University of Cihan-Sulaimaniya
Engineering Faculty
Architectural Engineering Department

# ENGINEERING MECHANICS

## Chapter 4: Force System Results

2nd Grade- Fall Semester 2023-2024

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## Chapter Description

- Aims
  - To explain the Moment of Force (2D-scalar formulation & 3D-Vector formulation)
  - To explain the Principle Moment
  - To explain the Moment of a Couple
  - To explain the Simplification of a Force and Couple System
  - To explain the Reduction of Simple Distributed Loading
- Expected Outcomes
  - Able to solve the problems of MOF and COM in the mechanics applications by using principle of moments
- References
  - Russel C. Hibbeler. Engineering Mechanics: Statics & Dynamics, 14th Edition

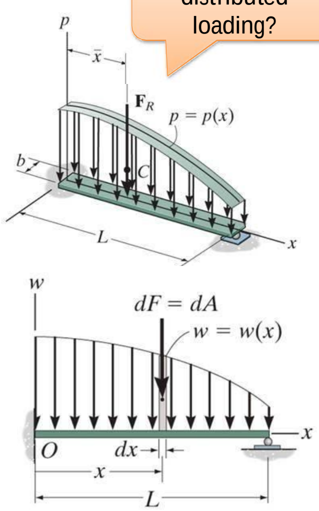
# Chapter Outline

1. Moment of Force (MOF) –Part I
2. Principle of Moment –Part II
3. Moment of Couple (MOC) Part III
4. Simplification of a Force and Couple System
5. Reduction of Simple Distributed Loading- part IV



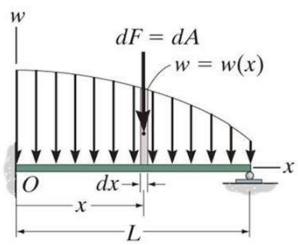
# 4.5 Reduction of a simple distributed loading

What is distributed loading?




- In many situations, a surface area of a body is subjected to a **distributed load**
- Such forces are caused by **winds, fluids, or the weight** of items on the body's surface
- Distributed loadings are defined by using a loading function $w = w(x)$ that indicates the intensity of the loading along the length of the member
- Intensity is measured in N/m
- The external effects caused by a coplanar **distributed load** acting on a body can be represented by a **single resultant force**
- The **magnitude** of the **resultant force** is equal to the **total area under the distributed loading diagram** $w = w(x)$
- The **location** of the **resultant force** is given by the fact that its line of action passes through the **centroid** or **geometric center** of this area

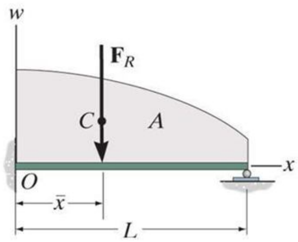
## Magnitude of resultant force




Consider an element of length dx.

The force magnitude dF acting on it is given as

$$dF = w(x)\,dx$$




The net force on the beam is given by

$$+\downarrow \; F_R = \int_L dF = \int_L w(x)\,dx = A$$

Here A is the area under the loading curve w(x).

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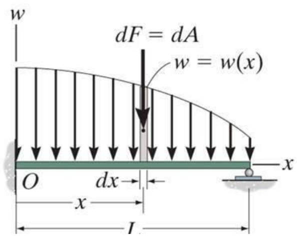
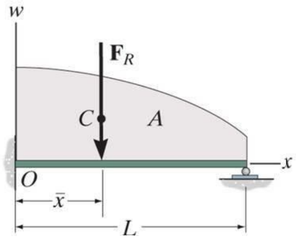
## Location of the resultant force




The force dF will produce a moment of (x)(dF) about point O.




Assuming that $F_R$ acts at x , it will produce the moment about point O as

$$\circlearrowleft + \; M_{RO} = (x)(F_R) = \int_L w(x)\,dx$$

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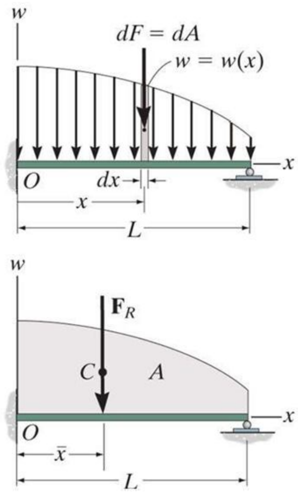
## Location of the resultant force




Comparing the last two equations, we get

$$\bar{x} = \frac{\int_L x w(x)\, dx}{\int_L w(x)\, dx} = \frac{\int_A x\, dA}{\int_A dA}$$

You will learn more detail later, but $F_R$ acts through a point “C,” which is called the geometric center or centroid of the area under the loading curve w(x).



