctly the solutions of a problems and recognize the situations where OR techniques can be used as decision making tools and to interpret correctly the conclusions which can be derived using these techniques.
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12-Learning Outcomes

At the end of this course the student will:-
1. understand and be able to construct CPM/PERT techniques to a set of activities.
2. learn how to deal with the problems arising due decision making under certainty , under uncertainty and under risk.
3. learn how to deal with the game theory.
4. understand and be able to deal with the inventory models.

Teaching and Learning Strategy

Contact hours: 2 theoretical weekly hours + Assignments

13- Assessment Method

The 100 marks will be divided into

The midterm examination	30%
Activities	10%
Final examination	60%

14-Text Books and References

1. **Text books:**

Hamdy, A.Taha; Operations Research an Introduction, 8th Edition, Pearson Education Inc., 2007.

2. **Reference books:**

Gupta, P. K. & D. S. Hira; Operations Research, 2nd Edition,S. Chand & Company (Pvt) Ltd, Ram Nagar, New Delhi, 1987.

15- The Syllabus

Chapter 1: Basics of Operations Research :-

#Introduction----#-Some Definitions of Operations Research

#-How Operations Research works---#--Some Technique of Operations Research

CH-2-Transportation Problem

1-Introduction --2-Transportation Method

3-Methods for finding Initial Solution—4- North-west Corner Method (NWCM)

5- Least Cost Method (LCM)---6- Vogel's Approximation Method (VAM)

7- Test for Optimality Method (MODI) 8-Examples 9-Problems

CH-3 – Assignment Problem

1-Introduction---2- The Hungarian Method ----3- Examples ---4- Problems

Chapter 4: Network Analysis:-

#1-Definitions , Activity, Event, Path, Dummy.#2-Arrow (network) Diagram Representation

3-Critical Path Method (C.P.M.)-

4-Network Analysis, CPM Computations.

5-Critical Path Method (C.P.M.) 6-Critical Path Calculations.

7-Program's Evaluation and Review Technique -(PERT) Computations.

8-PERT Computations---9-Cost Consideration in Project Scheduling

10-Determination of the Floats---11-Practice problem .

18. Practical Topics (If there is any)

19. Examinations:

-*Define -calculate -Explain -Test-Find-construct*

20. Extra notes:

Here the lecturer shall write any note or comment that is not covered in this template and he/she wishes to enrich the course book with his/her valuable remarks.

21. Peer review

This course book has to be reviewed and signed by a peer. The peer approves the contents of your course book by writing few sentences in this section.

(A peer is person who has enough knowledge about the subject you are teaching, he/she has to be a professor, assistant professor a lecturer or an expert in the field of your subject).

Operation Research

Chapter One

Basics of Operation Research

1-Introduction

2- Some Definitions of Operation Research

3- Methodology of Operation Research

4-Features of Operation Research

5-Applications and Scope of Operation Research

2023-2024

Prof.Dr. Obaid Mahmmood Mohsin

<u>Definitions:</u>

1- *Operation Research*: Is the systematic application of quantitative methods , techniques and tools to the analysis of problems involving the operation of systems.
2- *Operation Research*: In the most general sense , can be characterized as the application of scientific methods ,techniques and tools ,to problems involving the operation of systems so as to provide those in control of the operations with optimum solutions to the problems.
3- *Operation Research*: May be described as a scientific approach to decision-making that involves the operations of organizational system.

<u>Methodology of Operation Research</u>:

Every OR specialist my have his own way of solving problems. However ,for effective use of OR techniques ,it is essential to follow some steps that everybody agrees as being helpful in planning, organizing ,directing and controlling OR activities within an organization. A few such steps listed below:

Step-1: Analysis of the system and problem formulation : The analysis being by detailed observation of the organizational structure ,climate , communication and control system , the objective (or measure of effectiveness) and expectations of the organization. Such type of information will help in assessing the difficulty of the study in terms of costs ,time requirement, resource requirement , probability of success of the study and so on .

Step-2: Constructing a Mathematical Model: There are certain basic components which are required in every decision problem model.
Controllable (Decision) Variables: These are the issues or factors in the problem whose values are to be determined(in the form of numerical values) by solving the model .The possible values assigned to these variables are called decision alternatives (strategies or courses of action

).For example: in queuing theory , the number of service facilities is decision variable.

Uncontrollable Variables: They are the factors whose numerical value depends upon the external environment prevailing in the organization . The values of these variables are not under the control of the decision-maker and are also termed as state of nature .

*The Objective Function :*It is a representation of : (a) the criterion that express the decision-maker ,s manner of evaluating the desirability of alternative values of the decision variables, and (b) how that criterion is to be optimized (minimized or maximized) . For example ,in queuing theory the decision –maker may consider several criteria such as minimizing the average waiting time of customers , or average number of customers in the system at any time .

Constraints (or Limitations) : These are the restrictions on the values of the decision variables. These restrictions can arise due to limited resources such as space, money , manpower , material ,etc. The constraints may be in the form of equations or inequalities.

*Functional Relationships :*In a decision problem , the decision variables in the objective function and in the constraints are connected by a specific functional relationship. A general decision problem model can be written as :

Optimize (Max or Min)Z =f(x)

Subject to the constraints

$g_i(x)\ \{\leq, =, \geq\};\ i=1, 2, \ldots, m$

Where x*=a vector of decision variables* $(x_1, x_2, \ldots, x_n)$

$f(x)$*= criterion or objective function to be optimized*

$g_i(x)$ *=ith constraint*

b_i*= fixed amount of the jth resource*

A decision problem model is referred to as a linear model if all functional relationships among decision variables $x_1, x_2, x_3, \ldots, x_n$ *as well as* $f(x)$ *and* $g(x)$ *are of a linear form . But if one or more of the relationships are non-linear, the model is said to be non-linear model .*

*Step-3 Solution of the Model :*Opening a solution of the problem depends upon the particular model involved .In general , three methods discussed . In all these methods , values of decision variables are obtained that optimize (maximize or minimize) the given objective function or measure of effectiveness . Such a solution is called the optimal solution to the problem

Step-4 "Validation of the Model:

Step-5:Implementing the Solution:

Step-6 :Establishing Control over the solution:

Features of Operation Research Solution:

A solution that works but is quite expensive compared to the potential savings from its application should not be considered successful . Also a solution that is well within the budget but which does not accomplish the objective is not successful either. A few requirements of a good solution are as under "

1- *Technically Appropriate*: The solution should work technically, meet the constraints and operate in the problem environment.
2- *Reliable* :The solution must be useful for a reasonable period of time under the conditions for which it was designed.
3- *Economically Viable*: The economic value should be more than what it costs to develop and it should be seen as a wise investment in hiring OR talent..
4- *Behaviorally Appropriate* : The solution should be behaviorally appropriate and must remain valid for a reasonable period of time within the organization.
5- *Applications and Scope of Operation Research:* Some of the industrial / government/business problems which can be analyzed by OR approach have been arranged functional area wise as follow :

Finance and Accounting:

-Dividend policies , investment and portfolio management, auditing, balance sheet , cash flow analysis.

- Break-even analysis ,capital budgeting , cost allocation and control ,financial planning

-Claim and complaint procedure ,public accounting

<u>Marketing:</u>

-Selection of product-mix , marketing planning.

-Advertising and media planning export planning

-Sales effort allocation and assignment.

<u>Purchasing ,procurement and Exploration:</u>

Optimal buying and reordering under price quantity discount

-Bidding policies

-Vendor analysis

-Transporting planning

-Replacement policies

<u>Production Management:</u>

-Facilities Planning

-Manufacturing

-Maintenance and project scheduling

<u>Personal Management</u>

<u>Techniques and General Management</u>

<u>Government:</u>

Operation Research

Chapter Two

Transportation Problem

1-Introduction

2-Transportation Method

3-Methods for finding Initial Solution

- *North-west Corner Method (NWCM)*

- *Least Cost Method (LCM)*

- *Vogel's Approximation Method (VAM)*

4- Test for Optimality Method (MODI)

Examples -6
Problems -7

2023-2024

Prof.Dr. Obaid Mahmmood Mohsin

Introduction:

The Transportation problem deals with a situation in which a single product is to be transported from several sources (also called origin, supply or capacity centers) to several sinks (also called destination, demand or requirement centers) in general, let there be m sources $S_1, S_2, \ldots, S_m$ having a_i (i = 1, 2, …, m) units of supplies or capacity respectively to be transported among n destinations. $D_1, D_2, \ldots, D_n$ with b_j (j = 1, 2, …, n) units of requirements, respectively. Let C_{ij} be the cost of shipping one unit of the commodity from source i to destination j for each route. If X_{ij} represents the number of units shipped per route from source i to destination j, then the problem is to determine the transportation schedule so as to minimize the total transportation cost satisfying supply and demand conditions. Mathematically the problem may be stated as follows:

Minimize (total cost) $Z = \sum_{i=1}^{m} \sum_{j=1}^{n} \text{Cij Xij}$

Subject to the constraints:

$$\sum_{j=1}^{n} \text{Xij} = \text{ai} \qquad \text{i} = 1, 2, \ldots, \text{m} \;\; \text{(supply constraints)}$$

$$\sum_{i=1}^{m} \text{Xij} = bi\,,\;\; j = 1, 2, \ldots., n \;\; (demand\ constraints)$$

And $\qquad X_{ij} \geq 0$ for all i and j

Remark 1: A necessary and sufficient condition for the existence of a feasible solution to the transportation problem is:

Total supply = Total demand

$$\sum_{i=1}^{m} \text{ai} = \sum_{j=1}^{n} \text{bj} \;\; \text{(also called rim condition)}$$

2- when the total capacity equals requirement, the problem is called balanced transportation problem, otherwise it is known as unbalanced transportation problem.

3- the allocated cells in the transportation table having positive allocation are called occupied cells and empty cells are known as non-occupied cells.

4- basic feasible solution: the number of positive allocations (values of basic or decision variables) at any stage of feasible solution must be equal to the number (rows + columns -1) i. e. number of independent constraint equations, satisfying all the conditions.

5- when the number of positive allocations at any stage of the feasible solution is less than the required number (rows + columns -1), the solution is said to be degenerate solution, otherwise non-degenerate solution.

6- loops in transportation table: an ordered set of at least four cells in transportation table is said to form loop provided.

(i) Any two adjacent cells of the ordered set lie either in the same row or in same column.
(ii) An even number of at least four cells must participation a closed loop.
(iii) All cells that receive a plus or a minus sign, except the starting unoccupied cell, must be occupied cells. Each row and column in the loop should have only one plus and minus sign.
(iv) Closed loops may or not be square or rectangular in shape.

7- prohibited routes: when it is not possible to transport goods from certain sources to certain destinations, due to unfavorable weather conditions, road hazards, etc. the problem can be handled by assigning a very large cost say M or ∞ to each of the routes which are not available.

8- An unbalanced transportation problem is one in which the total supply of the factories (origins) and the total demand are not equal.

(i) When the total capacity of the origins exceeds the total requirement of destinations, a dummy destination is introduced in the transportation table which absorbs the excess capacity. The cost of shipping from each origin to this dummy destination is assumed to be zero. The insertion of a dummy destination establishes equality between the total origin capacities and total destination requirements. The problem is then amenable for solution by the transportation algorithm.

(ii) When total capacity of origins is less than the total requirement of destinations, a dummy origin is introduced in the transportation table to meet out the excess demand. The cost of shipping from the dummy origin to each destination is assumed to be zero. The introduction of a dummy origin in this case establishes the equality between the total capacity of origins and the total requirement of destinations. The problem is them amenable for solution by the transportation algorithm.

<u>The transportation method:</u>

The solution algorithm to a transportation problem may be summarized into the following steps:

Step1: formulate the problem and set up in the matrix form:

The formulation of the transportation problem is similar to the Lp problem formulation. Here the objective function is the total transportation cost and the constraints are the supply and demand available at each source and destination, respectively.

Step2: obtain an initial basic feasible solution:

In this chapter, we shall discuss following three different methods to obtain an initial solution:

(i) North west corner method (NWC)

(ii) Least cost method (LCM)

(iii) Vogel's Approximation (or penalty) method (VAM)

The initial solution by any of the three methods must satisfy the following conditions:

(i) The solution must be feasible, i. e. it must satisfy all the supply and demand constraints (also called rim conditions)

(ii) The number of positive allocation must be equal to m + n -1, when m is the number of rows and n is the number of columns.

Any solution that satisfies the above conditions is called non-degenerated basic feasible solution, otherwise, degenerate solution.

Step3: Test the initial solution for optimality:

We shall discuss the modified distribution (MODI) method to test the optimality of the solution obtained in step2. If the current solution is optimal, then stop otherwise, determine a new improved solution.

Step4: updating the solution: repeat step3 until an optimal solution is reached.

Methods for finding initial solution:

There are several methods available to obtain an initial basic feasible solution. Here we shall discuss only three different methods:

North-West Corner Method (NWCM):

It is a simple and efficient method to obtain an initial solution. This method does not take into account the cost of transportation on any route of transportation. The method can be summarized as follows:

Step1: start with the cell at the upper left (north-west) corner of the transportation matrix and allocated as much as possible equal to the minimum of the rim values for the first row and first column, i. e. min (a_1, b_1).

Step2: (a) if allocation made in step1 is equal to the supply available at first source (a_1 in first row), then move vertically down to the cell (2, 1) in the second row and first column and apply step1 again, for next allocation.

(b) if allocation made in step1 is equal to the demand of the first destination (b_1, in first column), then move horizontally to the cell (1, 2) in the first row and second column and apply step1 again for next allocation.

(c) if $a_1 = b_1$, allocate $x_{11} = a_1$ or b_1 and move diagonally to the cell (2, 2)

Step3: continue the procedure step by step till an allocation is made in the south-east corner cell of the transportation table.

Remark: if during the process of making allocation of a particular cell, supply equals demand, then next allocation of magnitude zero can be made in a cell either in the next row or column. This condition is known as degeneracy.

Example1:A company produces electrical transformers from three factions and distributes them to three regions in Iraq. The following table shows the production, demand and transformation costs between the factories (A, B, C) and regions (D, E, F)

Region / Factory	D	E	F	Production
A	10	14	8	20
B	12	10	12	30
C	8	12	10	40
Demand	40	30	20	90 / 90

Find:

(i) The initial solution by using (NWCM)
(ii) The total cost

Solution:

We must check $\Sigma d_i = \Sigma b_j$

$\Sigma a_i = 90$ and $\Sigma b_j = 90$

20 + 30 + 40 = 90

40 + 30 + 20 = 90

Region / Factory	D	E	F	Production
A	10 20	14	8	20
B	12 20	10 10	12	30
C	8	12 20	10 20	40
Demand	40	30	20	90 / 90

Number of basic variables = m + n -1 = 3 + 3 -1 = 5

$X_{11}, X_{21}, X_{22}, X_{32}, X_{33}$

$X_{11} = 20$, $X_{21} = 20$, $X_{22} = 10$, $X_{32} = 20$, $X_{33} = 20$

Number of non-basic variables = nm- (n + m -1) = (3) (3) – (3 + 3 -1) = 9 – 5 = 4

X_{12}, X_{13}, X_{23}, $X_{31} = 0$

Number of constraints = n + m = 6

Number of occupied cells = number of basic variables = m + n -1 = 5

Total cost = $\sum_{j=1}^{n} \sum_{i=1}^{m}$ Cij Xij

= (20) (10) + (20) (12) + (10) (10) + (12) (20) + (20) (10)

= 200 + 240 + 100 + 240 + 200 = **980**

Model formulation:

Let x_{ij} = number of the product (electrical transformers) to be transported from factory i (i = 1, 2, 3) (A, B, C) to region j (j= 1, 2, 3) (D, E, F).

The transportation problem is stated as an LP model as follows:

Minimize Z = $10X_{11} + 14X_{12} + 8X_{13} + 12X_{21} + 10X_{32} + 12X_{23} + 8X_{31} + 12X_{32} + 10X_{33}$

subject to the constraints (s. t.)

(i) Capacity constraints

$X_{11} + X_{12} + X_{13} + = 20$

$X_{21} + X_{22} + X_{23} = 30$

$X_{31} + X_{32} + X_{33} = 40$

(ii) Requirement constraints

X11 + X21 + X31 + = 40

X12 + X22 + X32 = 30

X15 + X23 + X33 = 20

And $X_{ij} \geq 0$

There are m * n = 9 decision variables, X_{ij} , i, j = 1, 2, 3

And m+ n = 6 constraints

Example2: A company has three production facilities S_1, S_2 and S_3 with production capacity of 7, 9, and 18 units per week of a product, respectively. These units are to be shipped to four warehouses D_1, D_2, D_3 and D_4 with requirement of 5, 6, 7 and 14 units per week respectively. The transportation costs per unit between factories to warehouses are given in the table below:

	D_1	D_2	D_3	D_4	capacity
S_1	19	30	50	10	7
S_2	70	30	40	60	9
S_3	40	8	70	20	18
Demand	5	8	7	14	34

Find: 1-The initial solution using (NWCM) 2-The total cost

Solution:

We check if Σbi = Σai

7 + 9 + 18 = 34 , 5+ 8 + 7 + 14 = 34

	D_1	D_2	D_3	D_4	capacity
S_1	19 5	30 2	50	10	7
S_2	70	30 6	40 3	60	9
S_3	40	8	70 4	20 14	18
Demand	5	8	7	14	34

No. of basic variables = m + n -1 = 3 + 4 -1 = 6

$X_{11} = 5$, $X_{12} = 2$, $X_{22} = 6$, $X_{23} = 3$, $X_{33} = 4$, $X_{34} = 14$

No. of non-basic variables = nm – (n + m-1) = (3) (4) – (3 + 4 – 1) = 12 – 6 = 6

X_{13}, X_{14}, X_{21}, X_{24}, X_{31}, $X_{32} = 0$

No. of constraints = n + m = 3 + 4 = 7

No. of occupied cells = no. of basic variables = m + n – 1 = 6

Total cost $=\Sigma\Sigma\ C_{ij}\ X_{ij}$

= (5) (19) + (2) (30) + (6) (30) + (3) (40) + (4) (70) + (14) (20)

= 95 + 60 + 180 + 120 + 280 + 280 = 1015

Least Cost Method (LCM):

Since the objective is to minimize the total transportation cost, we must try to transport as much as possible through those routes (cells) where the unit transportation cost is lowest. This method takes into account the minimum unit cost of transportation for obtaining initial solution and can be summarized as follows:

Step1: select the cell with the lowest unit cost in the entire transportation table and allocate as much as possible to this cell and eliminate (lineout) that row or column in which either supply or demand is exhausted. If both a row and a column are satisfied simultaneously, only one may be crossed out. In case the smallest unit cost cell is not unique, then select the cell where maximum allocation can be made.

