University of Cihan-Sulaimaniya
Engineering Faculty
Architectural Engineering Department

ENGINEERING MECHANICS

Chapter 2: Force Vectors (Static)

2nd Grade- Fall Semester 2023-2024

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Chapter Description

- Aims
  - To review the Parallelogram Law and Trigonometry
  - To explain the Force Vectors
  - To explain the Vectors Operations (Parlaw & Cartesian)
  - To express force and position in Cartesian Vectors
- Expected Outcomes
  - Able to solve the problems of force vectors in the mechanics applications by using Cartesian Coordinate System
- References
  - Russel C. Hibbeler. Engineering Mechanics: Statics & Dynamics, 14th Edition

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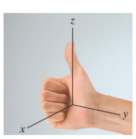
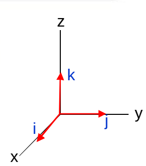
Chapter Outline

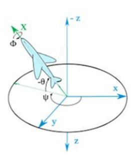
1. Scalars and Vectors – part I
2. Vectors Operations – part I
3. Vectors Addition of Forces – part I
4. **Cartesian Vectors – part II**
5. Force and Position Vectors – part III

2.4 Cartesian Vector

What is Cartesian Vector?







- It is a **coordinate system**
- Use to describe **position**
- Position can be defined by its **coordinate axis**
- It is a **unit vector** $\mathbf{u}_A = \mathbf{A} / A$
- Its magnitude is **1** and **dimensionless**
- It is denoted as **i,j,k**
- **i** is a unit vector pointing in the **x direction**
- **j** is a unit vector pointing in the **y direction**
- **k** is a unit vector pointing in the **z direction**
- **+ve** direction based on **right handed**
- It is important in **air transport**
- **Air Traffic controller** or **pilots** must know the **location** of every aircraft in the sky
- Without the coordinate system, the position or location of aircraft is **difficult** to know and may lead to **aircraft crashes**

Source:https://en.wikipedia.org/wiki/Axes_conventions

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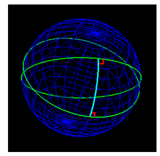
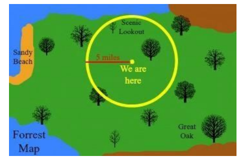
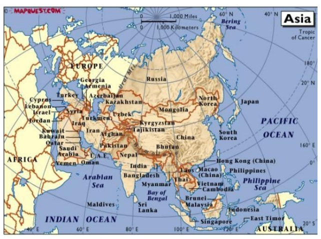
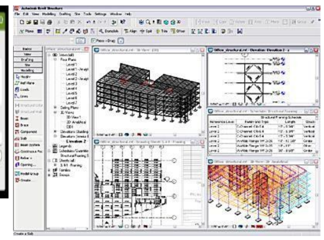
Application of Cartesian Vector





- Military Service
- Position of any body in the real world





- Location / Geographic /Latitude/longitude
- Mapping Project

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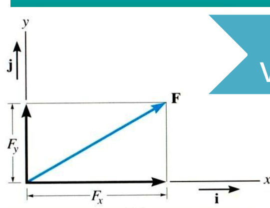
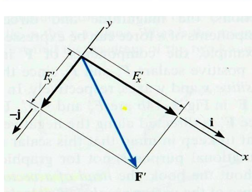
Application of Cartesian Vector

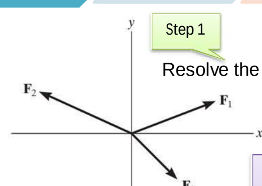
2-D vector → Resolve vector into Components → Addition vector



$$F = F_x\,i + F_y\,j$$



$$F' = F'_x\,i + (-F'_y)\,j$$



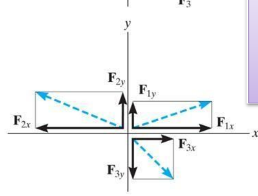
Step 1

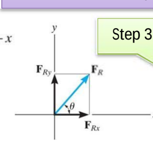
Resolve the vectors into X and Y components

Step 2

Then add them into respective components

$$F_R = F_1 + F_2 + F_3$$
$$= F_{1x}\,i + F_{1y}\,j - F_{2x}\,i + F_{2y}\,j + F_{3x}\,i - F_{3y}\,j$$
$$= (F_{1x} - F_{2x} + F_{3x})\,i + (F_{1y} + F_{2y} - F_{3y})\,j$$
$$= (F_{Rx})\,i + (F_{Ry})\,j$$





Step 3

$$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2}$$

$$\theta = \tan^{-1}\left|\frac{F_{Ry}}{F_{Rx}}\right|$$

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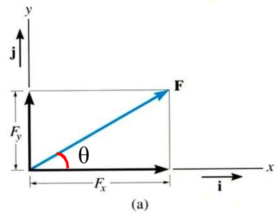
Application of Cartesian Vector

How to resolve into components Vector?

