University of Cihan-Sulaimaniya
Engineering Faculty
Architectural Engineering Department

ENGINEERING MECHANICS

Chapter 3: Equilibrium of a Particle

2nd Grade- Fall Semester 2023-2024

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Chapter Description

- Aims
  - To explain the Equilibrium Equation
  - To explain the Free Body Diagram
  - To apply the Equations of Equilibrium to solve particle equilibrium problems in Coplanar Force System (2-D &3-D)
- Expected Outcomes
  - Able to solve the problems of a particle or rigid body in the mechanics applications by using Equilibrium Equation
- References
  - Russel C. Hibbeler. Engineering Mechanics: Statics & Dynamics, 14th Edition

Chapter Outline

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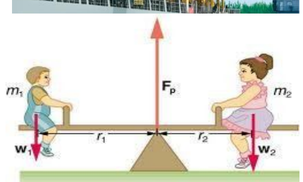
3.1 Equilibrium Equation

What is Equilibrium?

- **Equilibrium** means the **forces are balanced** but not necessarily equal
- In physic, it means equal balance which the **opposing forces or tendencies neutralize each other**

How to know the body is in Equilibrium?



- A body at rest or in uniform motion (velocity) is in equilibrium

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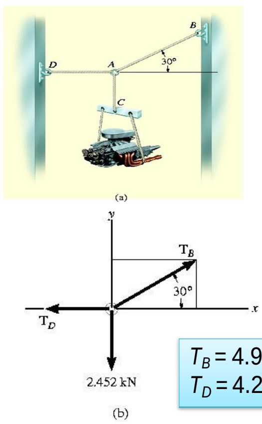
Condition for the Equilibrium of a Particle

How to know the body is in Equilibrium?



$T_B = 4.90kN$
$T_D = 4.25kN$

- Particle at *equilibrium* if
  - At rest
  -Moving at constant a constant velocity
- Newton's first law of motion

$$\sum \mathbf{F} = 0$$

where $\sum \mathbf{F}$ is the vector sum of all the forces acting on the particle

$+\rightarrow \sum F_x = 0; \quad T_B\cos30° - T_D = 0$
$+\uparrow \sum F_y = 0; \quad T_B\sin30° - 2.452kN = 0$

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Condition for the Equilibrium of a Particle