Step2: after adjusting the supply and demand for all uncrossed out rows and columns repeat the procedure with the next lowest unit cost among the remaining rows and columns of the transportation table and allocate as much as possible to this cell and eliminate (lineout) that row and column in which either supply or demand is exhausted.

Step3: repeat the procedure until the entire available supply at various sources and demand at various destination is satisfied. The solution so obtained need not be non-degenerate.

Example3:

Adairy firm has three plants located in a state. The daily milk production out each plant is as follows:

Plant:	1	2	3
Milk supply:	6	1	10

Each day, the firm must fulfil the needs of its four distribution centers. Minimum requirement at each center i as follows:

Center:	1	2	3	4
Milk supply:	7	5	3	2

Cost of shipping one million liter from each plant to each distribution center is given the following table:

Distribution center

plant		D_1	D_2	D_3	D_4	
	P_1	2	3	11	7	6
	P_2	1	0	6	1	1
	P_3	5	8	15	9	10
		7	5	3	2	17 / 17

Find initial basic feasible solution for given problem by using least cost method (LCM) and find the total cost.

Solution:

plant	D_1	D_2	D_3	D_4	
P_1	2 6	3	11	7	6
P_2	1	0 1	6	1	1
P_3	5 1	8 4	15 3	9 2	10
	7	5	3	2	17

(i) The lowest unit cost in table is (0) in cell (P_2, D_2) therefore maximum possible allocation which can be made here is (1). This exhausts the supply at plant (P_2), therefore row (2) is crossed out.

(ii) The next lowest unit cost is (2) in cell (P_1, D_1). The maximum possible allocation which can be made here is (6). This exhausts the supply at plant P_1, therefore row P_1 is crossed out.

(iii) Since the total supply at plant (P_3) is now equal to the unsatisfied demand at all the four distribution centers, therefore, maximum possible allocations satisfying the supply and demand conditions are made in cells (P_3, D_1), (P_3, D_2), (P_3, D_3) and (P_3, D_4).

The number of allocated cells in this case are six which is equal to the required number m + n-1 (3 + 4 – 1) = 6. Thus, this solution is non-degenerate.

The transportation cost associated with this solution is:

Total cost = (2) (6) + (0) (1) + (5) (1) + (8) (4) + (15) (3) + (9) (2)

= 12 + 0 + 5 + 32 + 45 + 18 = 112

Example4:

Solve the last example by using (NWCM) and find the total cost

Solution:

	D_1	D_2	D_3	D_4	a_i
P_1	2 6	3	11	7	6
P_2	1 1	0	6	1	1
P_3	5	8 5	15 3	9 2	10
bj	7	5	3	2	

(i) Comparing a_i and b_i, since $a_i < b_i$, allocate $X_{11} = 6$. This exhausts the supply at p_1 and leaves 1 unit as unsatisfied demand at D_1

(ii) Moe to cell (P_2, D_1) compare a_2 and b_1 (i: e 1 and 1). Since $a_2 = b_1$, allocate $X_{21} = 1$

(iii) Move to cell (P_3, D_2) since supply at P_3 is equal to the demand at D_2, D_3 and D_4 therefore, allocate $X_{32} = 5$, $X_{33} = 3$ and $X_{34} = 2$.

It may be noted that the number of allocated cells (also called basic cells) are (5) which is one less than the required number m + n – 1 (3 + 4 – 1) = 6. Thus, this solution is the degenerate solution.

The transportation cost associated with this solution is:

Total cost = (2) (6) + (1) (1) + (8) (5) + (15) (3) + (9) (2)

= 12 + 1 + 40 + 45 + 18 = 116

If allocated cells = basic variable = n + m – 1

$\Rightarrow$ Non-degenerate solution

If $\neq n + m - 1$

$\Rightarrow$ degenerate solution

<u>Vogel's Approximation Method (VAM)</u>

Vogel's Approximation (penalty or regret) method is a heuristic method and is preferred to the other two methods described above. In this method, each allocation is made on the basic of the opportunity (or penalty or extra) cost that would have been incurred if allocations in certain cells with minimum unit transportation cost were missed. In this method allocation made to that the penalty cost is minimized. The advantage of this is that it gives an initial solution which nearer to an optimal solution or is the optimal solution itself. The steps in (VAM) are as follows:

Step1: calculate penalties for each row (column) by taking the difference between the smallest and next smallest unit transportation cost in the same row (column). This difference indicates the penalty or extra cost which has to be paid if one fails to allocate to the cell with the minimum unit transportation cost.

Step2: select the row or column with the largest penalty and allocate as much as possible in the cell having the least cost in the selected row column satisfying the rim conditions. If there is a tie in the values of penalties, it can be broken by selecting the cell where maximum allocation can be made.

Step3: adjust the supply and demand and cross out the satisfied row or column. If a row and a column are satisfied simultaneously, only one of them is crossed out and remaining row (column) is assigned a zero supply (demand). Any row or column with zero supply or demand should not be used in computing future penalties.

Step4: Repeat step1to 3 until the entire available supply at various sources and demand at various destinations are satisfied.

Step5: find the initial feasible solution by using (VAM) and find the total cost. For the following

	D_1	D_2	D_3	D_4	Supply	Row penalty
S_1	20	22 40	17	4 80	120	13 13 - -
S_2	24 10	37	9 30	7 30	70	2 2 2 17
S_3	32 50	37	20	15	50	5 5 5 17
Demand	60	40	30	110	240 240	
Column penalty	4 4 8 8	15 - - -	8 8 11 -	3 3 8 8		

The number of allocated cells in table above are six, which is equal to the required number $m + n - 1 = 3 + 4 - 1 = 6$, therefore, this solution is non-degenerate.

The transportation cost associated with this solution:

Total cost = (22) (40) + (4) (80) + (24) (10) + (9) (30) + (7) (30) + (32) (50)

880 + 320 + 240 + 270 + 210 + 1600 = 3520

Example6:

A company has factories F_1, F_2 and F_3 which supply to warehouses at W_1, W_2 and W_3 weekly factory capacities are 200, 160 and 90 units, respectively. Weekly warehouse requirement are 180, 120 and 150 units respectively. Unit shipping costs are as follows:

Warehouse / factory	W_1	W_2	W_3	supply
F_1	16	20	12	200
F_2	14	8	18	160
F_3	26	24	16	90
Demand	180	120	150	450 450

Find:

(i) Initial feasible solution by using
a- North-west corner method (NWCM)

b- Lest cost method (LCM)

c- Vogel's Approximation method (VAM)

(ii) The type of solution for each

(iii) The total cost for each method

Solution:

a- (NWCM)

Warehouse / factory	W_1	W_2	W_3	supply
F_1	16 180	20 20	12	200
F_2	14	8 100	18 60	160
F_3	26	24	16 90	90
Demand	180	120	150	

Number of basic variables = number of occupied cells

= m + n -1 = 3 + + - 1 $\Longrightarrow$ The solution is non-degenerate

Total cost = (16) (180) + (20) (20) + (8) (100) + (18) (60) + (16) (90)

= 2880 + 400 + 800 + 1080 + 1440 = 6600

b- (LCM)

Warehouse / factory	W_1	W_2	W_3	supply
F_1	16 50	20	12 150	200
F_2	14 40	8 120	18	160
F_3	26 90	24	16	90
Demand	180	120	150	

Number of basic variables = m + n – 1 = 3 + 3 – 1 = 5

It is non-degenerate solution

Total cost = (16) (50) + (12) (150) + (14) (40) + (8) (120) + (26) (90)

= 800 + 1800 + 560 + 960 + 2340 = 6460

c- (VAM)

	W_1	W_2	W_3	supply	Row penalty
F_1	16 140	20	12 60	200	4 4 4
F_2	14 40	8 120	18	160	6 4 4
F_3	26	24	16 90	90	8 10 -
Demand	180	120	150		
Column panalty	2 2 2	12 - -	4 4 6		

Number of basic variables = m + n – 1 = 3 + 3 – 1 = 5

Non-degenerate solution

Total cost = (16) (140) + (12) (60) + (14) (40) + (8) (120) + (16) (90)

= 2240 + 720 + 560 + 960 + 1440 = 5920

<u>Test For Optimality: MODI Method</u> Step1: for an initial basic feasible solution with (m + n – 1) occupied cells, calculate u_i and v_j for rows and columns. The initial solution can be obtained by any of the three method discussed earlier.

To star with any one of u_i's or v_js is assigned the value zero. It is better to assign zero for a particular u_i or v_j where there are maximum number of allocations a row or column respectively as it will reduce arithmetic work considerably. Then complete the calculation of u_i's and v_js for other rows and columns by using the relation

$$C_{ij} = u_i + v_j \quad \text{for all occupied cells (i, j)}$$

Step2: for unoccupied cells, calculate opportunity cost by using the relationship

$$d_{ij} = C_{ij} – (u_i + v_j) \text{ for all i and j}$$

Step3: Examine sign of each d_{ij}

(i) If $d_{ij} > 0$, then current basic feasible solution optimal.

(ii) If $d_{ij} = 0$, then current basic feasible solution will remain unaffected but an alternative solution exists.

(iii) If one or more $d_{ij} < 0$, then an improved solution can be obtained by entering unoccupied cell (i, j) in the basis. An unoccupied cell having the largest negative q

Step4: construct a closed path (or loop) for the unoccupied cell with largest negative opportunity cost, start the closed path with the selected unoccupied cell and mark a plus sign (+) in this cell, trace a path along the rows (or columns) to an occupied cell, mark the corner with minus sign (-) and continue down the column (or row) to an occupied cell and mark the corner with plus sign (+) and minus sign (-) alternatively. Close the path back to the selected unoccupied cell.

Step5: select the smallest quantity among the cells marked with minus sign on the corner of closed loop. Allocate this value to the selected unoccupied cell and add it to other occupied cells marked with plus sign and subtract it from the occupied cells marked with minus signs.

Step6: obtain a new improved solution by allocating units to the unoccupied cells according to step5 and calculate the new total transportation cost.

Step7: test the revised solution further for optimality. The procedure terminates when all $d_{ij} \geq 0$ for unoccupied cells.

Remark: the loop starts ends at the selected unoccupied cell. It consists of successive horizontal and vertical (connected) lines whose end points must be occupied cells, except for a end point associated with entering unoccupied cell. This means that every corner element of the loop must be an occupied cell. It is immaterial whether the loop is traced in a clockwise or anti-clockwise direction. However, for a given solution only one loop can be constructed for each unoccupied cell.

Example7: For the following initial basic feasible solution by using (NWCM) find the optimal solution

	D_1	D_2	D_3	a_i	
S_1	5 9	1 3	8	12	$u_1 = 0$
S_2	2	4 7	0 7	14	$u_2 = 3$
S_3	3	6	7 4	4	$u_3 = 10$
B_j	9	10	11		
	$v_1 = 5$	$v_2 = 1$	$v_3 = -3$		

Basic variables $\Rightarrow X_{11}, X_{12}, X_{22}, X_{23}, X_{33}$

9 3 7 7 4

Non-basic variables $\Rightarrow X_{13}, X_{21}, X_{31}, X_{32} = 0$

Let u_1 (i = 1, 2, 3) for rows

v_1 (j = 1, 2, 3) for column

$c_{ij} = u_i + v_j$ let $u_1 = 0$

$c_{11} = u_1 + v_1$ $5 = 0 + v_1 \Rightarrow v_1 = 5$

$c_{12} = u_1 + v_2$ $1 = 0 + v_2 \Rightarrow v_2 = 1$

$c_{22} = u_2 + v_2$ $4 = u_2 + 1 \Rightarrow u_2 = 3$

$c_{23} = u_2 + v_3$ $0 = 3 + v_3 \Rightarrow v_3 = -3$

$c_{33} = u_3 + v_3$ $7 = u_3 + (-3) \Rightarrow u_3 = 10$

given table cost

5	1	8
2	4	0
3	6	7

Extracted table cost

5	1	-3
8	4	0
15	11	7

$C_{ij} = u_i + v_j$

Estimated table cost

0	0	11
-6	0	0
-12	-5	0

4

$\hat{C}_{ij} = C_{ij} - (u_i + v_j)$

We have values with sign (-) in the estimated cost then the solution is not optimal.

The entering variable $\longrightarrow$ X_{31} because $\hat{C}_{31} = -12$ the largest to determine the leaving varible

$$\begin{array}{ccccccc} + & - & + & - & + & - & \\ X_{31} \rightarrow & X_{11} \rightarrow & X_{12} \rightarrow & X_{22} \rightarrow & X_{23} \rightarrow & X_{33} \rightarrow & X_{31} \\ & 9 & & 7 & & & \end{array}$$

X_{33} is the leaving variable

	D_1	D_2	D_3	a_i	
S_1	5 5	1 7	8	12	$u_1 = 0$
S_2	2	4 3	0 11	14	$u_2 = 3$
S_3	3 4	6	7	4	$u_3 = -$
b_j	9	10	11		
	$v_1 = 5$	$v_2 = 1$	$v_3 = -3$		

Min (z) = (5) (5) + (1) (7) + (3) (4) + (11) (0) + (3) (4) = 56

$C_{ij} = u_i + v_j$ $\qquad u_1 = 0$

$C_{11} = u_1 + v_1 \Rightarrow 5 = 0 + v_1 \Rightarrow v_1 = 5$

$C_{12} = u_1 + v_2 \Rightarrow 1 = 0 + v_2 \Rightarrow v = 1$

$C_{22} = u_2 + v_2 \Rightarrow 4 = u_2 + 1 \Rightarrow u_2 = 3$

$C_{23} = u_2 + v_3 \Rightarrow 0 = 3 + v_3 \Rightarrow v_3 = 3$

$C_{31} = u_3 + v_1 \Rightarrow 3 = u_3 + 5 \Rightarrow u_3 = -2$

5	1	8
2	4	0
3	6	7

5	1	-3
8	4	0
3	-1	-5

$C_{ij} = u_i + v_j$

0	0	11
-6	0	0
0	7	12

$\hat{C}_{ij} = C_{ij} - (u_i + v_j)$

The solution is not optimal

X_{21} the intering variable

$$\overset{+}{X_{21}} \rightarrow \underset{5}{\overset{-}{X_{11}}} \rightarrow \overset{+}{X_{12}} \rightarrow \underset{3}{\overset{-}{X_{22}}} \rightarrow \overset{+}{X_{21}}$$

3 → out

	D_1	D_2	D_3	a_i
S_1	5 2	1 10	8	12
S_2	2 3	4	0 11	14
S_3	3 4	6	7	4
b_j	9	10	11	

Min Z = (5) (2) + (2) (3) + (3) (4) + (1) (10) + (0) (11) = 38

$C_{11} = u_1 + v_1 \Longrightarrow 5 = 0 + v_1 \Longrightarrow v_1 = 5$

$C_{12} = u_1 + v_2 \Longrightarrow 1 = 0 + v_2 \Longrightarrow v_2 = 1$

$C_{21} = u_2 + v_1 \Longrightarrow 2 = u_2 + 5 \Longrightarrow u_2 = -3$

$C_{23} = u_2 + v_3 \Longrightarrow 0 = -3 + v_3 \Longrightarrow v_3 = 3$

$C_{31} = u_3 + v_1 \Longrightarrow 3 = u_3 + 5 \Longrightarrow u_3 = -2$

5	1	8
2	4	0
3	6	7

5	1	3
2	-2	0
3	-1	1

$C_{ij} = u_i + v$

0	0	5
0	6	0
0	7	6

$\hat{C}_{ij} = C_{ij} - (u_i + v_j)$

All $\hat{C}_{ij} \geq 0$ ---So the solution is optimal

Example8:

Obtain an optimal solution to the transportation problem by (MODI) method

	D_1	D_2	D_3	D_4	Supply
S_1	19	30	50	10	7
S_2	70	30	40	60	9
S_3	40	8	70	20	18
Demand	5	8	7	14	34

Solution:

Applying (VAM) to obtain an initial basic feasible solution. This solution be:

	D_1	D_2	D_3	D_4	Supply
S_1	19 5	30	50	10 2	7
S_2	70	30	40 7	60 2	9
S_3	40	8 8	70	20 10	18
Demand	5	8	7	14	34

The number of occupied cells are m + n – 1 = 3 + 4 – 1 = 6 and initial solution is non-degenerate. Thus, an optimal solution can be obtained.