$\boldsymbol{F} = F_x \boldsymbol{i} + F_y \boldsymbol{j}$

$|F_X| = F_X = F \cos\theta$

$|F_Y| = F_Y = F \sin\theta$



The magnitude of ***F***

$$F = \sqrt{F_X^{\,2} + F_Y^{\,2}}$$

The direction of ***F***

$$\theta = \tan^{-1}\left|\frac{F_Y}{F_X}\right|$$

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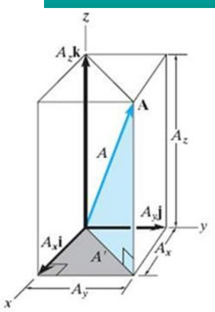
Application of Cartesian Vector



3-D vector → Resolve vector into components → Addition vector

The vector ***A*** can be resolved as

$\boldsymbol{A} = A_X \boldsymbol{i} + A_Y \boldsymbol{j} + A_Z \boldsymbol{k}$

Once the vectors are present in Cartesian term, it easy to add or subtract

Given

$\boldsymbol{A} = A_X \boldsymbol{i} + A_Y \boldsymbol{j} + A_Z \boldsymbol{k}$

$\boldsymbol{B} = B_X \boldsymbol{i} + B_Y \boldsymbol{j} + B_Z \boldsymbol{k}$

If ***A*** + ***B*** = ***C***

Sum of the vectors ***A*** and ***B*** can obtained vector ***C***

$$\mathbf{F}_R = \Sigma\mathbf{F} = \Sigma F_x\mathbf{i} + \Sigma F_y\mathbf{j} + \Sigma F_z\mathbf{k}$$

$$\boldsymbol{C} = (A_X \boldsymbol{i} + A_Y \boldsymbol{j} + A_Z \boldsymbol{k}) + (B_X \boldsymbol{i} + B_Y \boldsymbol{j} + B_Z \boldsymbol{k})$$
$$= (A_X + B_X)\boldsymbol{i} + (A_Y + B_Y)\boldsymbol{j} + (A_Z + B_Z)\boldsymbol{k}$$
$$= C_X \boldsymbol{i} + C_Y \boldsymbol{j} + C_Z \boldsymbol{k}$$

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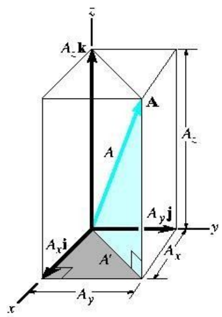
Application of Cartesian Vector

How to determine magnitude and direction angle in 3-D vector?

It should be noted that in 3-D vector information is given as:

- **Magnitude** and the **coordinate direction angles**, or
- **Magnitude** and **projection angles**

- Magnitude of a Cartesian Vector ***A*** in the x-y plane is A´



- From the colored triangle,

$$A = \sqrt{A'^2 + A_z^2}$$

- From the shaded triangle,

$$A' = \sqrt{A_x^2 + A_y^2}$$

- The magnitude of the position vector ***A***

-Combining the equations gives magnitude of **A**

$$A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

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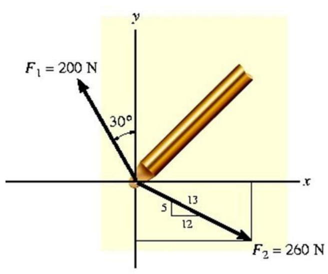
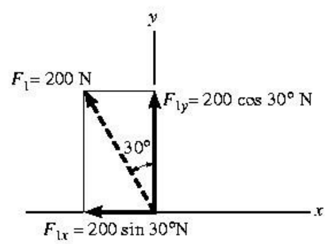
Example 2.4

Determine the X and Y components of F_1 and F_2 in Cartesian vectors



(a)