## Example 4.11

Determine the concentrated loads (which is a common name for the resultant of the distributed load)

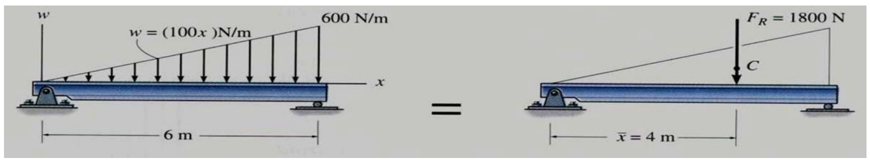

The rectangular load: *find the area of rectangular*

$F_R$ = 400 × 10 = 4,000 lb

$\bar{x} = \frac{10}{2} = 5$ ft *location is the centroid of rectangular*

## Example 4.12

Determine the concentrated loads (which is a common name for the resultant of the distributed load)




The triangular loading: *find the area of triangular*

$F_R = \frac{1}{2}(600)(6) = 1{,}800$ N and $\bar{x} = 6 - (1/3)\,6 = 4$ m

Please note that the centroid of a right triangle is at a distance one third the width of the triangle as measured from its base

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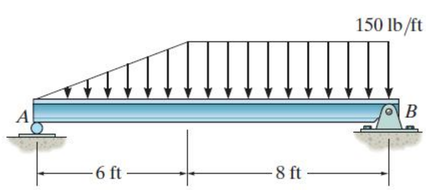
## Example 4.13

The loading on the beam as shown. Determine equivalent force and its location from point A.




1) The distributed loading can be divided into two parts. (one rectangular loading and one triangular loading).
2) Find $F_R$ and its location for each of the distributed loads.
3) Determine the overall $F_R$ of the point loadings and its location.

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## Solution Example 4.13

For the triangular loading of height 150 lb/ft and width 6 ft,

$F_{R1}$ = (0.5) (150) (6) = 450 lb

and its line of action is at $\bar{x}_1$ = (2/3)(6) = 4 ft from A

For the rectangular loading of height 150 lb/ft and width 8 ft,

$F_{R2}$ = (150) (8) = 1200 lb

and its line of action is at ¯$_2$= 6 + (1/2)(8) = 10 ft from A

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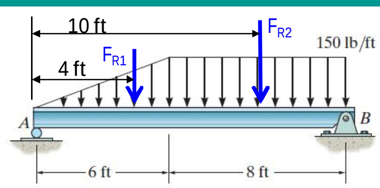
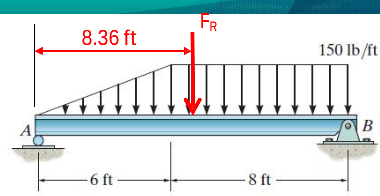
## Solution Example 4.13




The equivalent force and couple moment at A will be

$F_R$ = 450 + 1200 = 1650 lb

+↺ $M_{RA}$= 4 (450) +10(1200) = 13800 lb·ft

Since ($\bar{x}$ * $F_R$ ) has to equal $M_{RA}$ : 1650 * $\bar{x}$ =13800

Solve for $\bar{x}$ to find the equivalent force's location.

$\bar{x}$ = 8.36 ft from A.

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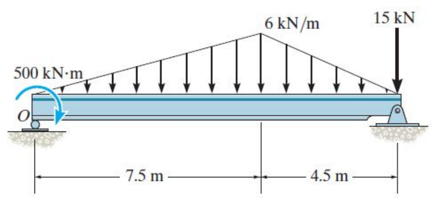
# Example 4.14

The loading on the beam as shown. Determine equivalent force and couple moment acting at point O




1) The distributed loading can be divided into two parts-two triangular loads
2) Find $F_R$ and its location for each of these distributed loads
3) Determine the overall $F_R$ of the point loadings and couple moment at point O

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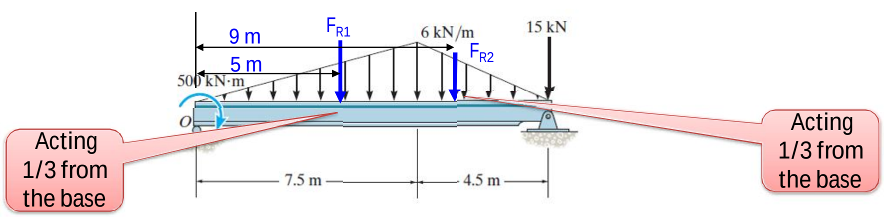
# Solution Example 4.14




For the triangular loading(right) of height 6 kN/m and width 7.5 m,

$F_{R1}$ = (0.5) (6) (7.5) = 22.5 kN

and its line of action is at $\bar{x}_1 = (2/3)(7.5) = 5$ m from O

For the triangular loading (left) of height 6 kN/m and width 4.5 m,

$F_{R2}$ = (0.5) (6) (4.5) = 13.5 kN

and its line of action is at $\bar{x}_2 = 7.5 + (1/3)(4.5) = 9$ m from O

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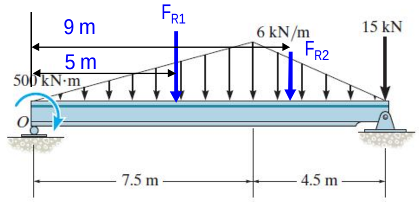
# Solution Example 4.14




For the combined loading of the three forces, add them.

$F_R = 22.5 + 13.5 + 15 = 51$ kN

The couple moment at point O will be

$+\circlearrowleft M_{RO} = 500 + 5\,(22.5) + 9\,(13.5) + 12\,(15) = 914$ kN·m

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# Conclusion of The Chapter 4

- Conclusions
  - The reduction of force simple loading has been identified
  - The external effects caused by a coplanar **distributed load** acting on a body can be represented by a **single resultant force**
  - The reduction of force analysis have been implemented to solve resultant force and moment problems in specified axis

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# Thank you

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