Total cost = min z = (5) (19) + (2) (10) + (7) (40) + (2) (60) + (8) (8) + (20) (10) = 779

In order to calculate the values of u_i's (i = 1, 2, 3) and v_j's (j = 1, 2, 3, 4) for each occupied cell, we arbitrarily assign $v_4 = 0$ to simplify calculations. Given

$V_4 = 0$, u_1, u_2 and u_3 can be computed immediately by using the relation $C_{ij} = u_i + v_j$ for occupied cells as shown below:

$C_{34} = u_3 + v_4 \Longrightarrow 20 = u_3 + 0 \Longrightarrow u_3 = 20$

$C_{24} = u_2 + v_4 \Longrightarrow 60 = u_2 + 0 \Longrightarrow u_2 = 60$

$C_{14} = u_1 + v_4 \Longrightarrow 10 = u_1 + 0 \Longrightarrow u_1 = 10$

$C_{11} = u_1 + v_1 \Longrightarrow 19 = 10 + v_1 \Longrightarrow v_1 = 9$

$C_{23} = u_2 + v_3 \Longrightarrow 40 = 60 + v_3 \Longrightarrow v_3 = -20$

$C_{32} = u_3 + v_2 \Longrightarrow 8 = 20 + v_2 \Longrightarrow v_2 = -12$

19	30	50	10
70	30	40	60
40	8	70	20

19	-2	-10	10
69	48	40	60
29	8	0	20

$C_{ij} = u_i + v_j$

0	32	60	0
1	-18	0	0
11	0	70	0

$\hat{C}_{ij} = C_{ij} - (u_i + v_j)$

- According to the optimality criterion for cost minimizing transportation problem, the current solution is not optimal, since the opportunity costs of the occupied cells are not all zero or positive. The value of d_{22} = -18 in cell (S_2, D_2) is indicating that the total transportation cost can be reduced in the multiple of by shifting an allocation to this cell.
- A closed loop (path) is traced along row S_2 to an occupied cell (S_3, D_3). A plus sign is placed in cell (S_2, D_2) and minus sign in cell (S_3, D_2). Now take a right angle and locate an occupied cell in column D_4. An occupied cell (S_3, D_4) exists at row S_3, and a plus sign is paced in this cell. Continuing the process and complete the closed path. The occupied cell (S_2, D_2) must be by passed otherwise it will violate the rules of constructing closed path.
- In order to maintain feasibility, examine the occupied cells with minus sign at the corners of closed loop, and select the one that has the smallest allocation. This determines the maximum number of units that can be shifted along the closed path. The minus signs are in cells (S_3, D_2) and (S_2, D_4). Cell (S_2, D_4) is selected because it has the smaller allocation, i. e 2. The value of this allocation is then added to cell (S_2, D_2) and (S_3, D_4) which carry plus

sign. The same value is subtracted from cells (S_2, D_4) and (S_3, D_2) because they carry minus signs -The revised solution is shown in the following table.

	D_1	D_2	D_3	D_4	Supply	u_i
S_1	19 5	30 +32	50 +42	10 2	7	
S_2	70 +19	30 2	40 7	60 +14	9	
S_3	40 +11	8 6	70 +52	20 12	18	
Demand	5	8	7	14		
v_j						

Min z = (19) (5) + (10) (2) + (30) (2) + (40) (7) + (8) (6) + (20) (12) +

= 95 + 20 + 60 + 280 + 48 + 240 = 743

- Test the optimality of the revised once again in the same way as discussed in earlier step.

The value of u_i s, v_js and d_{ij}s are shown in the following table:

	D_1	D_2	D_3	D_4	Supply	u_i
S_1	19 5	30 +32	50 +42	10 2	7	$u_1 = 0$
S_2	70 +19	30 2	40 7	60 +14	9	$U_2 = 32$
S_3	40 +11	8 6	70 +52	20 12	18	$U_3 = 10$
Demand	5	8	7	14	34	
v_j	$V_1 = 19$	$V_2 = -2$	$V_3 = 8$	$V_4 = 10$		

Since each of d_{ij}s is positive, the current basic feasible solution is optimal with minimum total transportation cost Min z = 743

Example9 The following table shows all the necessary information on the availability of supply to each warehouse, the requirement of each market and unit transportation cost from each warehouse to each market

Market / Warehouse	P	Q	R	S	supply
A	6	3	5	4	22
B	5	9	7	7	15
C	5	7	6	6	8
Demand	7	12	17	9	45

Find:

1- The initial feasible solution

(i) North-west corner method (NWCM)

(ii) Least cost method (LCM)

(iii) Vogel's Approximation method (VAM)

2- Is the solution non-degenerate or degenerate?

3- The total cost

4- Optimal solution and total co

Solution:

(i) (NWCM)

	P	Q	R	S	supply
A	6 7	3 12	5 3	4	22
B	5	9	2 14	7 1	15
C	5	7	8	6 8	8
Demand	7	12	17	9	45

Number of basic variables = m + n – 1 = 3 + 4 – 1 = 6

⇨ The solution is non-degenerate

Total cost = (6) (7) + (3) (12) + (5) (3) + (2) (14) + (7) (1) + (6) (8)

= 42 + 36 + 15 + 28 + 7 + 48 = 176

(ii) (LCM)

	P	Q	R	S	supply
A	6	3 12	5 1	4 9	22
B	5	9	2 15	7	15
C	5 7	7	8 1	6	8
Demand	7	12	17	9	45

Number of basic variable = m + n – 1 = 3 + 4 – 1 = 6

⇨ The solution is non-degenerate

Total cost = 36 + 5 + 36 + 30 + 35 + 8 = 150

(iii) (VAM)

	P	Q	R	S	supply	
A	6	3 12	5 2	4 8	22	1 1 1 2
B	5	9	2 15	7	15	3 4 - -
C	5 7	7	8	6 1	8	3 3 - -
Demand	7	12	17	9	45	1 1 1 1
	0 0 1 1	4 - - -	3 3 3 -	2 2 2 2		

Number of basic variables = 3 + 4 – 1 = 6

⇒ The solution is non-degenerate

Total cost = (3) (12) + (5) (2) + (4) (8) + (2) (15) + (5) (7) + (6) (1)

= 36 + 10 + 32 + 30 + 35 + 6 = 149

To test optimality of solution we calculate the value of u_i s (i = 1, 2, 3) and v_j s (j = 1, 2, 3, 4) for each occupied cell. We arbitrarily assign $u_1 = 0$, and using the relation $c_{ij} = u_i + v_j$ for occupied cells as shown below:

$c_{ij} = u_i + v_j$ $\quad u_i = 0$

$c_{12} = u_1 + v_2 \rightarrow 3 = 0 + v_2 \rightarrow v_2 = 3$

$c_{13} = u_1 + v_3 \rightarrow 5 = 0 + v_3 \rightarrow v_3 = 5$

$c_{14} = u_1 + v_4 \rightarrow 4 = 0 + v_4 \rightarrow v_4 = 4$

$c_{23} = u_2 + v_3 \rightarrow 2 = u_2 + 0 \rightarrow u_2 = -3$

$c_{34} = u_3 + v_4 \rightarrow 6 = u_3 + 4 \rightarrow u_3 = 2$

$c_{31} = u_3 + v_1 \rightarrow 5 = 2 + v_1 \rightarrow v_1 = 3$

	P	Q	R	S	supply	u_i
A	6	3 12	5 2	4 8	22	$u_1 = 0$
B	5	9	2 15	7	15	$u_2 = -3$
C	5 7	7	8	6 1	8	$u_3 = 2$
Demand	7	12	17	9	45	
v_j	$v_1 = 3$	$v_2 = 3$	$v_3 = 5$	$v_4 = 4$		

Given table cost

6	3	5	4
5	9	2	7
5	7	8	6

Extracted table cost

6	3	5	4
5	9	2	7
5	7	8	6

$C_{ij} = u_i + v_j$

Estimated table cost

0	0	0	0
0	0	0	0
0	0	0	0

$\hat{C}_{ij} = C_{ij} – (u_i + v_j)$

All $\hat{C}_{ij} \geq 0$

So the solution is optimal and total cost = 149

Problems

1- ABC limited has three production shops supplying a product to five werehouses. The cost of production varies from shop to shop and cost transformation from one shop to a warehouse also varies. Each shop has a specific production capacity and each warehouse has certain amount of requirement. The costs transportation are given below:

Warehouse / shop	i	ii	iii	iiii	iiiii	supply
A	6	4	4	7	5	100
B	5	6	7	4	8	125
C	3	4	6	3	4	175
Demand	60	80	85	105	70	400 / 400

Find:

(i) The initial basic feasible solution by using

a- NWC , b-LCM , c- VAM

(ii) The total cost for each method

2- Determine an initial basic feasible solution to the following transportation problem by using NWCM, LCM and VAM

Distinction / source	D_1	D_2	D_3	D_4	supply
S_1	21	16	15	3	11
S_2	17	18	14	23	13
S_3	32	27	18	41	19
Demand	6	10	12	15	43 / 43

3- Determine an initial basic feasible solution to the following transportation problem by using LCM and VAM

Distinction / source	D_1	D_2	D_3	D_4	supply
S_1	1	2	1	4	30
S_2	3	3	2	1	50
S_3	4	2	5	9	20
Demand	20	40	30	10	100 / 100

4- Determine an initial basic feasible solution to the following transportation problem by using NWCM, LCM and VAM.

Distinction / source	D_1	D_2	D_3	D_4	supply
A	11	13	17	14	250
B	16	18	14	10	300
C	21	24	13	10	400
Demand	200	225	275	250	950 / 950

5- Determine an initial feasible solution to the following transportation problem by using NWCM

Distinction / source	D_1	D_2	D_3	D_4	supply
Q_1	6	4	1	5	14
Q_2	8	9	2	7	16
Q_3	4	3	6	2	5
Demand	6	10	15	4	35 / 35

6- A manufacture has distribution centers at Q_1, Q_2 and Q_3. These centers have availability of 40, 20 and 40 units of this product, respectively. His retail outlets A, B, C, D and E require 25, 10, 20, 30 and 15 units, respectively. The transport cost per unit between each center outlet is given below:

Retail outlet / Production center	A	B	C	D	E
Q_1	55	30	40	50	40
Q_2	35	30	100	45	60
Q_3	40	60	95	35	30

Determine the optimal distribution to minimize the cost of transportation.

7- A company has three plants and four warehouses. The supply and demand in units and the corresponding transportation costs are given. The table below has been taken from the solution procedure of transportation problem.

Warehouse / plants	i	ii	iii	iiii	supply
A	5	10	5 10	10	10
B	6 20	8	7	2 5	25
C	4 5	2 10	5 5	7	20
Demand	25	10	15	5	55 / 55

Answer the following questions, given brief reasons

(i) Is this solution feasible

(ii) Is this solution degenerate

(iii) Is this solution optimum

8- A steel company has three open hearth furnaces and five rolling mills. Transportation costs for shipping steel from furnaces to rolling mills are shown in the following table:

	M_1	M_2	M_3	M_4	M_5	supply
F_1	4	2	3	2	6	8
F_2	5	4	5	2	1	12
F_3	6	5	4	7	7	14

What is the optimal shipping schedule?

Solutions

(1)

a- (NWCM)

	i	ii	iii	iiii	iiiii	supply
A	6 60	4 40	4	7	5	100
B	5	6 40	7 85	4	8	125
C	3	4	6	3 105	4 70	175
Demand	60	80	85	105	70	400 400

M + n – 1 = 3 + 5 – 1 = 7

The solution is degenerate.

Total cost = (6) (60) + (4) (40) + (6) (40) + (7) (85) + (3) (105) + (4) (70) +

= 360 + 160 + 240 + 595 + 315 + 280 = 1950

b- (LCM)

	i	ii	iii	iiii	iiiii	supply
A	6	4 15	4 85	7	5	100
B	5	6 55	7	4	8 70	125
C	3 60	4 10	6	3 105	4	175
Demand	60	80	85	105	70	

m + n – 1 = 3 + 5 – 1 = 7

The solution is non-degenerate

Total cost = (4) (15) + (4) (85) + (6) (55) + (8) (70) + (3) (60) + (4) (10) + (3) (105)

= 60 + 340 + 330 + 560 + 180 + 40 + 315 = 1825

c- (VAM)

	i	ii	iii	iiii	iiiii	supply	
A	6	4 15	4 85	7	5	100	1 1 0 0
B	5	6 20	7	4 105	8	125	1 2 2 -
C	3 60	4 45	6	3	4 70	175	0 1 1 0
Demand	60	80	85	105	70		
	2 - - -	1 1 1 1	2 2 - -	1 1 1 1	1 1 1 1		

m + n – 1 = 3 + 5 – 1 = 7

The solution is non-degenerate

Total cost = (5) (15) + (4) (85) + (6) (20) + (4) (105) + (3) (60) + (4) (45) + (4) (70)

75 340 + 120 + 420 + 180 + 180 + 280 = 1595

(2)

a- (NWCM)

	D_1	D_2	D_3	D_4	supply
S_1	21 6	16 5	15	3	11
S_2	17	18 5	14 8	23	13
S_3	32	27	18 4	41 15	19
Demand	6	10	12	15	43

m + n -1 = 3 + 4 – 1 = 6

The solution is non-degenerate

Total cost = 126 + 80 + 90 + 112 + 72 + 615 = 109

b- (LCM)

	D_1	D_2	D_3	D_4	supply
S_1	21	16	15	3 11	11
S_2	17 1	18	14 12	23	13
S_3	32 5	27 10	18	41 4	19
Demand	6	10	12	15	43

m + n – 1 = 3 + 4 – 1 = 6

The solution is non-degenerate

Total cost = 33 + 17 + 168 + 160 + 270 + 164 = 812 c- (VAM)

	D_1	D_2	D_3	D_4	supply	
S_1	21	16	15	3 11	11	1 2 - -
S_2	17 6	18 3	14	23 4	13	4 4 4 4
S_3	32	27 7	18 12	41	19	9 9 9 9
Demand	6	10	12	15	43	
	4 15 15 -	2 9 9 9	1 4 4 4	20 18 - -		

$m + n - 1 = 3 + 4 - 1 = 6$

The solution is non-degenerate

Total cost = 33 + 102 + 54 + 92 + 189 + 216 = 686

(3)

a- (LCM)

	D_1	D_2	D_3	D_4	supply
S_1	1 20	2	1 10	4	30
S_2	3	3 20	2 20	1 10	50
S_3	4	2 20	5	9	20
Demand	20	40	30	10	100

$m + n - 1 = 3 + 4 - 1 = 6$

The solution is non-degenerate

Total cost = 20 + 10 + 60 + 40 + 10 + 40 = 180

b- (VAM)

	D_1	D_2	D_3	D_4	supply	
S_1	1 20	2	1 10	4	30	0 0 1 1
S_2	3	3 20	2 20	1 10	50	1 1 1 1
S_3	4	2 20	5	9	20	1 2 3 1
Demand	20	40	30	10	100	
	2 2 -	0 0 0 1	1 1 1 1	1 3		

Total cost = 180

(4)

a- (NWCM)

	D_1	D_2	D_3	D_4	supply
A	11 200	13 50	17	14	250
B	16	18 175	14 125	10	300
C	21	24	13 150	10 250	400
Demand	200	225	275	250	950

Total cost = (11) (200) + (13) (50) + (18) (175) + (14) (125) + (13) (150) + (10) (250)

= 12200

b- (LCM)

	D_1	D_2	D_3	D_4	supply
A	11 200	13 50	17	14	250
B	16	18 175	14 125	10	300
C	21	24	13 150	10 250	400
Demand	200	225	275	250	950

Total cost = (11) (200) + (13) (50) + (18) (175) + (14) (125) + (13) (150) + (10) (250)

= 12200

c- (VAM)

$X_{11} = 200$ $X_{12} = 50$

$X_{22} = 175$ $X_{24} = 125$

$X_{33} = 275$ $X_{34} = 125$

Total cost = 12075

(5)

$X_{11} = 6$ $X_{12} = 8$

$X_{22} = 2$ $X_{23} = 14$

$X_{33} = 1$ $X_{34} = 4$

Total cost = 128

(6)

$X_1 = 2$ $X_{13} = 4$ $X_{15} = 2$ $X_{21} = 4$ $X_{31} = 1$ $X_{34} = 6$ $X_{35} = 1$

Total cost = 720

(7)

(i) the solution is feasible because is satisfies supply and demand constraints

(ii) the solution is non-degenerate because number of occupied cells = m + n – 1

(iii) solution is optimal.

(8)

Total requirement (30) < total capacity (34), add a dummy mill with requirement (34 – 30) = 4.

Degeneracy occur at the initial solution (VAM)

Answer:

$X_{12} = 4$, $X_{14} = 4$, $X_{24} = 2$, $X_{25} = 8$, $X_{31} = 4$, $X_{33} = 6$ and $X_{36} = 4$

Total cost = 80

Operation Research

Chapter Three

Assignment Problem

1-Introduction

2- The Hungarian Method

3- Examples

4- Problems

2023-2024

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Introduction:

The best person for the job is an apt description of the assignment model. The situation can be illustrated by the assignment of workers with varying degrees of skill to jobs. A job that happens to match a worker's skill costs less than one in which the operator is not as skillful. The objective of the model is to determine the minimum-cost assignment of workers to jobs. The general assignment model with (n) workers and (n)jobs is represented in following table. The element Cij represents the cost of assignment worker i to job j (i, j =1,2, 3, ..., n). There is no loss of generality in assuming that the numbers of workers always equal the number of jobs, because we can always add fictitious workers or fictitious jobs satisfy this assumption.

Assignment Model

		1	2	3	………………………..	n
worker	1	C11	C12	C13	.	C1n
	2	C21	C22	C23	.	C2n
	3	C31	C32	C33	.	C3n
	.					
	.					
	n	Cn1	Cn2	Cn3		Cnn

There is a simple solution algorithm called the (Hungarian Method)

The Hungarian Method:

We will use a set of examples to present the mechanics of the new algorithm.

Example-1

Joe Klyne's three children ,John ,Karen ,and Terri, want to earn some money to take care of personal expenses during a school trip to the local zoo .Mr. Kline has chosen three chores for his children : mowing the lawn ,painting the garage door , and washing the family car .To avoid anticipated sibling competition, he asks them to submit (secret)bids for what they feel is fair pay for each of the three chores .The understanding is that all three children will abide by their father's decision as to who gets which chore .The following table summarizes the bids received. Based on this information, how should Mr. Klyne assign the chores?