(b)

$$F_{1x} = -200\sin 30^{\circ}N = -100N = 100N \leftarrow$$

$$F_{1y} = 200\cos 30^{\circ}N = 173N = 173N \uparrow$$

Cartesian Vector Notation: $\mathbf{F}_1$ = {-100**i** +173**j** }N

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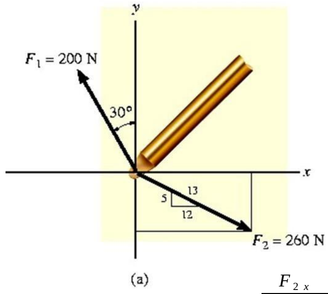
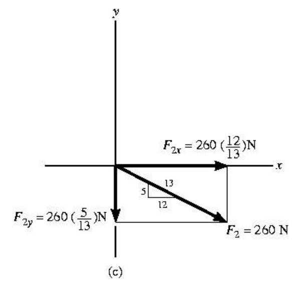
Example 2.4

Determine the X and Y components of F_1 and F_2 in Cartesian vectors





$$\frac{F_{2x}}{260\,N} = \frac{12}{13}$$

$$F_{2x} = 260\,N\left(\frac{12}{13}\right) = 240\,N \longrightarrow \qquad F_{2y} = 260N\left(\frac{5}{13}\right) = 100N \downarrow$$

Cartesian Vector Notation: $\mathbf{F}_2 = \{240\mathbf{i} - 100\mathbf{j}\}$N

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Example 2.5

Determine the magnitude and direction of the resultant force

Step 1

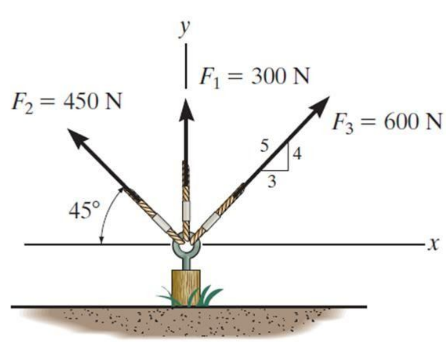
Resolve the forces into components x and y

Step 2

Then add the respective components to get the resultant forces

Step 3

Calculate the magnitude and direction from the resultant forces



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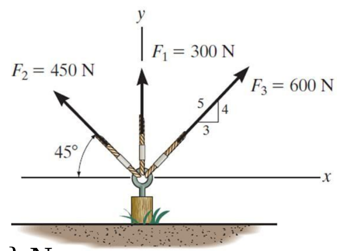
Solution Example 2.5

Step 1

Resolve the forces into components x and y



$\boldsymbol{F_1}$ = {0 $\boldsymbol{i}$ + 300 $\boldsymbol{j}$ } N

$\boldsymbol{F_2}$ = {– 450 cos (45°) $\boldsymbol{i}$ + 450 sin (45°) $\boldsymbol{j}$ } N

= {– 318.2 $\boldsymbol{i}$ + 318.2 $\boldsymbol{j}$ } N

$\boldsymbol{F_3}$ = { (3/5) 600 $\boldsymbol{i}$ + (4/5) 600 $\boldsymbol{j}$ } N

= { 360 $\boldsymbol{i}$ + 480 $\boldsymbol{j}$ } N

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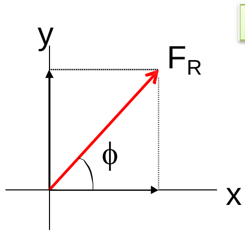
Solution Example 2.5

Step 2

- Summing up all the $\boldsymbol{i}$ and $\boldsymbol{j}$ components respectively:

$\boldsymbol{F_R}$ = { (0 – 318.2 + 360) $\boldsymbol{i}$ + (300 + 318.2 + 480) $\boldsymbol{j}$ } N

= { 41.80 $\boldsymbol{i}$ + 1098 $\boldsymbol{j}$ } N



Step 3

- Magnitude and direction:

$F_R = ((41.80)^2 + (1098)^2)^{1/2}$ = 1099 N

$\phi = \tan^{-1}(1098/41.80)$ = 87.8°

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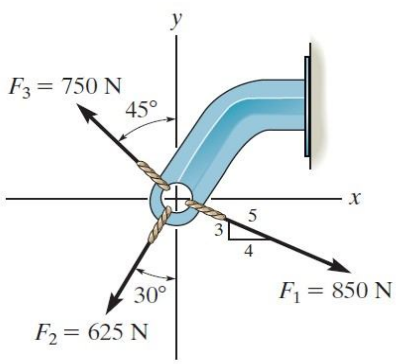
Example 2.6

