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Architectural Engineering Department
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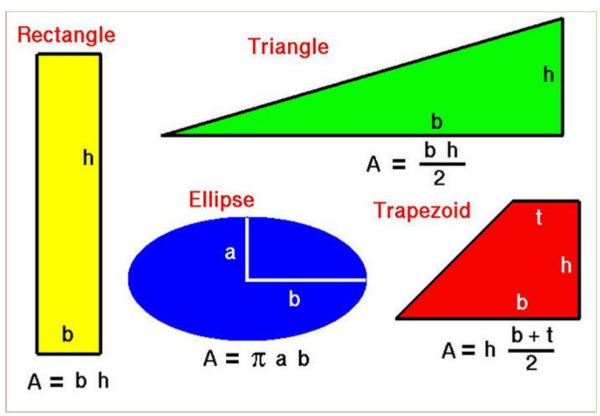
MSc in Structural Engineering

Engineering Surveying
Theory 5:Area

Introduction

One of the primary objects of most land surveys is to determine the area of the tract and volume of earth works. Areas are considered first of all, since the computation of areas is involved in the calculation of volumes.



Conversions (Review):

$1\ km^2 = 10^6\ m^2$

$1\ hectare = 10^4\ m^2$

$1\ km^2 = 100\ hectare$

$1\ acre = 4046.8452\ m^2$

$1\ acre = 42560\ ft^2$

$1\ Donum = 2500\ m^2$

Methods of Area Determination

A. Field Measurement:

1. Dividing the Area into Triangles:

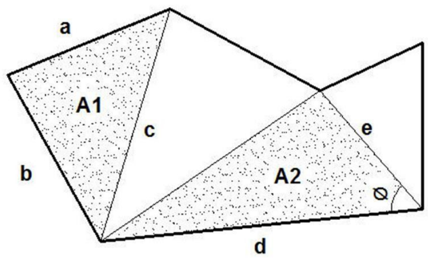
In this method, the area is divided into smaller triangles then the area of each segment is determined then the total area will be the summation of all.

The area of the triangle can be determined as the following:

$$A_1 = \sqrt{S(S-a)(S-b)(S-c)}$$

$$while\ \ S = \frac{1}{2}(a+b+c)$$

$$A_2 = \frac{1}{2}.d.e.\sin\theta$$



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Methods of Area Determination

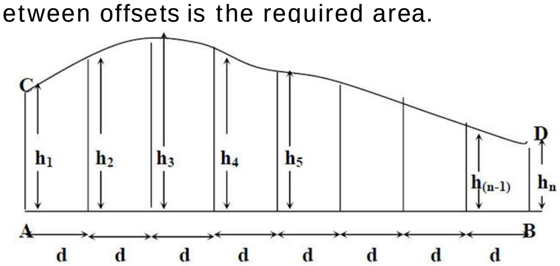
2. Trapezoidal rule:

Let the figure below represent a position of a tract laying between a traverse line AB and irregular boundary CD, offsets $h_1, h_2, h_3, \dots\dots\dots, h_n$ having been taken at the regular intervals d. The summation of the areas of the trapezoids comprising the total area is:

$$A = d\left[\left(\frac{h_1+h_n}{2}\right) + (h_2 + h_3 + \cdots h_{n-1})\right]$$

$$A = d[(average\ of\ end\ offsets) + (sum\ of\ intermediate\ offsets)]$$

Trapezoidal Rule : Add the average of the end offsets to the sum of the intermediate offsets. The product of the quantity thus determined the common interval between offsets is the required area.



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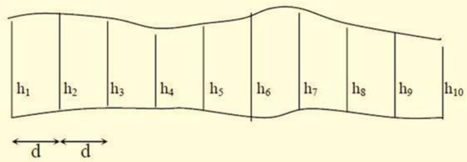
Example: Calculate the area of the plot shown in the figure below, if d=25 ft.