Klyne's Assignment problem

	Mow	paint	wash
John	15	10	9
Karen	9	15	10
Terri	10	12	8

Solution:

The assignment problem will be solved by the Hungarian method :

Step-1-: For the original cost matrix, identify each row's minimum, and subtract it from all the entries of the row.

Step-2: For the matrix resulting from step -1, identify each column's minimum ,and subtract it from all the entries of the column.

Step-3: Identify the optimal solution as the feasible assignment associated with the zero elements of the matrix obtained in step-2

Let Pi and qi be the minimum costs associated with row i and column j as defined in step -1 and-2 ,respectively. The row minimums of step-1 are completed from the original cost matrix.

Next, subtract the row minimum from each respective row to obtain the reduced matrix .The application of step-2 yields the columns we get the reduced matrix.

Step-1 of the H.M

	Mow	paint	wash	Row minimum
John	15	10	9	9
Karen	9	15	10	9
Terri	10	12	8	8

Step-2 of the H.M

	Mow	paint	wash
John	6	1	0
Karen	0	6	1
Terri	2	4	0
Column minimum	0	1	0

Step-3

	Mow	paint	wash
John	6	0	0
Karen	0	5	1
Terri	2	3	0

The cells with underscored zero entries provide the optimum solution:

This means that:

John gets the paint the garage door

Karen gets to Mow the lawn

Terri gets to wash the family car

The total cost to Mr. Klyne is (9+10+8)=27

This amount also will always equal (p_1 +p_2+p_3) +(q_1 +q_2 +q_3)=27

The given steps of the Hungarian Method work well in the preceding example because the zero entries in the final matrix happen to produce a feasible assignment (in the sense that each child is assigned a distinct chore) . In some cases the zeros created by step-1 and-2 may not yield a feasible solution directly, and further steps are needed to find the optimal (feasible) assignment.

Example-2:

Suppose that the situation discussed in example -1 is extended to four children and four chores the following table show the costs

Chore

child

	1	2	3	4	pi
1	1	4	6	3	1
2	9	7	10	9	7
3	4	5	11	7	4
4	8	7	8	5	5

The application of step-1 and -2 to the matrix p_1=1.p_2=7 ,p_3=4 ,p_4= 5 and the matrix

Chore

	0	3	5	2
	2	0	3	2
	0	1	7	3
	3	2	3	0
qi	0	0	3	0

q1=0 ,q2=0 ,q3 =3 ,q4 =0 this yields the following matrix

0	3	2	2
2	0	0	2
0	1	4	3
3	2	0	0

Number of lines =3 ≠ number of rows (columns)

The locations of the zero entries do not allow assigning unique chores to all the children.

This obstacle can be accounted for by adding the following step to the procedure outlined in ex.-1

Step-2a :If no feasible assignment (with all zero entries)can be secured from step-1 and -2 .

(i) Draw the minimum number of horizontal and vertical lines in the last reduced matrix that will cover all the zero entries.

(ii) Select the smallest uncovered entry, subtract it from every uncovered entry , then add it to every entry at the intersection of two lines.

(iii) If no feasible assignment can be found among the resulting zero entries, repeat step-2a, otherwise go to step -3 to determine the optimal assignment.

The application of step-2a to the last matrix produces the following matrix.

Chore

	1	2	3	4	
1	0	2	1	1	
2	3	0	0	2	
3	0	0	3	2	
4	4	2	0	0	

we see that the smallest un shaded entry equals (1) . This entry is added to the hold intersection cells and subtracted from the remaining shaded cell to produce the above matrix .

The optimum solution (shown by the underscored zeros) calls for assigning child-1 to chore-1

child-2 to chore-3

child-3 to chore-2

child-4 to chore-4

The associated optimal cost is (1+10+5+5 =21$)

The same cost is also determined by summing the pi,s and qj,s and the entry that was subtracted after the shaded cells were determined –that is

(1+7+4+5)+(0+0+3+0)+(1)=21$

Example-3

We have three machines A ,B , C , to fulfill three functions 1 ,2 ,3 ,. The following matrix show the production cost .

Use Hungarian method to assignment the machines

mw	1	2	3
A	16	8	14
B	10	4	8
C	8	2	10

Solution :

mw	1	2	3	pi
A	16	8	14	8
B	10	4	8	4
C	8	2	10	2

mw	1	2	3
A	8	0	6
B	6	0	4
C	6	0	8
qj	6	0	4

mw	1	2	3
A	2	0	2
B	0	0	0
C	0	0	4

Number of lines =3= number of columns(rows)

A---2 =8

B----3 = 8 24

C ---1 = 8

Also cost =(8 +4 +2) + (6 +0 +4)=2

Example-4

Use the following cost matrix to assignment

	A	B	C
1	27	43	24
2	24	50	12
3	15	40	6
4	21	46	15

Solution:

Because the number of columns (3) ≠ number of rows (4)---we add one column

	A	B	C	D
1	27	43	24	0
2	24	50	12	0
3	15	40	6	0
4	21	46	15	0
qj	15	40	6	0

	A	B	C	D
1	12	3	18	0
2	9	10	6	0
3	0	0	0	0
4	6	6	9	0

We subtract (3) from all values and add it to intersection

	A	B	C	D
1	9	0	15	0
2	6	7	3	0
3	0	0	0	3
4	3	3	6	0

	A	B	C	D
1	6	0	12	0
2	3	7	0	0
3	0	3	0	6
4	0	0	3	0

Example-5

Distribute four engineers (A ,B ,C ,D) to four production lines (1 ,2 ,3 ,4) , if you know that the costs are as in the following table:

Lines

Engineers

	1	2	3	4	pi
A	70	50	50	60	50
B	30	30	90	110	30
C	50	10	20	60	10
D	50	20	70	60	20

Solution:

Engineers

	1	2	3	4
A	20	0	0	10
B	0	0	60	80
C	40	0	10	50
D	30	00	50	40
qj	0	0	0	10

Engineers

	1	2	3	4
A	20	0	0	10
B	0	0	60	70
C	40	0	10	40
D	30	0	50	30

Number of lines =3 ≠ number of rows (columns)=4

Or 3 ≠ 4

20	10	0	0
0	10	60	70
30	0	0	30
20	0	40	20

Number of lines =4 = number of rows(columns)

This is optimal solution

B-----1 =30 ,,,, D----2 =20 ,,, C---3 =20 ,,, A---4 =60

Min cost=30+20+20+60=130

Note :Number of lines must be ≤ number of rows (columns) this means that

Number of lines never > number of rows (columns)

Example -6

The following be ***profit*** table or matrix

Goods

Markets	X	Y	Z
A	8	20	16
B	24	***31***	***34***
C	***8***	***22***	***18***

We subtract all values from the maximum value (34):

26	14	18	14
10	3	0	0
26	12	16	12

12	0	4
10	3	0
14	0	4
10	0	0

2	0	4
0	3	0
4	0	4

Number of lines ≠ number of rows (columns)----- 2≠ 3

0	0	2
0	5	0
2	0	2

Number of lines = 3 = number of rows (columns)

C---Y ,,A—X ,,B—Z

Max z= 8 +22 +34 =64

Problems

1- A computer center has three expert programmers .The center wants three application programmers to be developed .The head of the computer center , after studying carefully the programmers to be developed , estimate the computer time in minutes required by the experts for the application programmers as follows :

Programmers

		A	B	C
Programmers	1	120	100	80
	2	80	90	110
	3	110	140	120

A sign the programmers to the programs in such away that the total computer is minimum . Use Hungarian method.

2-A department has five employees with five jobs to be performed .The tine(in hours)each men will take the perform each job is given in the effectiveness matrix .

Employees

		1	2	3	4	5
Jobs	A	10	5	13	15	16
	B	3	9	18	13	6
	C	10	7	2	2	2
	D	7	11	9	7	12
	E	7	9	10	4	12

How should the jobs be allocated , one per employee, so as minimize the total man-hours ?

3-An a firm wish to allocate four workers A ,B ,C ,D on four machines 1 ,2 3 4 . The following table shows the operating costs .

Machines

		1	2	3	4
Workers	A	70	50	50	60
	B	30	30	90	110
	C	30	10	20	60
	D	50	20	70	60

Use Hungarian method to assignment the workers to machines

4- The modern mining company received five production orders . (1 ,2 ,3 ,4 ,5) . That must be processed on five machines (A ,B ,C, D , E) .The following table shows the costs of the orders on the machines .

Orders

		1	2	3	4	5
Machines	A	21	30	9	20	15
	B	12	24	23	28	17
	C	21	17	18	20	25
	D	14	17	12	20	12
	E	23	17	19	14	15

Use Hungarian method to assignment the orders to machines.

5- For the following costs table .use Hungarian method to assignment the orders to machines

Orders

		1	2	3	4
Machines	A	10	8	8	7
	B	6	6	12	12
	C	6	4	2	9
	D	8	5	10	9

Solutions

1-

pi

120	100	80	80
80	90	110	80
110	140	120	110

qj

40	20	0
0	10	30
0	30	10
0	10	0

40	10	0
0	0	30
0	20	10

Number of lines =3=number of rows (columns)

$$\left.\begin{array}{l} 1\text{---------C-----}80 \\ 2\text{--------B----}90 \\ 3\text{-----A-------}110 \end{array}\right\} 280$$

Cost= (80 + 80 +110) + (0 + 10 + 0) =280

2-

For rows

	1	2	3	4	5
A	5	0	8	10	11
B	0	6	15	10	3
C	8	5	0	0	0
D	0	4	2	0	5
E	3	5	6	0	8

For columns

	1	2	3	4	5
A	5	0	8	10	11
B	0	6	15	10	3
C	8	5	0	0	0
D	0	4	2	0	5
E	3	5	6	0	8

Number of lines =4 ≠ number of rows (columns) =5

7	0	8	12	11
0	4	13	10	1
10	5	0	0	0
0	2	0	0	3
3	3	4	0	6

A-------2 =5

B-------1 =3

C--------5 =2

D----3 = 9

E-----4 = 4

C =5 +3 +2 +9 +4 =23

3-

	1	2	3	4	PI
A	70	50	50	60	50
B	30	30	90	110	30
C	30	10	20	60	10
D	50	20	70	60	20

	1	2	3	4
A	20	0	0	10
B	0	0	60	80
C	20	0	10	50
D	30	0	50	40
qj	0	0	0	10

	1	2	3	4
A	20	0	0	0
B	0	0	60	70
C	20	0	10	40
D	30	0	50	30

Number of lines=3 ≠ 4

1	2	3	4
~~20~~	~~10~~	~~0~~	~~0~~
~~0~~	~~10~~	~~60~~	~~70~~
~~10~~	~~0~~	~~0~~	~~30~~
20	0	40	20

4=4

D----2 =20

C---3 =20

B---1 =30

A----4 =60

Cost= min C= 60 +30 +20 +20 = 130

4-

Orders

Machines		1	2	3	4	5	pi
	A	21	30	9	20	15	9
	B	12	24	23	28	17	12
	C	21	17	18	20	25	17
	D	14	17	12	20	12	12
	E	23	17	19	14	15	14

Orders

	1	2	3	4	5
A	12	21	0	11	6
B	0	12	11	16	5
C	4	0	1	3	8
D	2	5	0	8	0
E	9	3	5	0	1
qj	0	0	0	0	0

A---3 =9 ,,,, B---1 =12 ,,,,C-----2 =17 ,,,,E----4 =14 ,,,D-----5 =12

Min C= 9 + 12 + 17 + 14 + 2 =64

5-

Orders

Machines	1	2	3	4	pi
A	10	8	8	9	8
B	6	6	12	12	6
C	6	4	2	9	2
D	8	5	10	9	5

Orders

	1	2	3	4
A	2	0	0	1
B	0	0	6	6
C	4	2	0	7
D	3	0	5	4
qj	0	0	0	1

Orders

	1	2	3	4
A	2	0	0	0
B	0	0	6	5
C	4	2	0	6
D	3	0	5	3

C---3 =2 ,,,D—2 =5 ,,B—1 =6 ,,A—4 =9

Min C = 2+5+6+9 =22

Operation Research

Chapter Four

Network Analysis

1-Network Representation

2- Critical path (CRM) Computations

3- PERT Procedure

4-Examples

5-Problems

2023-2024

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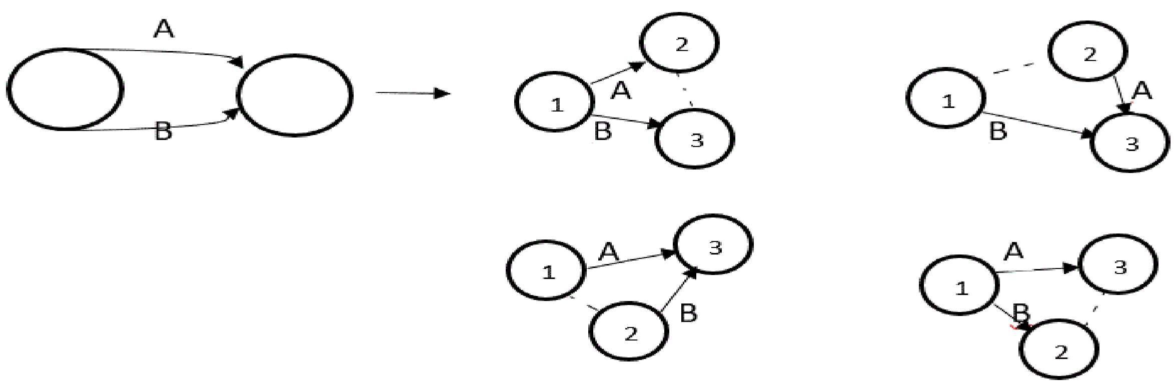
Network Representation :

Each activity of the project is represented by an are pointing in the direction of progress in the project . The nodes of the network establish the precedence relationships among the different activities. Three rules are available for constructing the network.

Rule-1 : Each activity is represented by one , and only one , arc ,

Rule-2 : Each activity must be identified by two distinct end nodes.

The following figure: shows how a dummy activity can be used to represent two concurrent activities . A and B .By definition, a dummy activity, which normally is depicted by a dashed arc ,consumes no time or resources .Inserting a dummy activity in one of the four ways shown in figure , we maintain the concurrence of A and B , and provide unique end nodes for the two activities (to satisfy rule-2)



Rule-3 : To maintain the correct precedence relationship ,the following questions must be answered as each activity is added to the network .

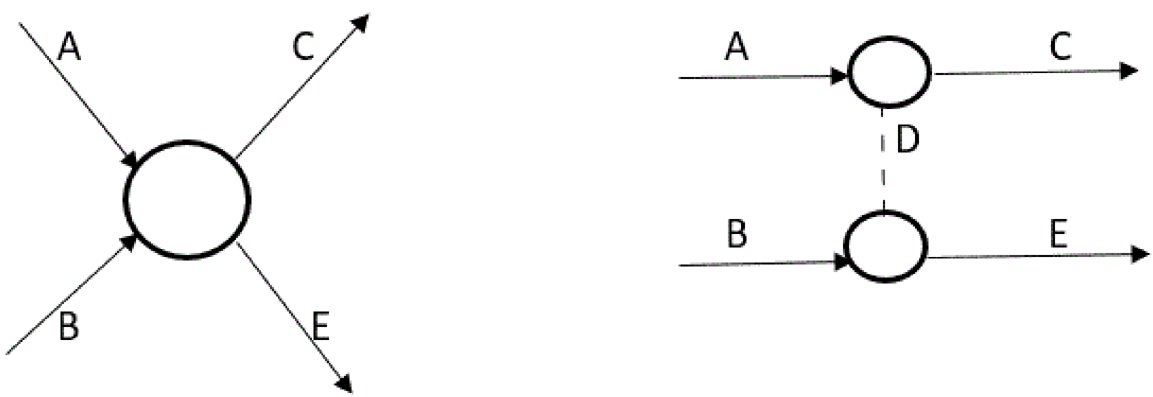
(a) What activities must immediately precede the current activity ?
(b) What activities must follow the current activity ?
(c) What activities must occur concurrently with the current activity?

The answers to the questions may require the use of dummy activities .For example ,consider the following segment of a project .

1-Activity C starts immediately after A and B have been completed.

2- Activity E starts only after B has been completed .

Part (a) in the following figure shows the incorrect representation of the precedence relationship because it requires both A and B to be completed before B can start .In part (b) , the use of a dummy activity rectifies the situation .



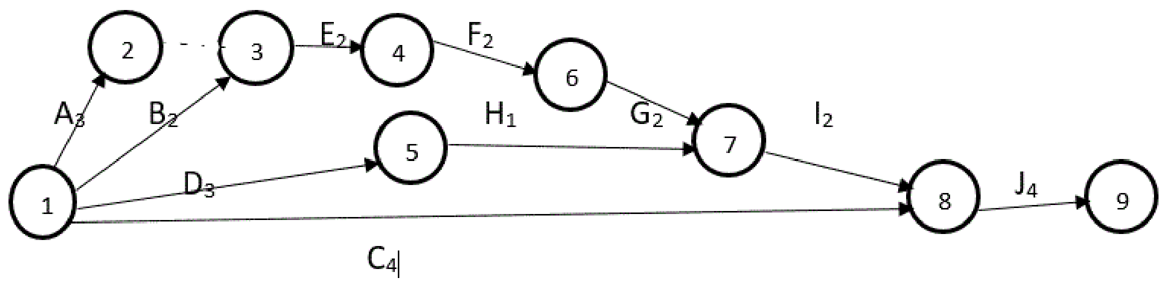
(a)   (b)

Example -1

For the following project draw network

Activity	Processor(s)	Duration(weeks)
A	---	3
B	---	2
C	---	4
D	----	3
E	A ,B	2
F	E	4
G	F	2
H	D	1
I	G ,H	2
J	C ,I	4

Solution:



__Critical Path (CPM)Computations:__

The end result in CPM is the construction of the time schedule for the project .To achieve this objective conveniently , we carry out special computations that produce the following information :

1-Total duration needed to complete the project .