Determine the magnitude and direction of the resultant force



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Solution Example 2.6

Step 1

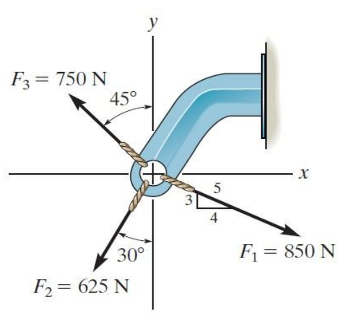
Resolve the forces into components

$\boldsymbol{F_3}$ = {-750 sin (45°) $\boldsymbol{i}$ + 750 cos (45°) $\boldsymbol{j}$ } N

{- 530.3 $\boldsymbol{i}$ + 530.3 $\boldsymbol{j}$ } N

$\boldsymbol{F_2}$ = {- 625 sin (30°) $\boldsymbol{i}$ – 625 cos (30°) $\boldsymbol{j}$ } N

= {- 312.5 $\boldsymbol{i}$ – 541.3 $\boldsymbol{j}$ } N



$\boldsymbol{F_1}$ = {850 (4/5) $\boldsymbol{i}$ – 850 (3/5) $\boldsymbol{j}$ } N

= { 680 $\boldsymbol{i}$ – 510 $\boldsymbol{j}$ } N

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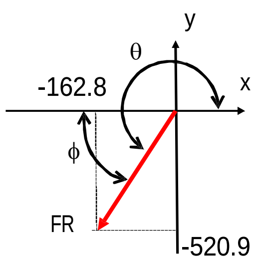
Solution Example 2.6

Step 2

- Summing all the $\boldsymbol{i}$ and $\boldsymbol{j}$ components, respectively:

$$F_R = \{ (680 - 312.5 - 530.3)\,\boldsymbol{i} + (-510 - 541.3 + 530.3)\,\boldsymbol{j} \}\text{N}$$
$$= \{- 162.8\,\boldsymbol{i} - 520.9\,\boldsymbol{j} \}\text{ N}$$



Step 3

- Magnitude and direction (angle):

$F_R = ((-162.8)^2 + (-520.9)^2)^{½} =$ 546 N

$\phi = \tan^{-1}(520.9 / 162.8) =$ 72.6°

From the positive x-axis, $\theta = 253°$

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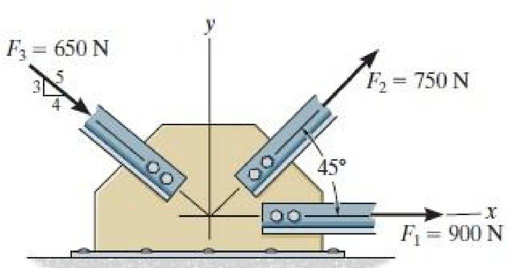
Example 2.7

Determine the magnitude of the resultant force acting on the plate and its direction, measured counterclockwise from the positive x axis.



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Solution Example 2.7

Rectangular Components: By referring to Fig. *a*, the *x* and *y* components of $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ can be written as

$$(F_1)_x = 900\text{ N} \qquad (F_1)_y = 0$$
$$(F_2)_x = 750\cos 45^\circ = 530.33\text{ N} \qquad (F_2)_y = 750\sin 45^\circ = 530.33\text{ N}$$
$$(F_3)_x = 650\left(\frac{4}{5}\right) = 520\text{ N} \qquad (F_3)_y = 650\left(\frac{3}{5}\right) = 390\text{ N}$$

Resultant Force: Summing the force components algebraically along the *x* and *y* axes, we have

$$\overset{+}{\rightarrow}\Sigma(F_R)_x = \Sigma F_x; \qquad (F_R)_x = 900 + 530.33 + 520 = 1950.33\text{ N}\rightarrow$$
$$+\uparrow\Sigma(F_R)_y = \Sigma F_y; \qquad (F_R)_y = 530.33 - 390 = 140.33\text{ N}\uparrow$$

The magnitude of the resultant force $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{1950.33^2 + 140.33^2} = 1955\text{ N} = 1.96\text{ kN} \quad \textbf{Ans.}$$

The direction angle θ of $\mathbf{F}_R$, measured clockwise from the positive *x* axis, is

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left(\frac{140.33}{1950.33}\right) = 4.12^\circ \quad \textbf{Ans.}$$