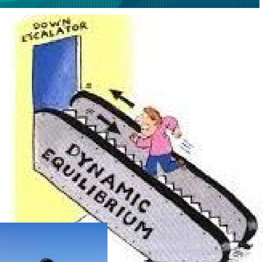
- Newton's second law of motion

$$\sum \mathbf{F} = m\mathbf{a}$$

- When the force fulfill Newton's first law of motion,

$$m\mathbf{a} = 0$$
$$\mathbf{a} = 0$$

therefore, the particle is moving in constant velocity or at rest



Static Equilibrium is when the body at rest

If the Dynamic Equilibrium, the body move and continue to move

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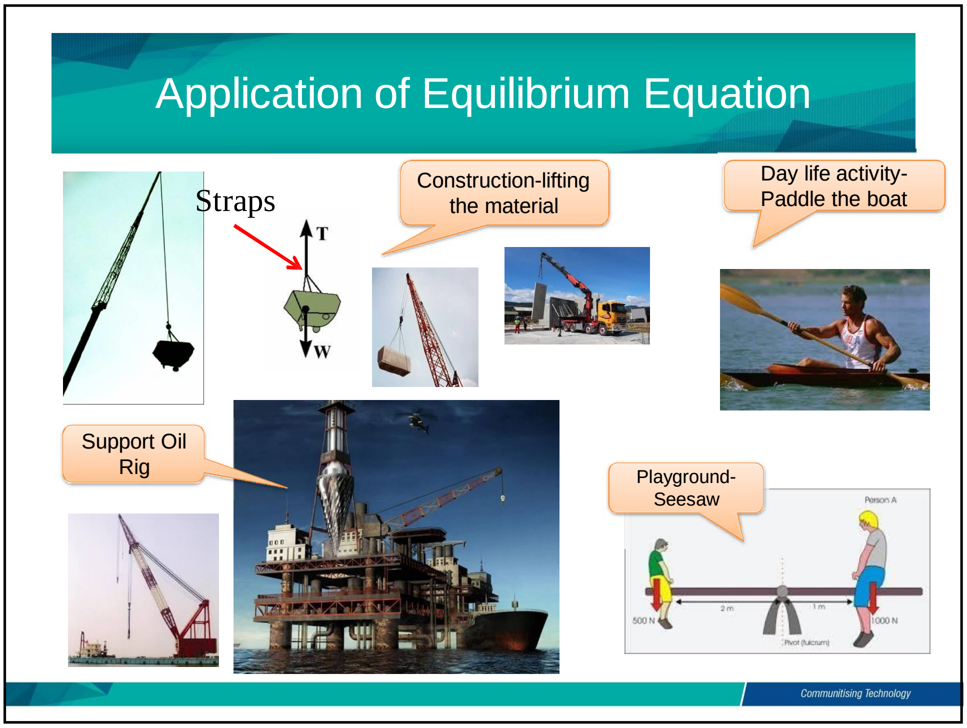
Application of Equilibrium Equation
Straps
T
W
Construction-lifting the material
Day life activity- Paddle the boat
Support Oil Rig
Playground- Seesaw
Person A
500 N
1000 N
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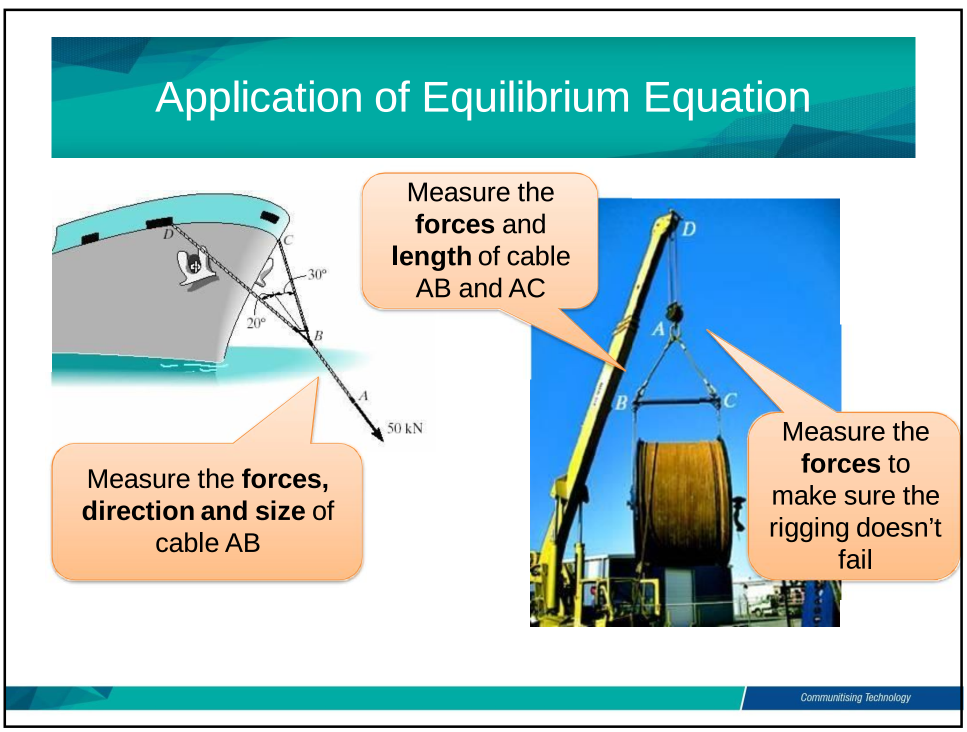
Application of Equilibrium Equation
Measure the **forces** and **length** of cable AB and AC
D
C
30°
20°
B
A
50 kN
Measure the **forces, direction and size** of cable AB
Measure the **forces** to make sure the rigging doesn't fail
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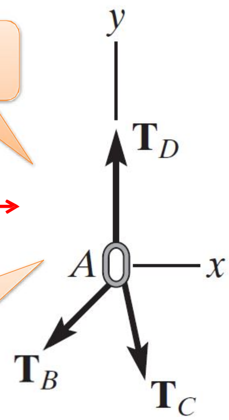
3.2 Free Body Diagram (FBD)

What is FBD?