2-Classification of the activities of the project as a critical and noncritical .

An activity is said to be *critical* if there is no leeway in determining its start and finish times . *A noncritical* activity allows some scheduling slack , so that the start time of the activity can be advanced or delayed within limits without affecting the completion date of the entire project .

To carry out the necessary computations , we define an event as a point in time at which activities are terminated and others are started . In terms of the network ,an event corresponds to a node .

Define:

Ej =Earliest occurrence time of event j

Lj =Latest occurrence time of event j

tij =Duration of activity (i ,j)

The definitions of the earliest and latest occurrences of event j are specified relative to the start and completion dates of the entire project .

The critical path calculations involve two passes:

The forward pass determines the earliest occurrence times of the events , and the *backward pass* calculates their latest occurrence times .

Forward pass (Earliest occurrence Time E): The computations start at node 1 and advance recursively to end node N .

Initial step . step E1=0 to indicate that the project start at time 0.

General step j , given that nodes p ,q ,….., and v are linked directly to node j by incoming activities (p ,j) ,(q , j) , ……, (v , j) and that the earliest occurrence times of events (nodes) p , q , …….,and v , have already been computed , then the earliest occurrence time of event j is computed as

$$Ej = \max [Ep + tpj , Eq + tqj , \ldots\ldots, Ev + tvj]$$

The forward pass is complete when E_N at node N has been computed . By definition Ej represents the longest path (duration)to node j.

Backward pass (Latest occurrence time L) : Following the completion of the forward pass , the backward pass computations start at node N and end at node 1 .

Initial step . set $L_N=E_N$ to indicate that the earliest and latest occurrence of the last node of the project are the same general step j , given that nodes p ,q , …. , and v are linked directly to node j by outgoing activities (j,p), (j ,q) , …., and (j ,v) and that the latest occurrence times of nodes p ,q , … ,v have already been computed , the latest occurrence time of node j is computed as :

$$Lj = \min [Lp - tjp , Lq - tjq , \ldots\ldots, Lv - tjv]$$

The backward pass is complete when L1at node 1 is computed.

At this point L1= E1=0

Based on the preceding calculations , an activity (i ,j) will be critical if it satisfies three conditions .

1- $E_i = L_i$

2- $E_j = L_j$

3- $L_j - L_i = E_j - E_i = t_{ij}$

The three conditions state that the earliest and latest times of end nodes I and j are equal and the duration tij fits tightly in the specified time span ..An activity that does not satisfy all three conditions is thus noncritical. By definition , the critical activities of a network must constitute an uninterrupted path that spans the entire network from start to finish .

PERT Procedure :

*Step-1:*Draw the project network

Step-2: Compute the expected duration of each activity

$$t_e = [t_0 + 4t_m + t_p] / 6$$

Step-3: Compute the expected variance σ^2 of each activity

Step-4: Compute the earliest start , earliest finish ,latest start , latest finish , and total float of each activity.

Step-5: Determine the critical path and identify critical activities.

Step-6: Compute the expected variance of the project length (also called the variance of the critical path) σ^2 which is the sum of the variances of all the critical activities .

Step-7: Compute the expected standard deviation of the project length σ_c and calculate the standard normal deviation

$$[T_s - T_E] / \sigma_c$$

Where: T_s = specified or scheduled time to complete the project

T_E = Normal expected duration (duration of the project)

σ_c =Expected standard deviation of the project length .

Float (slack) of an Activity and Event :

The float (slack) or free time is the length of time to which a non-critical activity and / or an event can be delay or extended without delaying the total project completion time .

Total float : It is the amount of time by which an activity can be delayed without affecting project completion time. The time within which an activity must be scheduled is computed from its latest finish (or start), LF and earliest finish (or start) EF times . That is , for each activity (i,j) the total float is equal to the latest permissible time of an end event minus the earliest possible of time of start event minus the activity duration .That is :

$$\text{Total float } (TF_{ij}) = (L_j - E_j) - t_{ij}$$

$$\text{Or} \quad = LS_{ij} - ES_{ij} = LF_{ij} - EF_{ij}$$

$$= (L_j - t_{ij}) - E_i \quad \text{or} \quad L_j - (E + t_{ij})$$

$$= T_{LS} - E_i \quad \text{or} \quad L - T_{EF}$$

Free Float : It is calculated to know how much an activity's completion time may be delayed without causing any delay in its immediate successor activities .That is ,a delay in performing an activity without affecting float of subsequent activities.

Thus , free float for a non-critical activity is defined as the time by which the completion of activity can be delayed without causing any delay iv its immediate succeeding activities . Free float for each activity (i, j) is computed as follows :

$$\text{Free float } (FF_{ij}) = (E_j - E_i) - t_{ij}$$

$$= \text{Min } \{ ES_{ij} \text{ for all immediate successors of activity } (i,j) \} - EF_{ij}$$

Remarks

1-Latest occurrence time of an event is always greater than or equal (≥) to its earliest occurrence time i.e ($L_i \geq E_i$)

$$TF_{ij} \geq (E_j - E_i) - t_{ij} \qquad \text{or } TF_{ij} \geq FF_{ij}$$

This implies that the value of free float may range from zero to total float and can never exceed to total float value.

$$\text{Free float} \leq \text{Total float}$$

2-The calculation of various float can help the decision-maker in identifying the underutilized resources , flexibility in the total schedule and possibilities of redeployment of resources.

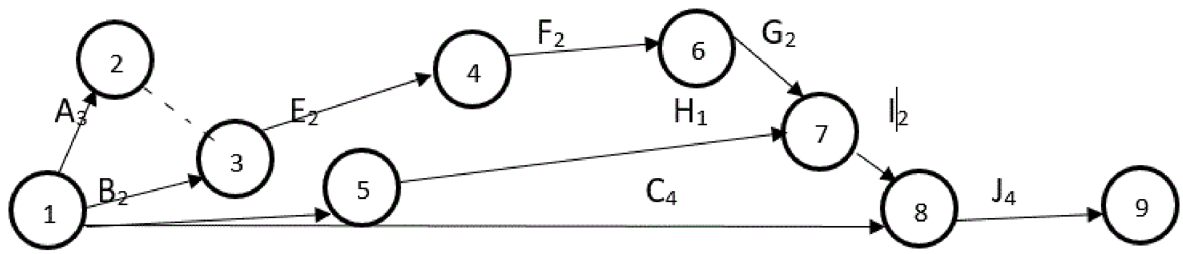
3-Once the float of an activity is disturbed float of all other activities of the project is changed and should be recalculated.

Example -2

A publisher has a contract with an outher to publish a textbook .The (simplified) activities associated with the production of the textbook are given below .The outher is required to submit to the publisher a hard copy and associated network for the project

Activity	*Predecessor(s)*	*Duration (week)*
A ;manuscript proof reading by editor	----	*3*
B: Sample pages preparation	-----	*2*
C: Book cover design	----	*4*
D: Artwork preparation	------	*3*
E: Author's approval of edited manuscript and sample pages	*A,B*	*2*
F :Book formatting	*E*	*4*
G :Author's review of formatted pages	*F*	*2*
H : Author's review of artwork	*D*	*1*
I :Production of printing plates	*G,H*	*2*
J :Book production and binding	*C,I*	*4*

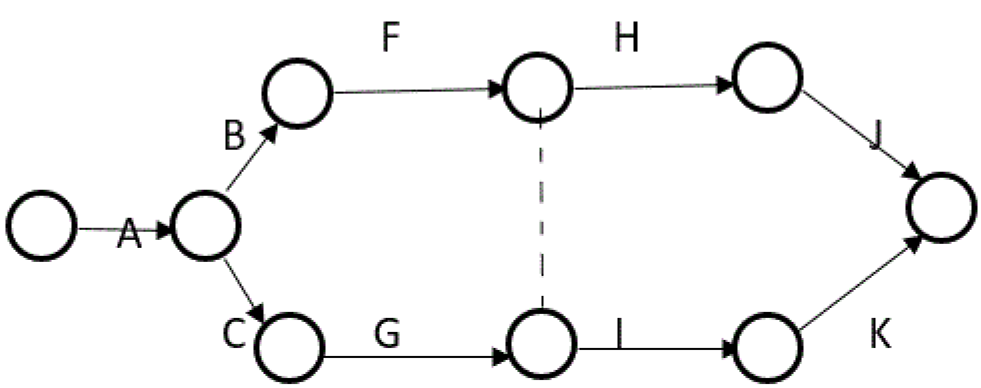
Solution:



Example-3

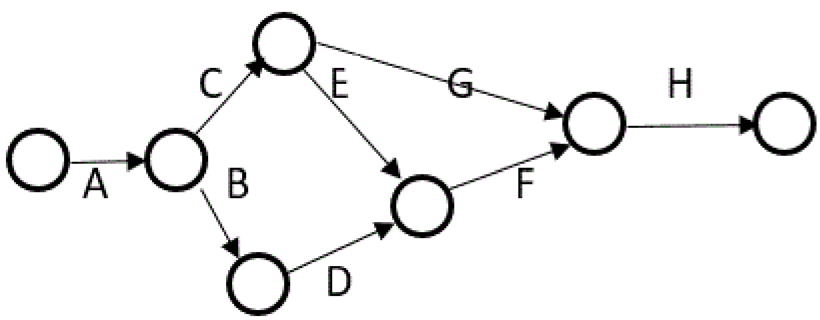
Draw the network for the following

activity	predecessor
B	A
C	A
F	B
G	C
H	F
H	G
I	G
J	H
K	I



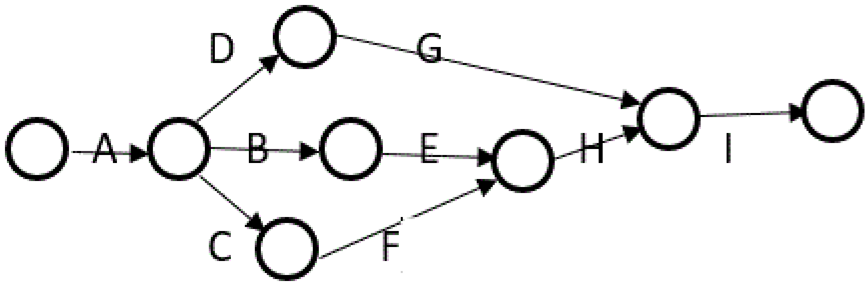
Example -4

activity	predecessor
A	---
B	A
C	A
D	B
E	C
F	D ,E
G	C
H	F ,G



Example-5

activity	predecessor
A	----
B	A
C	A
D	A
E	B
F	C
G	D
H	E ,F
I	G ,H



Example-6:

For the following:

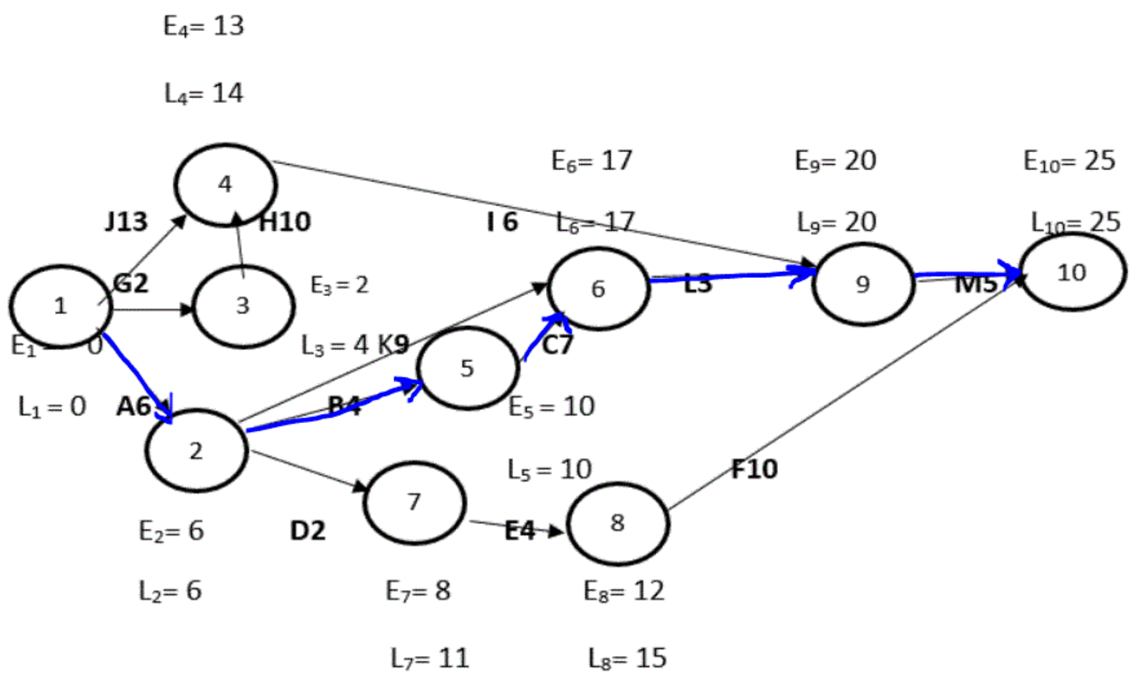
Find:

(i) Draw an arrow diagram for this project and find CP.

(ii) For each non-critical activity, find the total and free float.

activity	predecessor	Duration (days)
A	--	6
B	A	4
C	B	7
D	A	2
E	D	4
F	E	10
G	--	2
H	G	10
I	J ,H	6
J	--	13
K	A	9
L	C ,K	3
M	I ,L	5

Solution:



The critical path in the network diagram figure above has been shown by double lines by joining those events where E-values and L-values are equal. The critical path of the project is :

1-2-5-6-9-10 and the critical activities are A ,B ,C, L ,and M

The total project time is 25 days

Activity i,j	Duration tij	Earliest time St. Ei	Earliest time finish Ei +tij	Latest time Lj-tij	Latest time Lj	Float Total (Ej-tij)-Ei	Float free (Ej-Ei)-tij
1-3	2	0	2	2	4	2	0
1-4	13	0	13	1	14	1	0
2-6	9	6	15	8	17	2	2
2-7	2	6	8	9	11	3	0
3-4	10	2	12	4	14	2	1
4-9	6	13	19	14	20	1	1
7-8	4	8	12	11	15	3	0
8-10	10	12	22	15	25	3	3

Example-7:

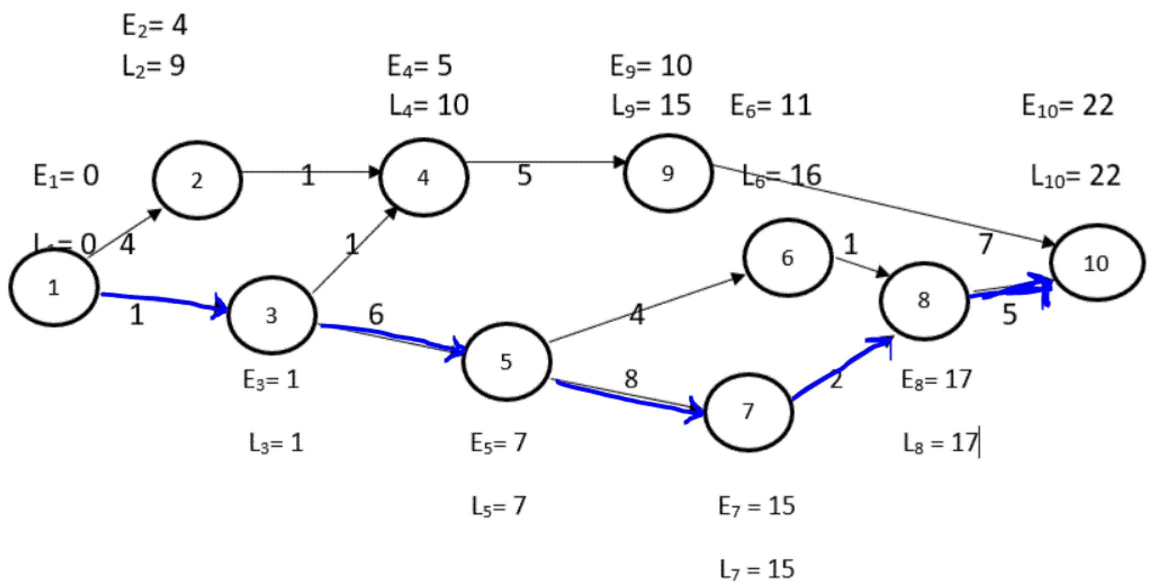
A project schedule has the following characteristics:

Activity	Times(week)	Activity	Times(week)
1-2	4	5-6	4
1-3	1	5-7	8
2-4	1	6-8	1
3-4	1	7-8	2
3-5	6	8-10	5
4-9	5	9-10	7

(i) Construct the network diagram

(ii) Compute E and L for each event, and find the critical path.