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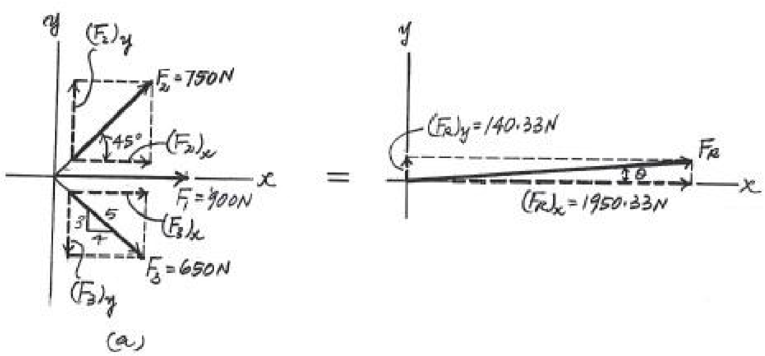
Solution Example 2.7



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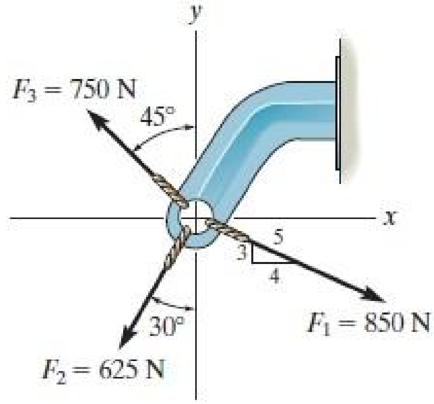
Example 2.8

Determine the magnitude of the resultant force and its direction measured counterclockwise from the positive x axis.



Solution Example 2.8

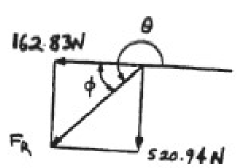
$$\xrightarrow{+} F_{Rx} = \Sigma F_x; \qquad F_{Rx} = \frac{4}{5}(850) - 625 \sin 30^\circ - 750 \sin 45^\circ = -162.83 \text{ N}$$

$$+\uparrow F_{Ry} = \Sigma F_y; \qquad F_{Ry} = -\frac{3}{5}(850) - 625 \cos 30^\circ + 750 \cos 45^\circ = -520.94 \text{ N}$$

$$F_R = \sqrt{(-162.83)^2 + (-520.94)^2} = 546 \text{ N} \qquad \textbf{Ans.}$$

$$\phi = \tan^{-1}\left(\frac{520.94}{162.83}\right) = 72.64^\circ$$

$$\theta = 180^\circ + 72.64^\circ = 253^\circ \qquad \textbf{Ans.}$$



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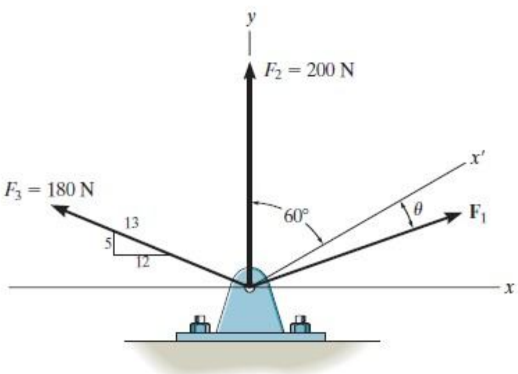
Example 2.9

Three forces act on the bracket. Determine the magnitude and direction θ of F_1 so that the resultant force is directed along the positive x' axis and has a magnitude of 800 N.



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Solution Example 2.9

$$\overset{+}{\rightarrow} F_{Rx} = \Sigma F_x; \quad 800 \sin 60^\circ = F_1 \sin(60^\circ + \theta) - \frac{12}{13}(180)$$

$$+\uparrow F_{Ry} = \Sigma F_y; \quad 800 \cos 60^\circ = F_1 \cos(60^\circ + \theta) + 200 + \frac{5}{13}(180)$$

$$60^\circ + \theta = 81.34^\circ$$

$$\theta = 21.3^\circ$$

$$F_1 = 869 \text{ N}$$

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Conclusion of The Chapter 2 part II

- Conclusions
  - The Cartesian vector have been identified and determined in the mechanics
  - The 2-D and 3-D vector has been represent in a Cartesian coordinate system
  - The magnitude and direction of 2-D of resultant forces have been determined in a Cartesian coordinate system

Thank you

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