Offset	h_1	h_2	h_3	h_4	h_5	h_6	h_7	h_8	h_9	h_{10}
Length (ft)	55	60	53	48	54	68	65	57	55	50



Solution:

$$A = d\left[\left(\frac{h_1+h_n}{2}\right) + (h_2 + h_3 + \cdots h_{n-1})\right]$$

$$A = d[(average\ of\ end\ offsets) + (sum\ of\ intermediate\ offsets)]$$

$$\text{Area} = d\left[\frac{(h_1 + h_{10})}{2} + h_2 + h_3 + h_4 + h_5 + h_6 + h_7 + h_8 + h_9\right]$$

$= 25[\,(55 + 50\,)/2 + 60 + 53 + 48 + 54 + 68 + 65 + 57 + 55\,]$

$= 25[\,(52.5) + (460)\,] = 25\,(512.5) =$ **12812.5 ft²**

$= 12812.5/9 =$ **1423.611 yd²** $= 1423.611/4840$ acres $=$ **0.294 acres**

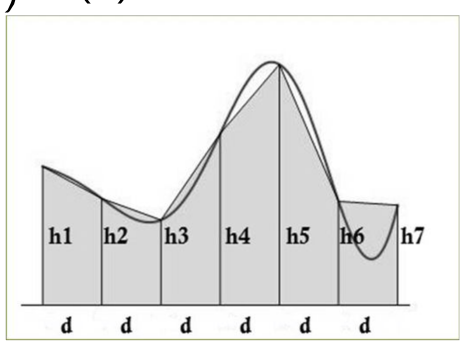
Methods of Area Determination

3. Average Height:

$$A = \left(\frac{\sigma\, h}{n}\right) * (n-1)d$$

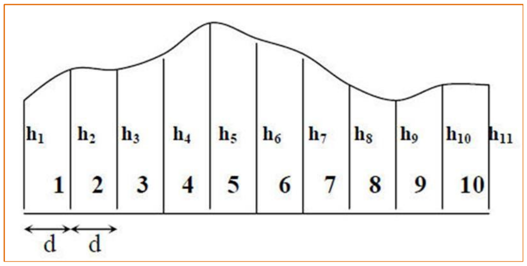
Example: *for the figure below, determine the area.*

$$A = \left(\frac{h_1+h_2+h_3+h_4+h_5+h_6+h_7}{7}\right) * 6(d)$$



Methods of Area Determination

4. Simpson`s rule (Simpson`s one-third rule):

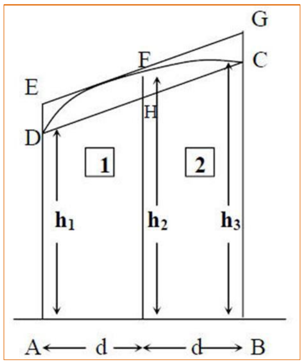


$A_{1,2} = ABCFDA$

$A_{1,2} = Trapezoid\ ABCHDA + Area\ CHDFC$

$A_{1,2} = \left[\frac{(h_1+h_3)}{2}.2d\right] + \left[\frac{2}{3}\left(h_2 - \frac{(h_1+h_3)}{2}\right).2d\right]$

$A_{1,2} = \frac{d}{3}(h_1 + 4h_2 + h_3)$



Similarly for the next intervals:

$$A_{3,4} = \frac{d}{3}(h_3 + 4h_4 + h_5)$$

The summation of these partial areas for (n –1) intervals, n being an odd number and representing the number of offsets , is;

$$Area = \frac{d}{3}[\{h_1 + h_n\} + 2\{h_3 + h_5 + h_7 + \ldots\ldots + h_{n-2}\} + 4\{h_2 + h_4 + h_6 + \ldots\ldots + h_{n-1}\}]$$

$$= d/3\,[\,(sum\ of\ first\ and\ last\ offsets) + 2(sum\ of\ remaining\ odd\ offsets) + 4(sum\ of\ the\ even\ offsets)\,]$$

$$A_{total} = \frac{d}{3}[h_1 + h_n + 4*\ h_{even} + 2*\ h_{odd}]$$

Example

Example :In a survey the following offsets were taken to a fence from a traverse line .

Chainage (ft)	0	20	40	60	80	100	120	140	160	180	200
Offset (ft)	55	60	58	62	70	65	63	58	54	57	56

Find the area between the fence and the traverse line in acres by the Simpson's One-third Rule.