Solution:



The critical path of the project is shown by thick bold lines: 1-3-5-7-8-10

For each non-critical activity ,total float calculations are shown in the following table :

Activity i,j	Duration tij	Earliest time St. Ei	Earliest time finish Ei +tij	Latest time Lj-tij	Latest time Lj	Total Float (Lj-tij)-Ei
1-2	4	0	4	5	9	5
1-3	1	0	1	0	1	0
2-4	1	4	5	9	10	5
3-4	1	1	2	9	10	8
3-5	6	1	7	1	7	0
4-9	5	5	10	10	15	5
5-6	4	7	11	12	16	5
5-7	8	7	15	7	15	0
6-8	1	11	12	16	17	5
7-8	**2**	15	17	15	17	0
8-10	5	17	22	17	22	0
9-10	7	10	17	15	22	5

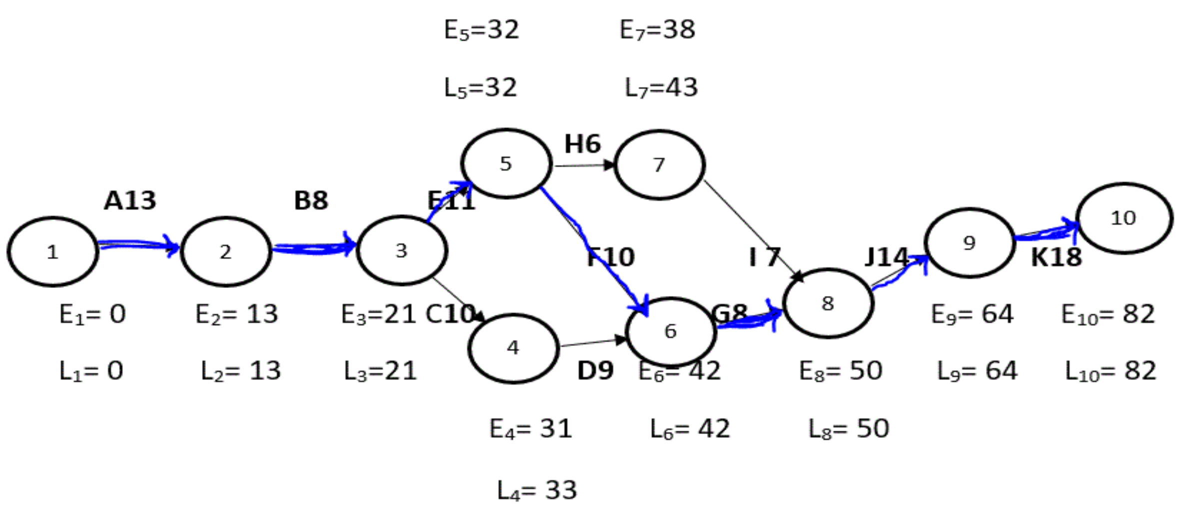
E:xample-8

Draw network diagram for following activities and find critical path and total slack of activities

Job	A	B	C	D	E	F	G	H	I	J	K
Job (days) time	13	8	10	9	11	10	8	6	7	14	18
Immediate predecessor	--	A	B	C	B	E	D,F	E	H	G,I	J

Solution:

The network diagram for the given set of activities is shown in figure below:



The critical path---- A-B-E-F-G-J-K shown by double lines. It connects that events E values = L values

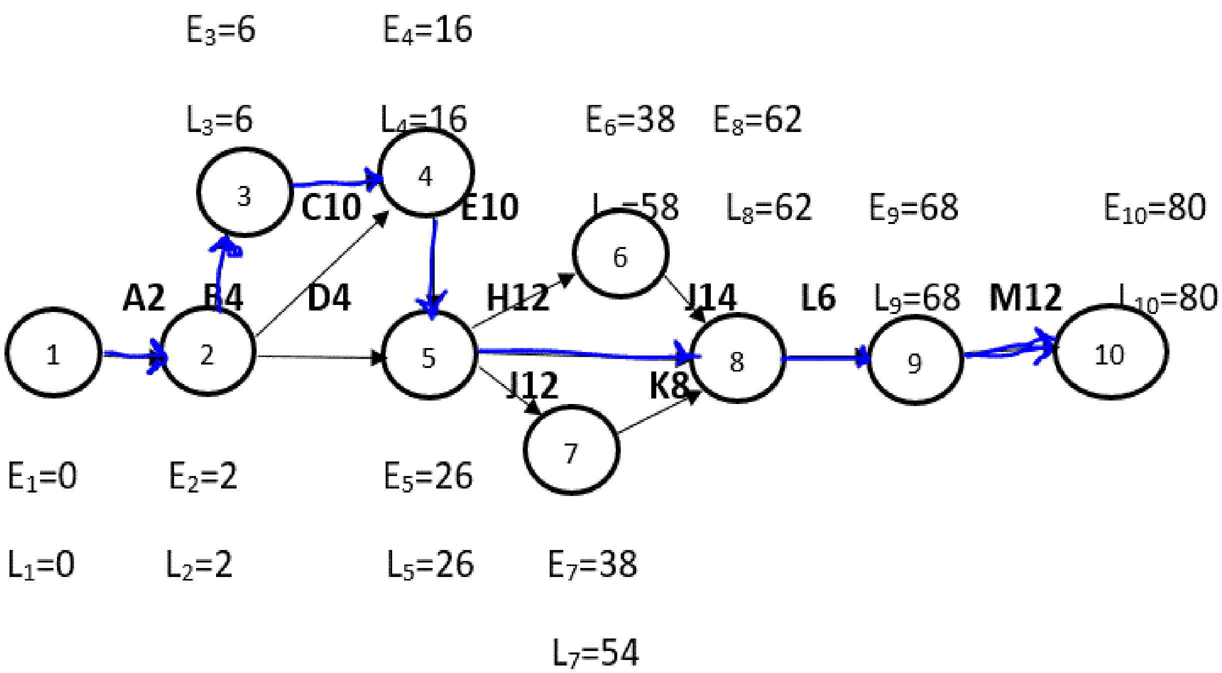
The calculation of earliest time, latest time, total float and free float for each non-critical activity are given in following table:

Activity i,j	Activity sequence	Duration tij	Earliest time St. Ei	Earliest time finish Ei +tij	Latest time Lj-tij	Latest time Lj	Float Total (Lj-tij)-Ei	Float Free (Ej-Ei)-tij
A	1-2	13	0	13	0	13	0	0
B	2-3	8	13	21	13	21	0	0
C	3-4	10	21	31	23	33	2	0
D	4-6	9	31	40	33	42	2	0
E	3-5	11	21	32	21	32	0	0
F	5-6	10	32	42	32	42	0	0
G	6-8	8	42	50	42	50	0	0
H	5-7	6	32	38	37	43	0	0
I	7-8	7	38	45	43	50	5	0
J	**8-9**	**14**	50	64	50	64	5	5
K	9-10	18	64	82	64	82	0	0

Example-9:

Draw a network diagram for the following activities and calculate earliest and latest starting and finishing times of each activity and the total and free floats for non-critical activities.

Activity sequence	Activity	Duration(day)
1-2	A	2
2-3	B	4
3-4	C	10
2-4	D	4
4-5	E	10
2-5	F	5
5-8	G	36
5-6	H	12
6-8	I	4
5-7	J	12
7-8	K	8
8-9	L	6
9-10	M	12



The critical path of the project is:-

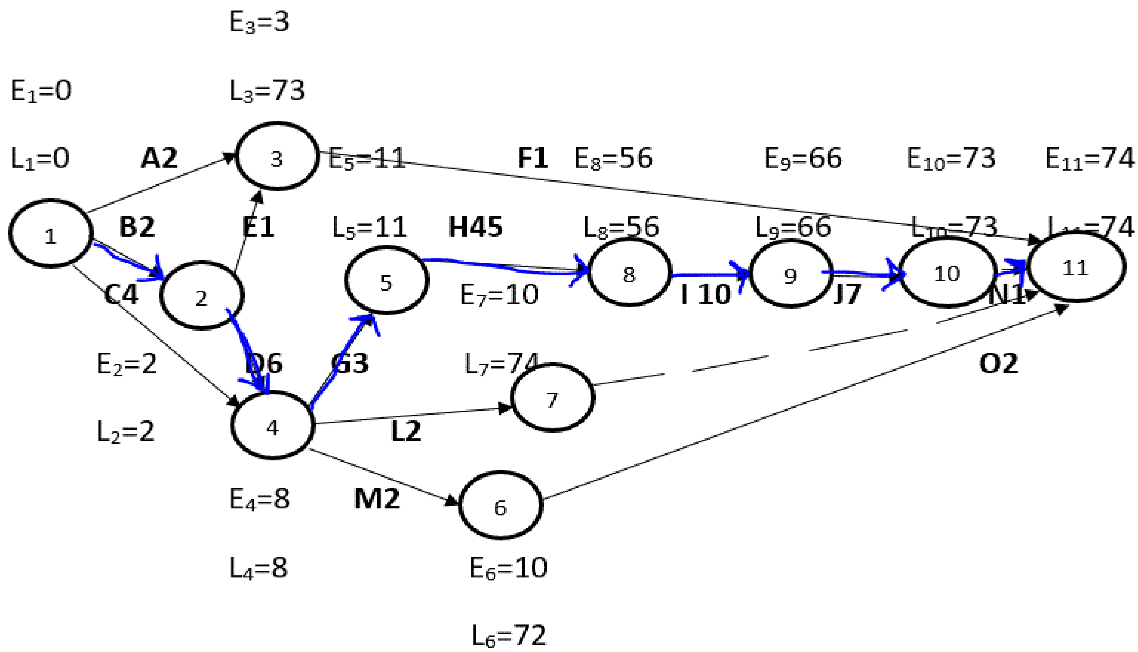
1-2-3-4-5-8-9-10 and critical activities are ,,, A ,B ,C , E ,G ,L ,and , M

Activity i,j	Activity sequence	Duration tij	Earliest time St. Ei	Earliest time finish Ei +tij	Latest time Lj-tij	Latest time Lj	Float Total (Lj-tij)-Ei	Float Free (Ej-Ei)-tij
A	1-2	2	0	2	0	2	0	0
B	2-3	4	2	6	2	6	0	0
C	3-4	10	6	16	6	16	0	0
D	2-4	4	2	6	12	16	10	10
E	4-5	10	16	26	16	26	0	0
F	2-5	5	2	7	21	26	19	19
G	5-8	36	26	62	26	62	0	0
H	5-6	12	26	38	46	58	20	20
I	6-8	4	38	42	58	62	20	20
J	**5-7**	**12**	26	38	42	54	16	16
K	7-8	8	38	46	54	62	8	8
L	8-9	6	62	68	62	68	0	0
M	9-10	12	68	80	68	80	0	0

Example-10:

Draw a network diagram for the following activities and calculate earliest and Latest times of each activity and the total and free floats for the non-critical activities

Activity	Preceding activities	Duration(days)
A	--	2
B	--	2
C	--	4
D	B	6
E	B	1
F	A,E	1
G	C,D	3
H	G	45
I	H	10
J	I	7
K	C,D	6
L	C,D	2
M	C,D	2
N	J	1
O	M	2



The critical path of the project has been shown by double lines joining all those events where E-values and L-values are equal .

The critical path is----------,,, B –D –G –H- I –J – N with project duration of 74 days

To find earliest and latest times and total ,free float for each non-critical activity

are shown in following table

Activity i,j	Activity sequence	Duration tij	Earliest time St. Ei	Earliest time finish Ei +tij	Latest time Lj-tij	Latest time Lj	Float Total (Lj-tij)-Ei	Float Free (Ej-Ei)-tij
A	1-2	2	0	2	0	2	0	0
B	1-3	2	0	2	71	73	71	1
C	1-4	4	0	4	4	8	4	4
E	2-3	1	2	3	72	73	70	0
D	2-4	6	2	8	2	8	0	0
F	3-11	1	3	4	73	74	70	70
G	4-5	3	8	11	8	11	0	0
M	4-6	2	8	10	70	72	62	0
L	4-7	2	8	10	72	74	64	60
K	**4-11**	**6**	8	14	68	74	60	60
H	5-8	45	11	56	11	56	0	0
O	6-11	2	10	12	72	74	62	62
Dummy	7-11	0	10	10	74	74	64	64
I	8-9	10	56	66	56	66	0	0

J	9-10	7	66	73	66	73	0	0
N	10-11	1	73	74	73	74	0	0

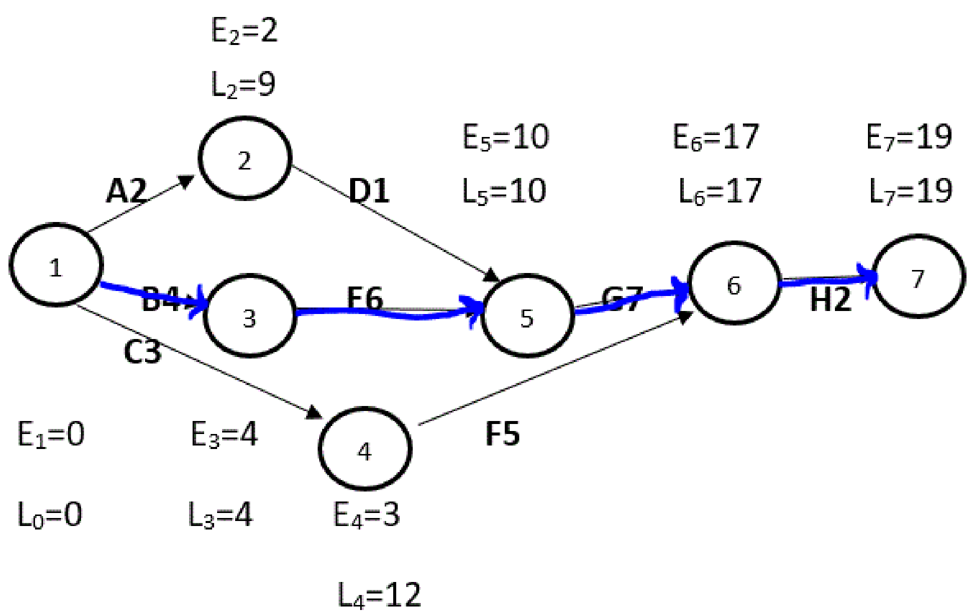
Example-11: A small project is composed of 7 activities whose time estimates are listed in the table below:

Activity	Predecessors	Estimated duration (weeks)		
		optimistic	most likely	pessimistic
A	---	1	1	7
B	---	1	4	7
C	---	2	2	8
D	A	1	1	1
E	B	2	5	14
F	C	2	5	8
G	D ,E	3	6	15
H	F ,G	1	2	3

(a) Draw the project network and determine the expected project time.
(b) What duration will have 95 percent confidence of project completion .
(c) If the average duration for activity F increases to 14 weeks ,what will be its effect on the expected project completion time which will have 95 percent confidence .

Solution:

Activity seq.	Activity	t_0	t_m	t_p	$t_e=[t_0+4t_m+t_p]/6$	$\sigma^2=[1/6(tp-to)]^2$
1-2	A	1	1	7	2	1
1-3	B	1	4	7	4	1
1-4	C	2	2	8	3	1
2-5	D	1	1	1	1	0
3-5	E	2	5	14	6	4
4-6	F	2	5	8	5	1
5-6	G	3	6	15	7	4
6-7	H	1	2	3	2	0.33



The E-values and L-values based on expected time (t_e) of each activity

(a) Critical path 1- 3 -5 -6 -7 and the critical activities --,, B—E—G—H .
Expected project length =4 +6 +7 +2 =19 weeks
Variance of the project length (week) is the sum of the variances of each critical activity , that is :
Variance =1 +4 +4 +0.33 =9.33
Standard deviation $\sigma= \sqrt{9.33} =3.05$

(b) Project will be completed with 95 percent confidence is given by
$Z= [T_s-T_e]/\sigma_e$ or $1.645 = [T_s-19]/3.05$ or $Ts =1.645*3.05 +19=24$ weeks(approx.)
Hence project may be completed in 24 weeks with 95%cofidence.

© when the average duration of activity F increases to 14 weeks , the path C—F—H also becomes critical. Thus, new standard deviation of project length is : $\sqrt{9.33 +2} = 3.36$ and the project will be completed with 95% confidence is given by:$1.645 = [T_s -19]/3.36$ or $T_s = 19 +1.645* 3.36=24.52$ weeks

Hence the project completion time of 24.52 weeks will have 95% confidence

Example—12:

A civil engineering firm has to bid for the construction of a dam . The activities and time estimates are given below :

Activity	Estimated duration (in days)		
	optimistic	Most likely	pessimistic
1-2	14	17	25
2-3	14	18	21
2-4	13	15	18
2-8	16	19	28
3-4 dummy	--	--	--
3-5	15	18	27
4-6	13	17	21
5-7 dummy	--	--	--
5-9	14	18	20
6-7 dummy	--	--	--
6-8 dummy	--	--	--
7-9	16	20	41
8-9	14	16	22

The policy of the firm with respect to submitting bids is to bid the minimum amount that will provide a 95% of probability of at best breaking even . The fixed cost for the project are eight lakh and the variable cost are 9000 every day spent working on the project. The duration is in days and the costs are in terms of rupees.

What amount should the firm bid under this policy? (you may perform the calculations on duration ,etc .. up two decimal places)

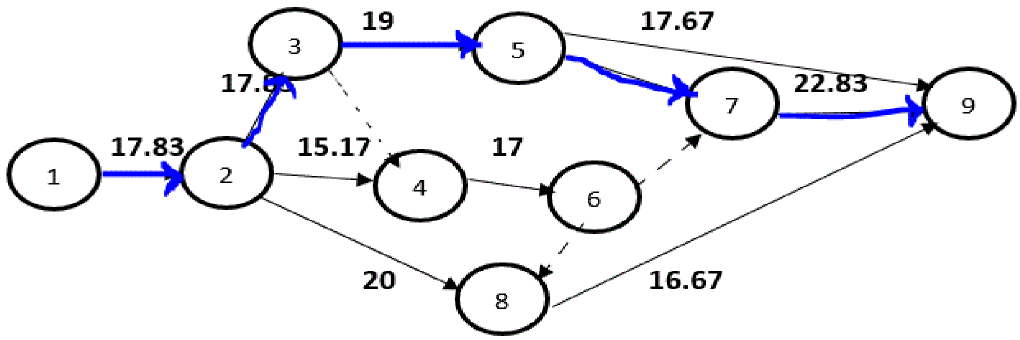
Solution:

The calculations of expected duration and variance of each activity are given in following table:

activity	Optimistic to	Most likely tm	Pessimistic tp	$t_e=[t_0+4t_m+t_p]/6$	$\sigma^2=[1/6(tp-to)]^2$
1-2	14	17	25	17.83	3.36
2-3	14	18	21	17.83	1.36
2-4	13	15	18	15.17	--
2-8	16	19	28	20.00	--
3-4	--	--	--	--	--
3-5	15	18	27	19.00	4
4-6	13	17	21	17.00	--
5-7	--	--	--	--	--
5-9	14	18	20	17.67	--

6-7	--	--	--	--	--
6-8	--	--	--	--	--
7-9	16	20	41	22.83	17.36
8-9	14	16	22	16.67	--

The network diagram for the given activities is shown below:



The various paths and their lengths as follows:

	path	duration
1	1-2-3-5-7-9	77.49 **
2	1-2-3-5-9	72.33
3	1-2-3-4-6-7-9	75.49
4	1-2-3-4-6-8-9	69.33
5	1-2-8-9	54.50
6	1-2-4-6-8-9	66.67
7	1-2-4-6-7-9	72.83

Thus , the critical path is

1-2-3-5-7-9 with maximum project duration of 77.49 days.