FBD is a sketch to show only the **forces** acting on **selected** body



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3.2 Free Body Diagram (FBD)

- Best representation of all the unknown forces (∑**F**) which acts on a body
- A sketch showing the particle "free" from the surroundings with all the forces acting on it

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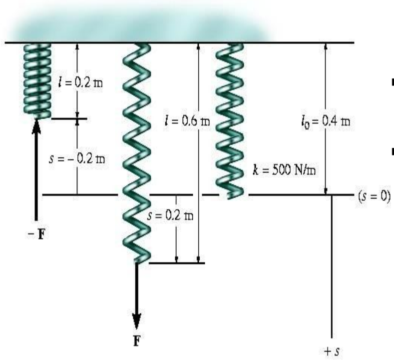
Spring



- Linear elastic spring: **change** in **length** is **directly proportional** to the **force** acting on it
- *spring constant or **stiffness k***: defines the ***elasticity*** of the spring
- **Magnitude of force** when spring is **elongated** or **compressed**

$$F = ks$$

- where ***s*** is determined from the difference in spring's deformed length ***l*** and its undeformed length l_o

$s = l - l_o$

$$s = l - l_o$$

- If ***s*** is positive, **F** "pull" onto the spring
- If ***s*** is negative, **F** "push" onto the spring

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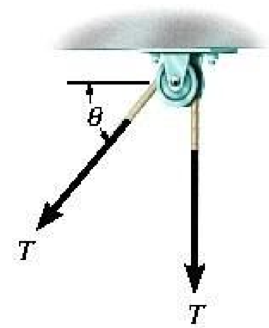
Cables and Pulley

- Cables (or cords) are assumed to have negligible weight and they cannot stretch
- A cable only support tension or pulling force
- Tension always acts in the direction of the cable
- Tension force in a continuous cable must have a constant magnitude for equilibrium
- For any angle θ, the cable is subjected to a constant tension ***T*** throughout its length



Cable is in tension

With a frictionless pulley and cable $T_1 = T_2$.

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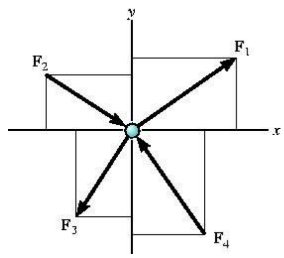
3.3 Coplanar Systems 2-D



- A particle is subjected to coplanar forces in the x-y plane
- Resolve into **i** and **j** components for equilibrium

  $\sum \mathbf{F}_x = 0$

  $\sum \mathbf{F}_y = 0$
- Scalar equations of equilibrium require that the algebraic sum of the x and y components to equal o (zero)

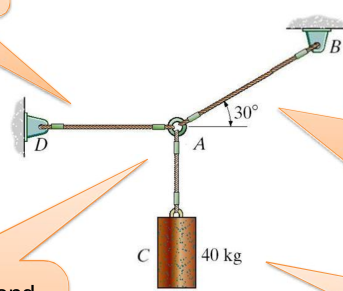
Scalar Notation

- Sense of direction = an algebraic sign that corresponds to the arrowhead direction of the component along each axis
- For unknown magnitude, assume arrowhead sense of the force
- Since magnitude of the force is always positive, if the scalar is negative, the force is acting in the opposite direction

Step to draw FBD

Step 1: Sketch outline shape

Step 2: Show all the forces that act on body and indicate the active (set the body in motion) or reactive forces (tend to resist the motion)



Step 3: Labeled the known forces (magnitude and direction)

Step 4: Apply EE and calculate the unknown forces(can be write in letters)

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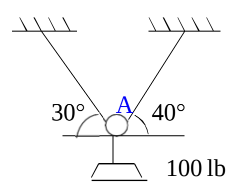
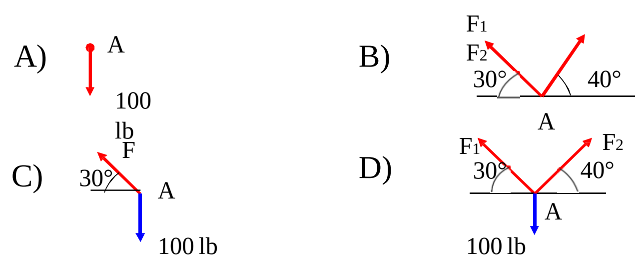
Select the correct FBD of Particle A





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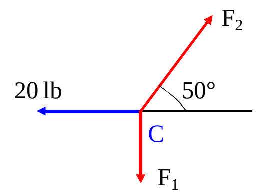
FBD

Using this FBD of Point C, the sum of forces in the x-direction (ΣF_X) is__. Use a sign convention of + → .