Solution :

d = (200 – 180) = (180 – 160) = ... = (40 – 20) = (20 – 0) = 20 ft

h_1=55, h_2= 60, h_3= 58, h_4= 62, h_5= 70, h_6= 65, h_7= 63, h_8= 58, h_9= 54, h_{10}= 57, h_{11}= 56

Area = d/3[(sum of first and last offsets) + 2(sum of remaining odd offsets) + 4(sum of even offsets)]

$= d/3[\{h_1 + h_n\} + 2\{h_3 + h_5 + h_7 + ... + h_{(n-2)}\} + 4\{h_2 + h_4 + h_6 + ... + h_{(n-1)}\}]$

$= d/3[\{h_1 + h_n\} + 2\{h_3 + h_5 + h_7 + h_9\} + 4\{h_2 + h_4 + h_6 + h_8 + h_{10}\}]$

= 20/3[{55 + 56} + 2{58 + 70 + 63 + 54} + 4{60 + 62 + 65 + 58 + 57}]

= 20/3[{111} + 2{245} + 4{302}] = 20/3[{111} + {490} + {1208}]

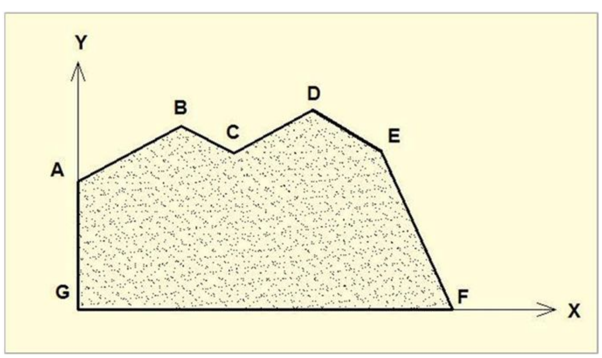
= 20/3[1809] = **12060 ft²** = 12060/9 yd² = **1340 yd²**

= 1340/4840 acres = **0.2768 acres**

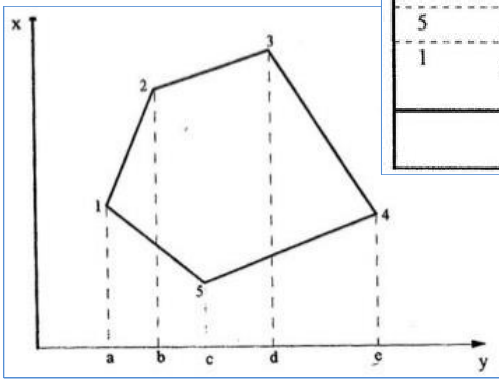
Methods of Area Determination

5. Using Coordinate of land corners: In this method, the coordinates (x,y) of all the points should be known, then by using the formula below, the area can be determined.

$A = \frac{1}{2}[(X_A.Y_B + X_B.Y_C + X_C.Y_D + X_D.Y_E + X_E.Y_F + X_F.Y_G + X_G.Y_A) - (Y_A.X_B + Y_B.X_C +$



Methods of Area Determination



Point	y	x	Positive product (Solid product)	Negative product (Dashed product)
1	y_1	x_1		
2	y_2	x_2	$x_1 y_2$	$y_1 x_2$
3	y_3	x_3	$x_2 y_3$	$y_2 x_3$
4	y_4	x_4	$x_3 y_4$	$y_3 x_4$
5	y_5	x_5	$x_4 y_5$	$y_4 x_5$
1	y_1	x_1	$x_5 y_1$	$y_5 x_1$
			Sum 1	Sum 2

$$\text{Area} = \frac{1}{2}\left(|\text{Sum1 - Sum 2}| \right)$$

Methods of Area Determination

Example: Find the area of the following closed loop traverse (ABCDEA):

Station	y (ft)	x (ft)
A	-57.41	-231.66
B	-311.26	-79.49
C	-31.66	123.48
D	62.35	309.11
E	172.76	-19.44

Solution:

Point	y	x	Positive product (Solid product)	Negative product (Dashed product)
1	-57.41	-231.66		
2	-311.26	-79.49	72,106.49	4,563.52
3	-31.66	123.48	2,516.65	-38,434.38
4	62.35	309.11	7,698.98	-9,786.42
5	172.76	-19.44	53,401.84	-1,212.08
1	-57.41	-231.66	1,116.05	-40,021.58
			136,840.01	-84,890.94