The variance of project duration is obtained by summing variances of critical activities

Variance σ^2 = 3.36 +1.36 +4 +17.36 =26.08

Standard deviation $\sigma = \sqrt{26.08} = 5.12$

To calculate the project duration which will have 95 % chances of its completion , we find the value of z corresponding to 95% area under normal distribution curve which is 1.645. Thus

$Z = [T_s - T_e]/\sigma$ or $1.645 = [T_s - 77.49]/5.12$

$T_s = 1.645 * 5.12 + 77.49 = 86$ days

Since the fixed cost of the project is Rs 8 lakhs and the variable cost is Rs 9000 per day , amount to bid

= Rs 8lakh +Rs 9000 * 86 = Rs 1574000

Example—13:

The owner of a chain of fast-food restaurants is considering a new computer system for accounting and inventory control . A computer company sent the following information about the computer system installation

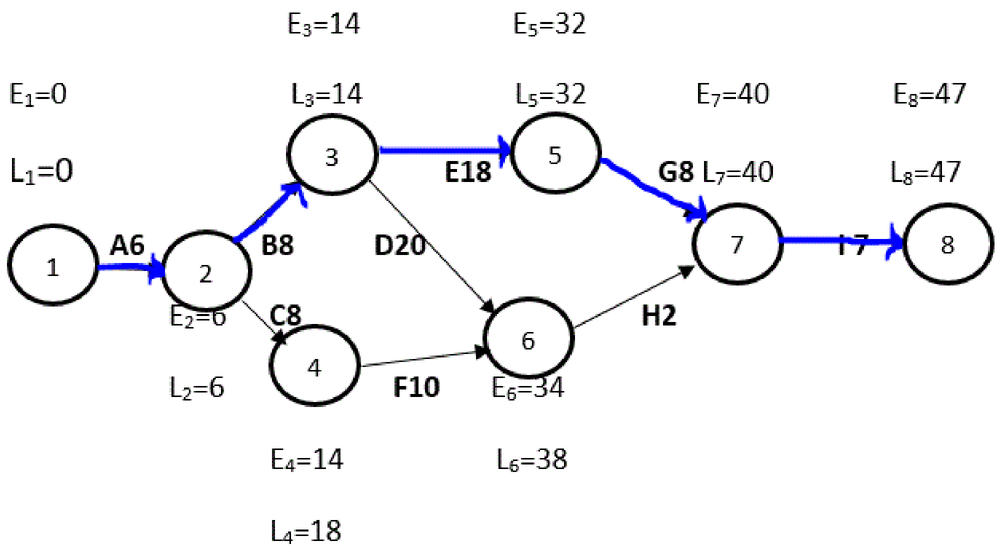
Activity	Description	Predecessors	Estimated time (day)		
			to	tm	tp
A	Select the computer model	--	4	6	8
B	Design input /output system	A	5	7	15
C	Design monitoring system	A	4	8	12
D	Assemble computer hardware	B	15	20	25
E	Develop the main programs	B	10	18	26
F	Develop input / output routines	C	8	9	16
G	Create database	E	4	8	12
H	Install the system	D ,F	1	2	3
I	Test and implement	G ,H	6	7	8

(a) Construct an arrow diagram for this problem .

(b) Determine the critical path and compute the expected completion time.
(c) Determine the probability of completing the project in 55 days.

Solution:

(a) The network diagram for the given activities is shown as follows:



(b) The calculation for expected time (te) and variance (σ^2) for activities are shown in the table below:

$T_e = [t_o + 4t_m + t_p]/6$ and $\sigma^2 = [(t_p - t_o)/6]^2$

Activity	to	tp	tm	te	σ^2
A	4	6	8	6	0.44
B	5	7	15	8	2.77
C	4	8	12	8	-----
D	15	20	25	20	-----
E	10	18	26	18	7.11
F	8	9	16	10	--
G	4	8	12	8	1.77
H	1	2	3	2	--
I	6	7	8	7	0.11

The critical path is shown by thick line , where E-values and L-values are same . The critical path is :

1 -2 -3 -5 -7 -8 and critical activities are ---,,, A ,B ,E ,G ,and I.

The earliest completion time of the project is 47 days

Variance , σ^2 =0.44 +2.77 +7.11 +1.77 +0.11 =12.20

Standard deviation , $\sigma = \sqrt{12.20} = 3.492$

© the probability of the project being completed in 55 days is calculated as follows:

$P(X \leq 55) = P[Z \leq \{(55-47)/3.492\}] = P[Z \leq 2.29]$

From the area under normal curve , the area between z=0 and z=2.29 is equal to 0.4981. Thus

$P(X \leq 55) = P[Z \leq 2.29] = 0.5 + 0.4981 = 0.9981.$

Problems

1- For each of the following draw the network

Activity	*Predecessor*
A	--
B	--
C	--
D	A ,B
E	C
F	D ,E
G	A ,B
H	C
J	G
K	F
L	

2-

Activity	*Predecessor*
A	--
B	A
C	A
D	A
E	B
F	D
G	B
H	E ,C ,F
I	D
J	G ,H
K	I ,J

3-

Activity	*Predecessor*
A	--
B	A
C	A
D	B ,C
E	C
F	C

4-

Activity	*Predecessor*
A	--
B	--
C	--
D	A ,B
E	A ,C

5-

Activity	*Predecessor*
A	S
B	A ,K
C	B ,H
D	H
E	G ,K
F	C ,D ,E
G	S
H	K
K	S
S	--

6- Tasks A ,B ,C ,.....,H , I constitute a project . The notation x< y means that the task x must be finished before y can being ,with the notation.

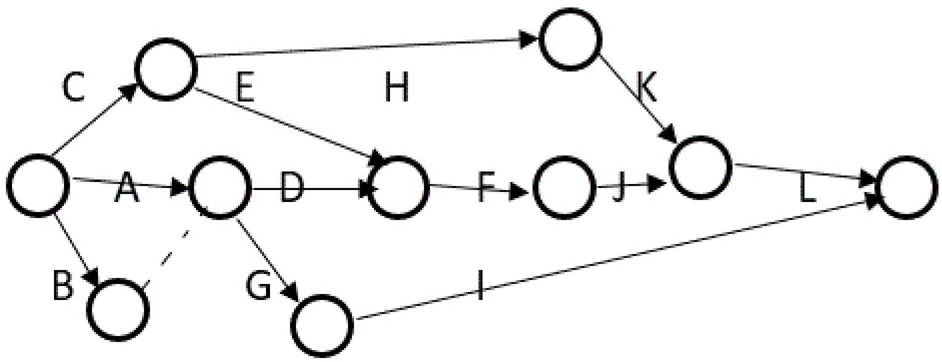
A<B,A<E ,B<F ,D <F ,C <G ,C <H ,F <I ,G <I

Draw a graph to represent the sequence of tasks and find the CP when the time (in days) of completion of each task is as follows:

Task :	A	B	C	D	E	F	G	H	I
Time :	8	10	8	10	16	17	18	14	9

Solutions

1-



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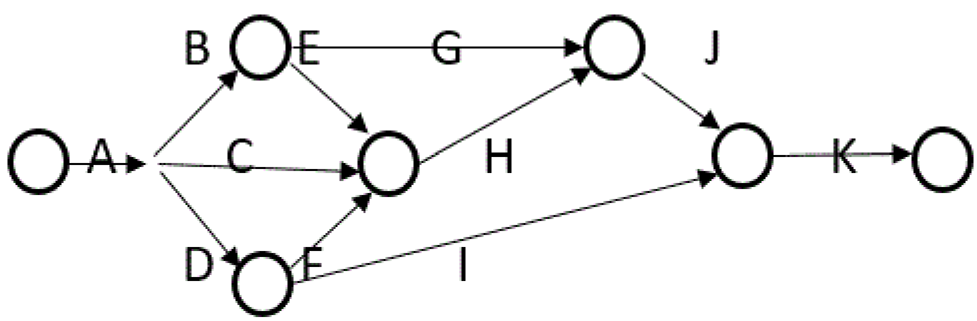

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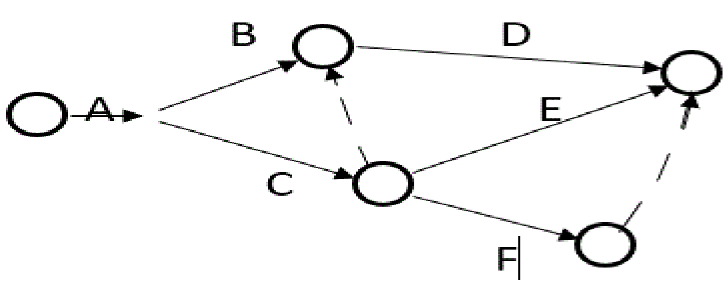
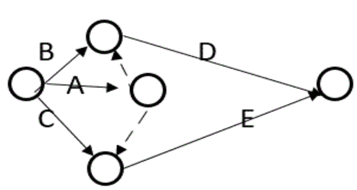
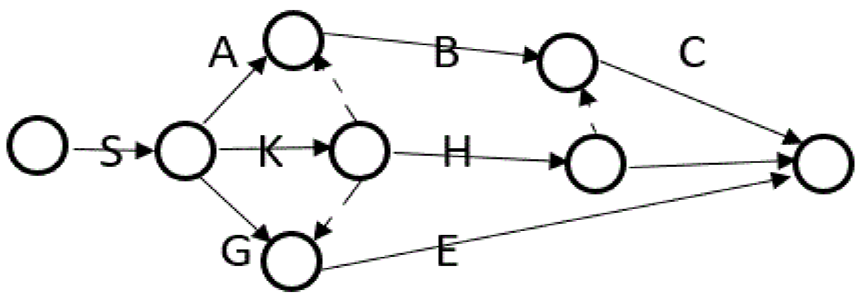


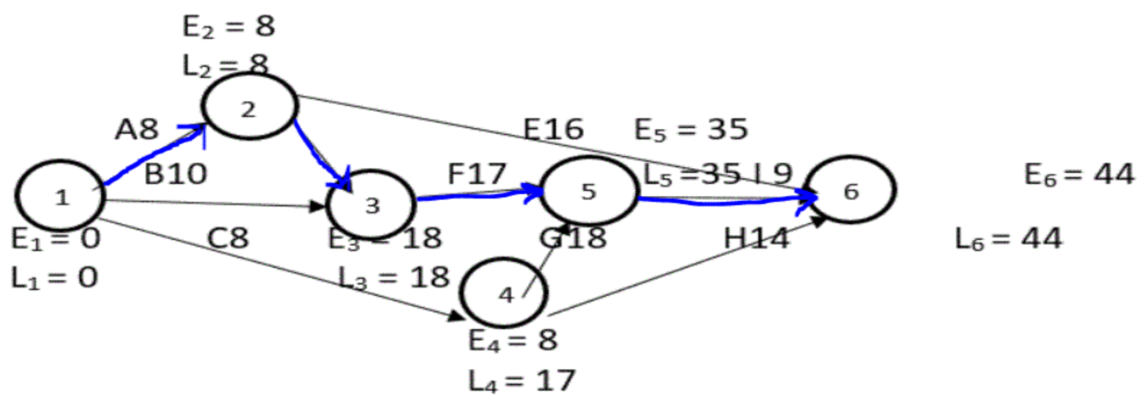
4-



5-



6- The network diagram based on the given sequence of activities is shown as below:



The critical path ,, 1 –2 –3 --5 --6 ,, when E-values and L-values are equal.

For each non-critical activity the total float calculations are shown as following:

Task	tij	Earliest time		Latest time		Total float (Lj-tij)-Ei
		Ei	Ei+tij	Lj-tij	Lj	
A	8	0	8	0	8	0
B	10	0	10	8	18	8
C	8	0	8	9	17	9
D	10	8	18	8	18	0
E	16	8	24	28	44	20
F	17	18	35	18	35	0
G	18	8	26	17	35	9
H	14	8	22	30	44	22
I	9	35	44	35	44	0

Operation Research

Chapter Five

Decision Theory

1-Introduction

2- Basic Definitions

3- Decision Making Under Certainty

4- Decision Making Under Uncertainty

5- Decision Making Under Risk

6- Decision Trees

7- Examples

8-Problems

2022-2023

Prof.Dr. Obaid Mahmmood Mohsin

Introduction:

Decision models useful in helping decision-maker make the possible decisions are classified according to the degree of certainty . The scale of certainty can range from complete certainty to complete uncertainty .The region which falls between these two extreme points corresponds to the decision- making under risk (probabilistic problem).

Irrespective of the type of decision model, there are certain essential characteristics which are common to all as listed below.

Decision Alternatives:

There is a finite number of decision alternatives available with the decision-maker at each point in time when a decision is made. The number and type of such alternatives may depend on the previous decisions made and on what has happened subsequent to those decisions. These alternatives are also called ***course of action (actions ,acts or strategies)*** and are under control and known to the decision –maker. These may be described numerically such as , conducting a market survey to know the likely demand of an time .

State of nature:

A Possible future condition(consequence or event) resulting from the choice of a decision alternative depends upon certain factors beyond the control of the decision-maker. These factors are called states of nature (future). For example, if the decision is to carry an umbrella or not, the consequence (get wet or do not) depends on what action nature takes.

The states of nature are mutually exclusive and collectively exhaustive with respect to any states of nature is such as employees strike ,etc.

Payoff:

A numerical value resulting from each possible combination of alternatives and states of nature is called pay off. The pay off values are always conditional values because of unknown states of nature. A tabular arrangement of these conditional outcome (pay off)values is known as *pay off matrix* as shown in following table:

State of nature	Courses of Action(Alternatives)			
	S1	S2	………………	Sn
N1	P11	p12		p1n
N2	P21	P21		P2n
.	.	.	……..	
.				
.	.	.	……	
Nm	Pm1	pm2		pmn

Decision-Making Environments:

Decisions are made based upon the data available about the occurrence of events as well as the decision situation (or environment) . we will study three types of decision –making environment :

Decision –Making under Certainty:

In this case the decision-maker has complete knowledge (perfect information) of consequence of every decision choice (course of action or alternative)with certainty , obviously , he will select an alternative that yields the largest return (pay off)for the known future (state of nature)

Decision –Making under Uncertainty:

In this case the decision-maker is unable to specify the probabilities with which the various states of nature (futures) will occur . Thus , decisions under uncertainty are taken with even less information than decisions under risk . For example , the probability that Mr. X will be the prime minister of the country 15 years from now is not known .

Several methods for arriving at an optimal solution under uncertainty are discussed below:

1-Criterion of optimism (maximax or minimin):

The working method is summarized as follows:

(i) Locate the maximum (or minimum) pay off values corresponding to each alternative (or course of action) ,then

(ii) Select an alternative with best anticipated pay off value (maximum for profit and minimum for cost)

Since in this criterion the decision-maker selects an alternative with largest (or lowest) possible pay off value , it is called an optimistic decision criterion.

Example --1:

A food products company is contemplating the introduction of are evolutionary new product with new packaging or replace the existing product at much higher price(S1) or a moderate change in the composition of the existing product with a new packaging at a small increase in price (S2) or a small change in the composition of the existing product except the word 'New' with an egligible increase in price (S3).The three possible state of nature or events are : (i) high increase in sales (N1) , (ii) no change in sales (N2) and (iii) decrease in sales (N3) , the marketing dependent of the company worked out the pay off in terms of yearly net profits for each of the strategies of three events(expected sales) .This is represented in the following table :

strategies	States of nature		
	N1	N2	N3
S1	700000	300000	150000
S2	500000	450000	0
S3	300000	300000	300000

Which strategy should the concerned executive choose on the bais of (i) maximax criterion

Solution: The pay off matrix is rewritten as follows :

States of nature	strategies S1	S2	S3
N1	700000	500000	300000
N2	300000	450000	300000
N3	150000	0	300000
Maximum column	***700000***	500000	30000

The maximum of the column is 700000 , hence the company should adopt strategy S1.

2-Criterion of Pessimism (minimax or maximin) :

The working method is summarized as follows :

(i) Locate the minimum (or maximum in case of profit) pay off in case of loss (or cost) values corresponding to each alternative , then

(ii) Select an alternative with the best anticipated pay off value (maximum for profit and minimum for loss or cost).

Since in this criterion the decision –maker is conservative about the future always anticipate worst possible outcome (minimum for profit and maximum for cost or loss) , it is called a pessimistic decision criterion . The criterion is also known as 'Wald's ' criterion .

Example--2:

For the last example which strategy should be concerned executive choose on the bias of (ii) maxmin criterion

States of nature	strategies S1	S2	S3
N1	700000	500000	300000
N2	300000	450000	300000
N3	150000	0	300000
Column minimum	150000	0	***300000***

The maximum of column minima is 300000 , hence , the company should adopt strategy S3 .

