University of Cihan-Sulaimaniya
Engineering Faculty
Architectural Engineering Department

ENGINEERING MECHANICS

Chapter 5: Equilibrium of Rigid Body

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Chapter Description

- Aims
  - To transform the rigid body into free-body diagram
  - To apply the equation of equilibrium in the rigid body
- Expected Outcomes
  - Able to determine the forces involved in the rigid body using equation of equilibrium
- References
  - Russel C. Hibbeler and Kai Beng Yap (2013) Engineering Mechanics: Statics & Dynamics, 13th Edition

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Chapter Outline

1. Introduction of Equilibrium
2. Free-Body Diagrams
3. Equations of Equilibrium
4. Example Calculation

1.1 Introduction of Equilibrium

What is equilibrium?

A body is in the static motion, not move, not rotate, or moving with constant velocity

A body exposed to the 3 forces there are:

1) External Force
2) Couple moment system

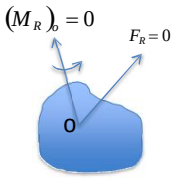
Affected by gravitational, electrical, magnetic, or contact force caused by adjacent bodies

3) Internal Force → Interaction between particles within the bodies

Equilibrium equation of a body at point O:

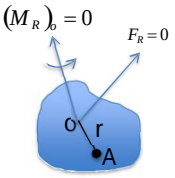
$$F_R = \sum F = 0 \quad \textit{(zero)}$$

$$(M_R)_o = \sum M_{\substack{O \\ F_R = 0}} = 0 \quad \textit{(zero)}$$



Equilibrium equation of a body at point A:

$$\sum M_A = r \times F_R + (M_R)_o = 0$$



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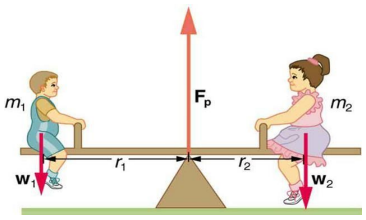
5.2 Free-Body Diagrams (FBDs)

What is FBDs?

1. Sketch all the forces and couple moments surroundings apply on a body.
2. Primary importance to solve the problems in mechanics



Source: https://www.boundless.com

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5.2 Free-Body Diagrams

Support Reactions:

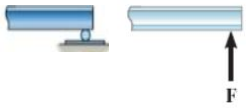
1) Force caused by the supports and points which contacted to body subjected to coplanar force systems

2) If a support prevents the translation of a body in a given direction, means that a force is developed on the body in that direction

3) If rotation is prevented, a couple moment exerted on the body

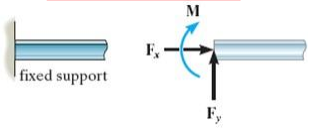
5.2 Free-Body Diagrams

Roller



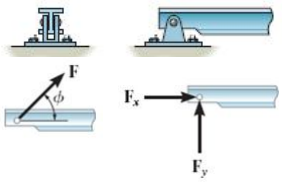
Prevent from translating in the vertical direction. Therefore, only vertical force will be exerted in this direction

Fixed Support



Prevent from translation and rotation of a beam. Thus, All forces and moment at this support must be developed

Pin



Prevent from translating in any direction. Therefore, it involves resultant force which comes from component Fx and Fy

Procedure of FBDs:

1) Draw the outline of body shape
2) Indicate all dimensions of the body
3) Allocate all forces and couple moments act on the body
4) Label their magnitudes and directions

Example Problem:

Draw the FBDs for the Figure below:

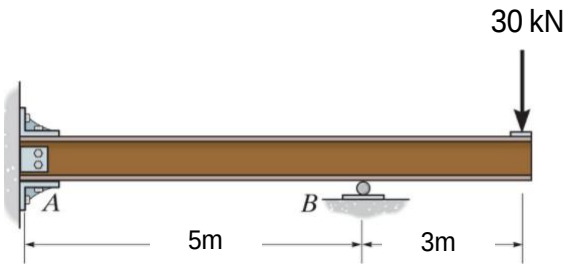


Figure 1: Fixed beam

Source: http://www.chegg.com

Example Solution:

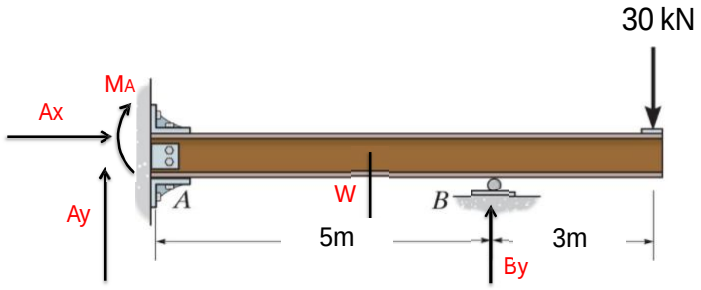


Figure 1: Fixed beam

Source: http://www.chegg.com

5.3 Equations of Equilibrium

- For equilibrium of a rigid body in 2D,

$$\sum F_x = 0$$
$$\sum F_y = 0$$
$$\sum M_O = 0$$

- $\sum F_x$ is sum of all forces in x-axis
- $\sum F_y$ is sum of all forces in y-axis

- $\sum M_O$ is sum of the couple moments and moments of forces due to point origin (o)

Procedure of Equilibrium Equation:

1) After draw FBDs, apply equation of equilibriums

$$\sum F_x = 0$$
$$\sum F_y = 0$$
$$\sum M_O = 0$$

2) For the moment at point O, all the forces must be considered and sign of the moment based on the rotation
3) Use 3 equilibrium equations in determining third unknown
4) Negative result shows the direction of the determined force in opposite

EXAMPLE CALCULATION

Example Problem 1:

Determine the horizontal and vertical components of reaction on the beam as shown in Figure 2 below:

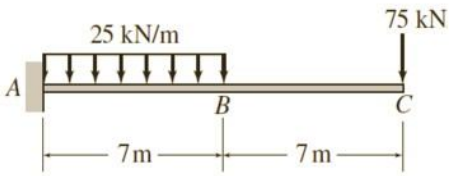
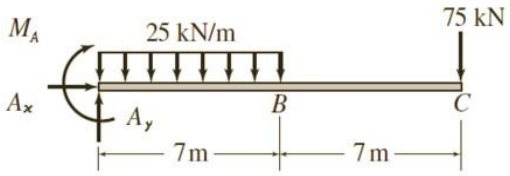


Figure 2: Beam with load

Solution:

1) FBDs



2) Find the force at support system using equilibrium equation

Answer: Ax = 0 kN, Ay = 250 kN, M_A = 1662.5 kN

Conclusion of The Chapter 5

- Conclusions
  - The FBDs diagram have been introduced and applied to solve the equilibrium problems for the rigid body