Area = 0.5(| 136840.01 – (-84890.94) |)
= 110865.48 ft^2

Methods of Area Determination

B. Map Measurement:

1. Dividing the Area into Triangles:

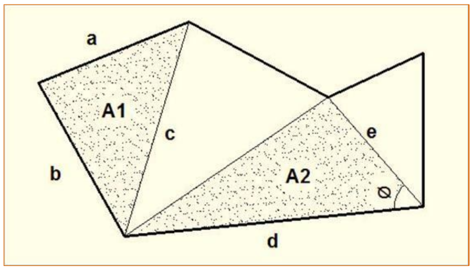
In this method, the area is divided into smaller triangles then the area of each segment is determined then the total area will be the summation of all.

the area of the triangle can be determined as the following:

$$A_1 = \sqrt{S(S-a)(S-b)(S-c)}$$

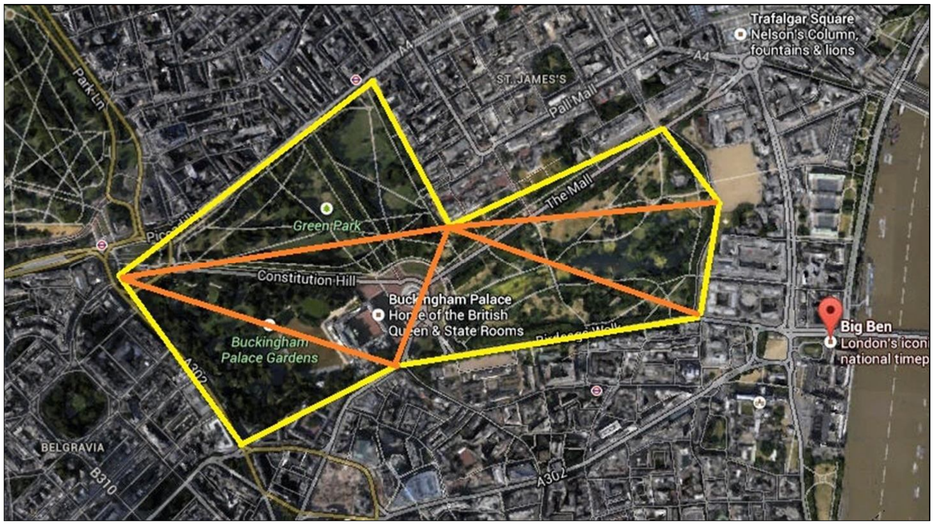
$$while: \; S = \frac{1}{2}(a+b+c)$$

$$A_2 = \frac{1}{2}.d.e.\sin\theta$$



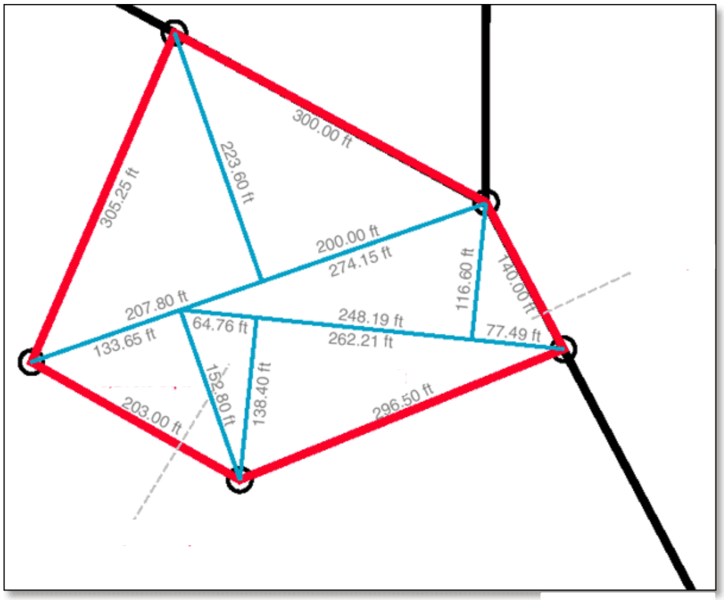
Example

Example:



HW

HW: Find area for this map which has all required dimensions?