3-Equally likely Decision (Laplace criterion):

The working method is summarized as follows :

(i) Assign equal probability value to each state of nature by using the formula: 1/(number of state of nature)

(ii) Compute the expected (or average) pay off for each alternative (course of action), by adding all the pay offs and dividing by the number of possible states of nature or by applying the formula.:
Probability of state of nature / and pij pay off value for the combination of alternative and state of nature j.

(iii) Select the best expected pay off value (maximum for profit and minimum for cost)
Example--3:
For the last example: which strategy should be concerned executive choose on the bias of (iii) Laplace criterion.
Solution:
Since we do not know the probabilities of state of natures ,assume that they are equal. For this example, we would assume that each state of nature has probability 1/3 of occurrence .thus ,

strategy	Expected Return (Rs)
S1	(700000+300000+150000)/3=***383333.33***
S2	(500000+300000+300000)/3=316666.66
S3	(300000+300000+300000)/3=300000

Since the largest expected return is from strategy S1 ,the executive must select strategy S1.

4-*Criterion of Realism (Hurwicz criterion):*

The Hurwicz approach suggest that the decision-maker must select an alternative that maximizes:

H(criterion of realism) = α (maximum in column)+ (1- α) (minimum on column)

The working method is summarized as follows :

(i) Decide the coefficient of optimism α and then coefficient of pessimism (1-α)

(ii) For each alternative select the largest and lowest pay off value and multiply these with α and(1-α) values , respectively . then calculate the weighted average , H by using a bove formula.

(iii) Select an alternative with best anticipated weighted average pay off value.

Example--4 : for the last example:

Which strategy should be concerned executive on the bias of (iv) H (criterion of realism) (use α= 0.65)

Solution:

strategy	Best pay off	worst pay off	H
S1	700000	150000	(0.65)(700000) +(0.35)(150000_=507500
S2	500000	0	325000
S3	300000	300000	300000

Since the largest value is from S1 =507500, The executive must select strategy S1.

<u>5- Criterion of Regret (savage-criterion):</u>

The working method is summarized as follows :

(i) Form the given pay off matrix ,develop an opportunity-loss (or regret) matrix.
-find the best pay off corresponding to each of nature and
-subtract all other entries (pay off values) in that row from this value.

(ii) for each course of action (strategy) identify the worst or maximum regret value . Record this number as a new row.

(iii)select the course of action (alternative) with the smallest anticipated opportunity-loss value.

Example--5 :use the last example-which strategy should be concerned executive choose on the bias (v)maximax regret criterion.

Solution :

Minimax regret criterion : opportunity loss table is shown below:

State of nature	Strategies		
	S1	S2	S3
N1	700000-700000=0	700000-50000=200000	700000-300000=400000
N2	450000-300000=150000	450000-450000=0	450000-300000=150000
N3	300000-150000=150000	300000-0=300000	300000-300000=0
Column maximum	***150000***	300000	400000

Hence , the company should adopt minimum opportunity loss strategy ,S1.

Example ---6:

For the following table of profit:

alternatives	State nature			
	S1	S2	S3	S4
a1	55	10	20	70
a2	45	20	12	40
a3	60	25	9	10
a4	14	8	10	60

Required:

A specific decision from among the available alternatives for the purpose of profit maximization on the bias:

(i) Criterion of optimism (maximax)
(ii) = = pessimism(maximin)
(iii) Laplace criterion.
(iv) Criterion of realism (Hurwicz criterion) let α =0.7
(v) = = regret (savage criterion)

Solution:

alternatives	State nature				max	min
	S1	S2	S3	S4		
a1	55	10	20	70	**70***	10
a2	45	20	12	40	45	**12***
a3	60	25	9	10	60	9
a4	14	8	10	60	60	8

(i) Criterion of optimism (maximax)=70------a1

(ii) = = pessimism (maximin)=12 ---- is a2

(iii) Laplace criterion

a1= [55 +10+20+70]/4=155/4=***38.75***

a2 =[45+20+12+40]/4 =117/4 =29.25

a3 =[60+25+9+10]/4 =104/4 =26

a4 =[14+8+10+60]/4 =92/4 =23

Laplace criterion=38.75 ------ a1

(iv)

strategy	Best pay off	worst pay off	H
a1	70	10	(0.7)(70)+(0.3)(10)=***52 ****
a2	45	12	(0.7) (45)+(0.3) (12)=35.1
a3	60	9	(0.7)(60)+(0.3) (9)=44.7
a4	60	8	(0.7) (60) + (0.3)(8) =44.4

Criterion of realism (H criterion) =52-----a2

(v)

alternatives	State nature				max
	S1	S2	S3	S4	
a1	70-55=15	70-10=60	70-20=50	70-70=0	50
a2	45-45=0	45-20=25	45-12=33	45-40=5	33
a3	60-60=0	60-25=35	60-9=51	60-10=50	51
a4	60-14=46	60-8=52	60-10=50	60-60=0	***52****

Since the largest expected return is =52-------is a4

Decision-making under Risk:

In this case the decision-maker has less than complete knowledge with certainty of the consequence of every decision choice (course of action) .This means there is more than one will occur. For, example, probability of getting head in the toss of a coin is 0.5.

Expected Monetary Value(EMV):

The expected value (EMV) for a given course of action is the weighted average pay off ,which is the probabilities associated with each state of nature.

Mathematically EMV is stated as follows:

EMV (course of action, Sj) = $\sum^{m}_{j=1} p_{ij}\, p_i$

Where m=number of possible state of nature

p_i = probability of occurrence of state of nature

p_{ij} =pay off associated with state of nature Ni and course of action , Sj

steps for calculating EMV:

The various steps involved in the calculation of EMV are as follows:

(i)construct a pay off matrix listing all possible courses of action and state of nature. Enter the conditional pay off values associated with each possible combination of the occurrence of each state of nature.

(ii)calculate the EMV for each course of action by multiplying the conditional pay offs by the associated probabilities and add these weighted values for each course of action.

(iv) Select the course of action that yields the optimal EMV.

Example--7: An investor is given the following investment alternatives and percentage rates of return

	State of nature(market conditions)		
	low	medium	high
Regular shares	7%	10%	15%
Risky shares	-10%	12%	25%
property	-12%	18%	30%

Over the past 300 days , 150 have been medium market conditions and 60 days have high market increases. On the bias is of these data , state the optimum investment strategy for the investment.

Solution:

According to the given information, the probabilities of low , medium and high market conditions would be

90/300 or 0.30 , 150/300 or 0.50 , and 60/300 or 0.20

Respectively . the expected pay off for each of the alternatives are shown below:

		strategy		
Market condition	probability	Regular shares	Risky shares	property
low	0.3	0.07 *0.30	0.10 *0.30	0.15 *0.30
medium	0.5	-0.10* 0.50	0.12 *0.50	0.25 * 0.50
high	0.20	-0.12 *0.20	0.18 * 0.20	0.30 *0.20
Expected return		0.136	0.126	***0.230***

Since the expected return of 0.23 percent is the highest for ***properly ,*** the investor should invest in this alternative .

Example—8:
the following pay off table is given :

alternatives (Act)	State of nature (event)			
	E1	E2	E3	E4
A1	40	200	-200	100
A2	200	0	200	0
A3	0	100	0	150
A4	-50	400	100	0

Suppose that the probability of events of this table are:

P(E1) =0.20 , P(E2) =0.15 , P(E3) =0.40 , P(E1) =0.25

Calculate the expected pay off and expected loss of each action.

Solution :

Computations of expected pay off and loss shown below:

		Conditional pay off				Expected pay off			
event	probability	A1	A2	A3	A4	A1	A2	A3	A4
	(1)	(2)	(3)	(4)	(5)	(1) (2)	(1) (3)	(1) (4)	(1) (5)
E1	0.20	40	200	0	-50	8	40	0	-10
E2	0.15	200	0	100	400	30	0	15	60
E3	0.40	-200	200	0	100	-80	80	0	40
E4	0.25	100	0	150	0	25	0	37.5	0
Expected Pay off						-17	120	52.5	90

The opportunity loss or regret table can be obtained from the given pay off table by subtracting all the pay off elements of an event from the highest pay off for that event as shown below:

		Computation of expected loss							
		Opportunity loss			Expected loss				
event	probability	A1	A2	A3	A4	A1	A2	A3	A4
	(1)	(2)	(3)	(4)	(5)	(1) (2)	(1) (3)	(1) (4)	(1) (5)
E1	0.20	200-40=160	0	200	250	32	0	40	50
E2	0.15	200	400	300	0	30	60	45	0
E3	0.40	400	0	200	100	160	0	80	40
E4	0.25	50	150	150	150	12.5	37.5	0	37.5
Expected loss						234.5	97.5	165	127.5

Example---9:

Mr. X quite of ten flies from town A to town B . He can use the airport bus which costs (25$) but if he takes it , there is a (0.08) chance that he will miss the flight. The stay in a hotel costs (270$)with a (0.96)chance of being on time for the flight. For (350$) he can use a taxi which will make (0.99) chance of being no time for the flight . If Mr .X catches the plane on time , he will coclude a business transaction which will produce a profit of (1000$), otherwise he will lose it . which mode of transport should Mr . X use ?answer on the basis of the EMV criterion.

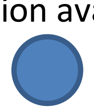
Solution:

Computation of EMV of various courses of action is shown below :

	Courses of action								
Strategy of nature	bus			Stay in hotel			taxi		
	cost	Prob.	Exp. val.	cost	Pro b.	Exp.val	cost	Prob.	Exp.val.
Catches the flight	1000025=9975	0.92	9177	1000-270=9730	0.96	9340.8	10000-350=9650	0.99	9553.50
Mis the flight	-25	0.08	-20	-270	0.04	-10.80	-350	0.01	-3.50
Expected monetary value(EMV)			9175			9330			9550

Comparing the EMV associated with each course of action indorses that course of action ***taxi*** is the logical alternative because it has the highest EMV .

<u>*Decision Tree Analysis:*</u>

The decision tree analysis involves construction of a diagram showing all the possible courses of action, states of nature and the probabilistic associated with the states of nature . the decision diagram looks very much like a drawing of tree , therefore ,also called decision –tree. A decision tree consists of nodes , branches , probability estimates ,and pay offs . There are two types of nodes : *decision nodes and chance nodes* . A decision node is usually represented a square  and indicates places where a decision-maker must make a decision. Each branch leading away from a decision node represents one of the several possible courses of action available to the decision-maker . The *chance node* is represented by a circle  and indicates a point at which the decision –maker will discover the response to his decision , i.e. different possible outcomes (states of nature , competitors actions , etc.) which can result from a chosen course of action .

Branches emanate from and connect various nodes are either decisions or states of nature . There are two types of branches: *Decision branches and chance branches*. Each branch leading away from a decision node represents a course of action or strategy that can be chosen at this decision point, whereas a branch leading away from a chance node represents the state of nature of a set of chance factors . Associated probabilities are indicated along side of respective chance branch. These probabilities are the likelihood that the chance outcome will assume the value assigned to the particular branch , Any branch that makes the end of the decision tree , i.e . it is not followed by either a decision or chance node , is called *a terminal branch* . Aterminal branch can represent either a course of action or a chance outcome. The terminal points of a decision tree are said to be mutually exclusive points so that exactly one course of action will be chosen .

Example –10:

Grow fast company is evaluating ***four*** alternatives single-period investment opportunities whose returns are based on the state of the economy. The possible states of the economy and the associated probability distribution is as follow:

State : fair good great

Probability: 0.2 0.5 0.3

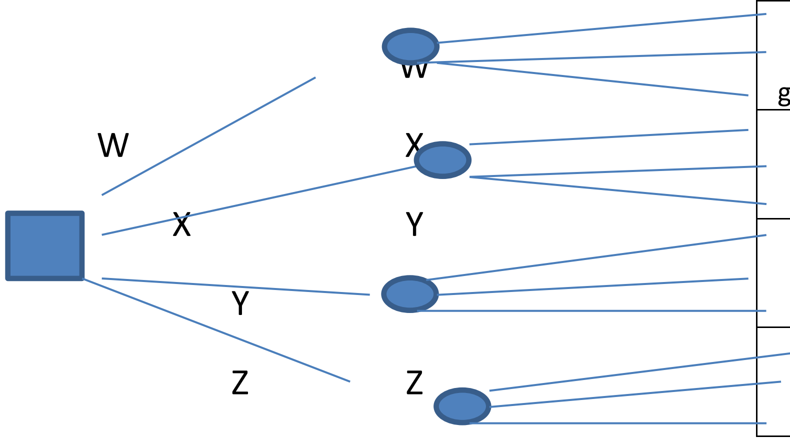
The return for each investment opportunity and each state of the economy are as follows:

State of economy			
alternative	fair	good	great
W	1000	3000	6000
X	500	4500	6800
Y	0	5000	8000
Z	-4000	6000	8500

Using the decision-tree approach , determine the expected return for each alternative , which alternatives investment proposed would you recommend if the expected monetary value criterion is to be employed.

Solution:

The decision-tree diagram for the problem is given as follow:



state	return	probability	EMV
Fair	1000	0.2	
Good	3000	0.5	3500
great	6000	0.3	
Fair	500	0.2	
Good	4500	0.5	4390
great	6800	0.3	
Fair	0	0.2	
Good	5000	0.5	***4900***
great	8000	0.3	
Fair	-4000	0.2	
Good	6000	0.5	4750
great	8500	0.3	

Since the expected return for alternative Y is the maximum, alternative it should be selected investment .

Example-11:

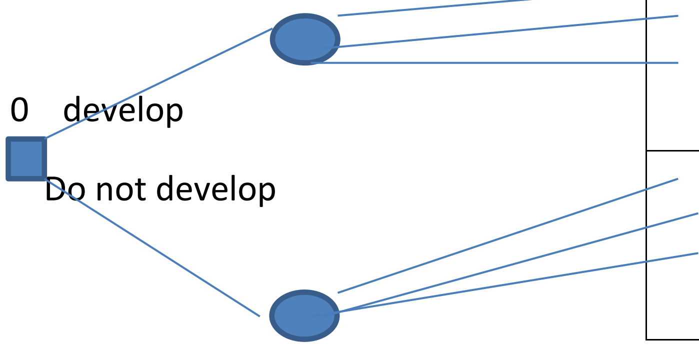
You are given the following estimates concerning are search and development program :

Decision Di	Probability of decision pi given research R P(Di/R)	Outcome number	Prob. Of outcome Xi given Di p(Xi /Di)	Pay off value of outcome Xi
develop	0.5	1 2 3	0.6 0.3 0.1	600 -100 0
Do not develop	0.5	1 2 3	0.0 0.0 1.0	600 -100 0

Construct and evaluate the decision tree diagram for the above data. Show your working for evaluation

Solution:

The decision tree of the given problem along with necessary calculation is shown as follows :



probability	Pay off	Expected pay off
0.5 * 0.6	600	180
0.5 *0.3	-100	-15
0.5 *0.1	0	0
		165
0.5 *0	600	0
0.5 *0	-100	0
0.5 *0	0	0
		0

Example---12:

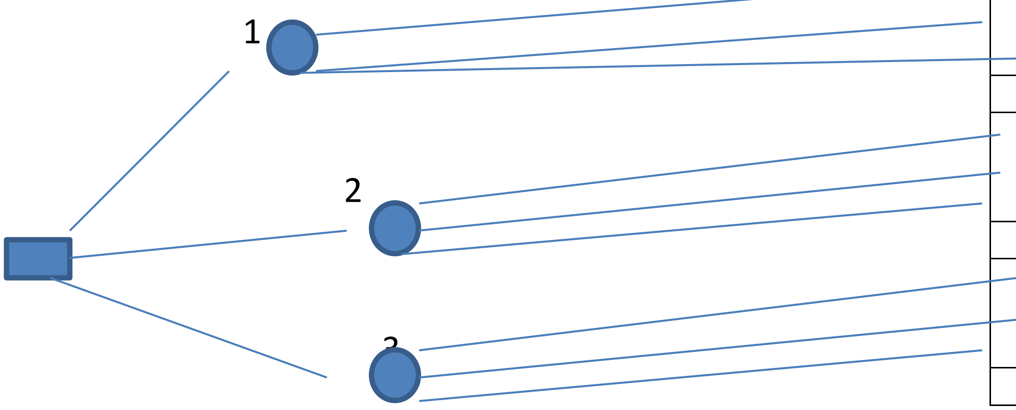
A glass factory specializing in crystal is developing a substantial backlog and the firm's management is considering three courses of action :*Arrange for sub-contracting (S1), begin overtime production (S2) , and construct new facilities (S3)* . The correct choice depends largely upon future demand which may be *low , medium ,or high* .By consensus , management ranks the respective probabilities as (0.10 ,0.50 and 0.40). A cost analysis reveals effect upon the profits that is shown in the table below:

		Course of action		
demand	probability	S1(subcontracting)	S2(begin over time)	S3(construct facilities)
Low(L)	0.10	10	-20	-150
Medium (M)	0.50	50	60	20
High(H)	0.40	50	100	200

Show this decision situation in the form of a decision tree and indicate the most preferred decision and corresponding expected value.

Solution:

A decision tree which represents possible courses of action and states of nature are shown in following figure . In order to analyze the tree, we start working back word from the end branches : the most preferred decision at the decision node (0) is found by calculating expected value of each decision branch and selecting the path (course of action) with high value.



Expected pay off
0.10 *10 =1
0.50* 50=25
0.40 *50 =20
46
0.10 *-20 =-2
0.50 *60 =30
0.40 * 100=4
68
0.10 * -150 =-15
0.50 *20 =10
0.40 *200=80
75

Since node (3) has the highest EMV (75) , therefor the decision at node (0) will be to choose the course of action   S3 .