A) $F_2 \sin 50° - 20 = 0$

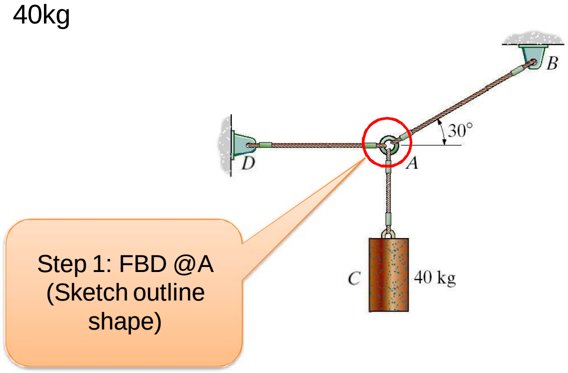
B) $F_2 \cos 50° - 20 = 0$

C) $F_2 \sin 50° - F_1 = 0$

D) $F_2 \cos 50° + 20 = 0$

Example 3.1

This is an example of a 2-D or coplanar force system. If the whole assembly is in equilibrium, then particle A is also in equilibrium. Determine the tensions in the cables for a given weight of cylinder = 40kg



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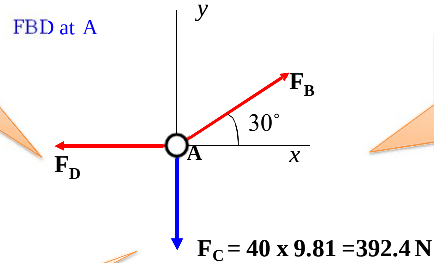
Solution Example 3.1

Step 4: Apply EE and calculate the unknown forces(can be write in letters)



Step 2:Show all the forces that act on body and indicate the active (set the body in motion) or reactive forces (tend to resist the motion)

Step 3: Labeled the known forces (magnitude and direction)

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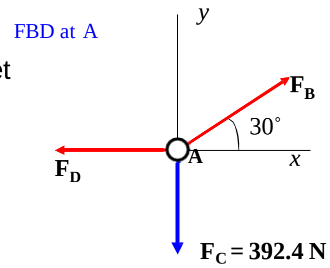
Solution Example 3.1

Step 4: Apply EE and calculate the unknown forces(can be write in letters)

Since particle A is in equilibrium, the net force at A is zero.

So $\boldsymbol{F_B} + \boldsymbol{F_C} + \boldsymbol{F_D} = 0$

or $\Sigma \boldsymbol{F} = 0$



In general, for a particle in equilibrium,

$\Sigma \boldsymbol{F} = 0$ or $\Sigma F_x \boldsymbol{i} + \Sigma F_y \boldsymbol{j} = 0 = 0\,\boldsymbol{i} + 0\,\boldsymbol{j}$ (a vector equation)

Or, written in a scalar form,
$\Sigma F_x = 0$ and $\Sigma F_y = 0$

- Two scalar equations of equilibrium (E-of-E)
- Used to solve for up to two unknowns

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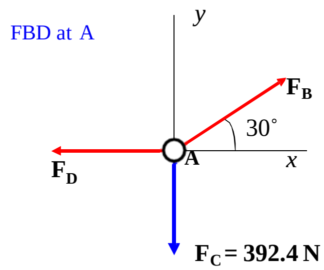
Solution Example 3.1

Write the scalar E-of-E:

$+\rightarrow \Sigma F_x = F_B \cos 30^o - F_D = 0$

$+\uparrow \Sigma F_y = F_B \sin 30^o - 392.4 \text{ N} = 0$

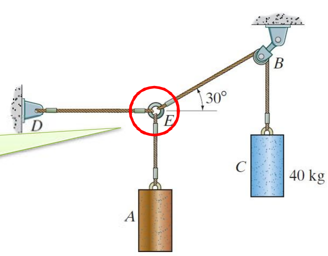


Solving the second equation, $F_B = 785$ N →

From the first equation, $F_D = 680$ N ←

Example 3.2

This is an example of a 2-D or coplanar force system. If the whole assembly is in equilibrium, then particle E is also in equilibrium. Determine the tensions in the cables DE,EA and EB for a given weight of cylinder = 40kg



Step 1: Draw FBD @ point E

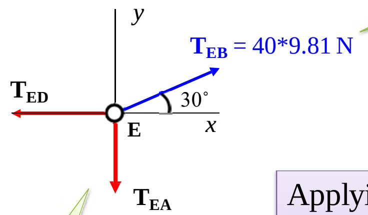
Step 2

Step 3

Solution Example 3.2

FBD at point E

Step 2



Step 3

Step 4

Applying the scalar E-of-E at E,

$$+\rightarrow \sum F_x = -T_{ED} + (40*9.81)\cos 30° = 0$$

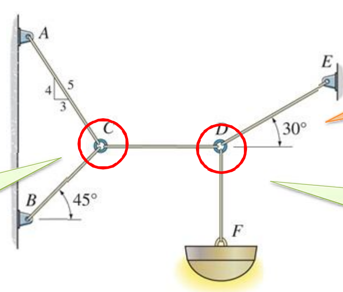
$$+\uparrow \sum F_y = (40*9.81)\sin 30° - T_{EA} = 0$$

Solving the above equations,

$T_{ED} = 340$ N $\leftarrow$ and $T_{EA} = 196$ N $\downarrow$

Example 3.3

This is an example of a 2-D or coplanar force system. If the whole assembly is in equilibrium, then particle C and D are in equilibrium. Determine the force in each cables for a given weight of lamp = 20kg



How many unknown forces?

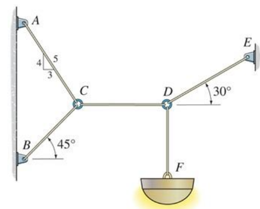
Step 2: Draw FBD @ point C to solve F_{CB} & F_{CA}

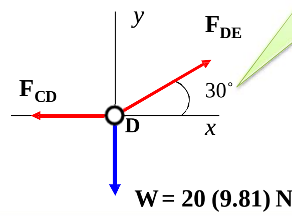
Step 1: Draw FBD @ point D to solve F_{CD} & F_{DE}

Solution Example 3.3

FBD at point D

Step 1: Draw FBD @ point D to solve F_{CD} & F_{DE}





Applying the scalar E-of-E at D,

$+\uparrow \sum F_y = F_{DE} \sin 30° - 20\,(9.81) = 0$

$+\rightarrow \sum F_x = F_{DE} \cos 30° - F_{CD} = 0$

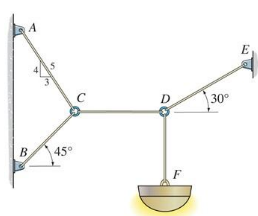
Solving the above equations,

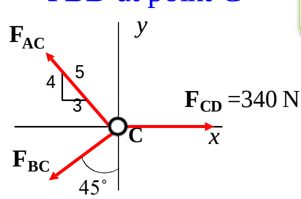
$F_{DE} = 392$ N and $F_{CD} = 340$ N

Solution Example 3.3

FBD at point C

Step 2: Draw FBD @ point C to solve F_{CB} & F_{CA}





Applying the scalar E-of-E at C,

$+\rightarrow \sum F_x = 340 - F_{BC} \sin 45° - F_{AC}\,(3/5) = 0$

$+\uparrow \sum F_y = F_{AC}\,(4/5) - F_{BC} \cos 45° = 0$

Solving the above equations,

$F_{BC} = 275$ N ↙ and $F_{AC} = 243$ N ↖

Conclusion of The Chapter 3

- Conclusions
  - The Equilibrium and FBD have been identified
  - The Equilibrium Equation have been implemented to solve a particle problems in Coplanar Forces Systems

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Thank you

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