Methods of Area Determination

2. Using Graphical Paper:

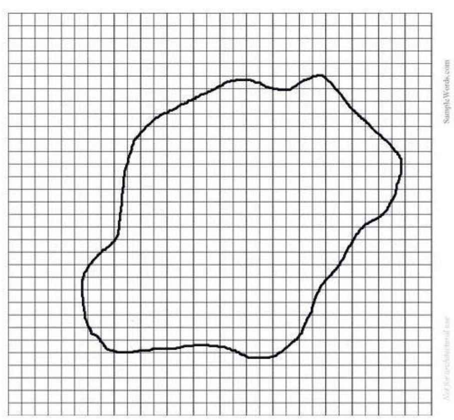
$$\mathbf{A = n.A_i}$$

Where:

$A = Total\ Area$

n = Number of squares

$A_i = Area\ of\ one\ square$



Methods of Area Determination

3. Planimeter:

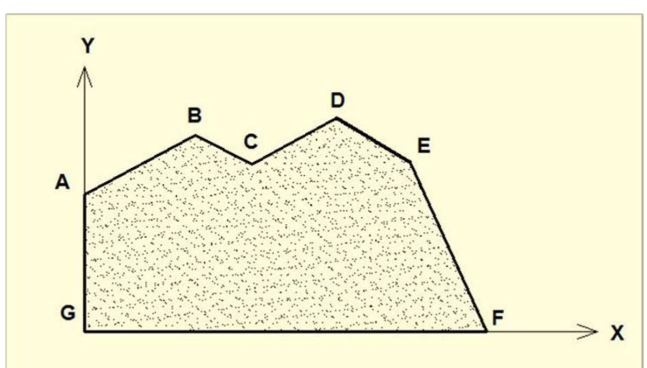
A planimeter (also known as a platometer) is a measuring instrument used to determine the area of an arbitrary two-dimensional shape.

- https://www.youtube.com/watch?v=pvGuGaImTek

Methods of Area Determination

4. Using Coordinate of land corners: In this method, the coordinates (x , y) of all the points should be known, then by using the formula below, the area can be determined.

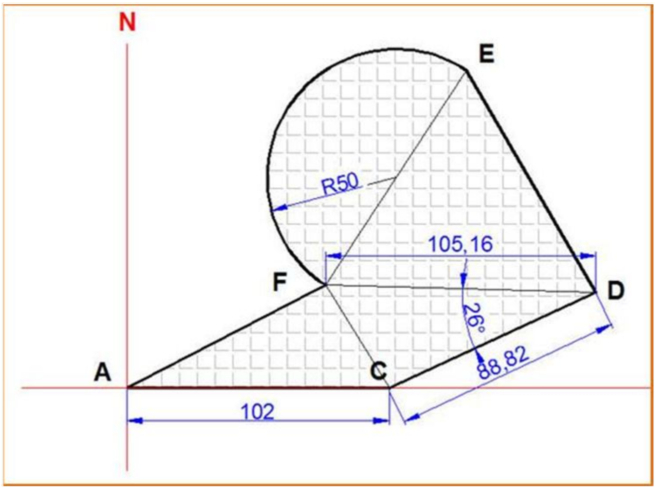
A = $\frac{1}{2}[(X_A.Y_B + X_B.Y_C + X_C.Y_D + X_D.Y_E + X_E.Y_F + X_F.Y_G + X_G.Y_A) - (Y_A.X_B +$



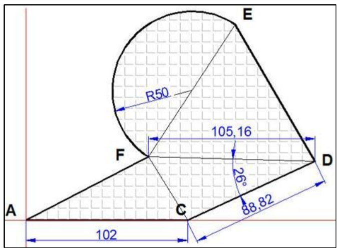
Example

Example: Calculate the shaded area shown in the figure below.

Given: A (503.42, 1710.48), F (580.62, 1750.60), ED=EF



Solution:



$Area_{(half\,circle)} = \frac{D^2\,\pi}{8} = \frac{100^2\,\pi}{8} = 3927$

$Area_{(EFD)}$:

$S = \frac{1}{2}(100 + 100 + 105.16) = 152.58$

$Area_{(EFD)} = \sqrt{152.58 * (152.58 - 100) * (152.58 - 100) * (152.58 - 105.16)} = 4472.5$

$Area_{(CDF)} = \frac{1}{2} * 105.16 * 88.82 * \sin 26 = 2047.26$

$Angle_{(A)} = tan^{-1}\frac{\Delta y}{\Delta x} = tan^{-1}\frac{1750.60 - 1710.48}{580.62 - 503.42} = tan^{-1}\frac{40.12}{77.2} = 27.46^{o}$

$- Length_{(AF)} = \sqrt{40.12^2 + 77.2^2} = 87$

$Area_{(AFC)} = \frac{1}{2} * 87 * 102 * \sin 27.46 = 2046$

$Total\,area = 12492.5$