

University of Cihan-Sulaimaniya  
Engineering Faculty  
Architectural Engineering Department



# ENGINEERING MECHANICS

## Chapter 2: Force Vectors (Static)

2<sup>nd</sup> Grade- Fall Semester 2023-2024

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## Chapter Description

- Aims
  - To review the Parallelogram Law and Trigonometry
  - To explain the Force Vectors
  - To explain the Vectors Operations
  - To express force and position in Cartesian Vectors
- Expected Outcomes
  - Able to solve the problems of force vectors in the mechanics applications by using Parallelogram law and Trigonometry
- References
  - Russel C. Hibbeler. Engineering Mechanics: Statics & Dynamics, 14<sup>th</sup> Edition

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## Chapter Outline

1. Scalars and Vectors – part I
2. Vectors Operations – part I
3. Vectors Addition of Forces – part I
4. Cartesian Vectors – part II
5. Force and Position Vectors – part III




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## 2.1 Scalars and Vectors

**What is Scalars?**

A quantity that has only a magnitude



Source: <http://www.automobiledimensions.com/>


**Length** of a mini car is **3.821 mm**

↓

**quantity**                      **magnitude**

**What is Vectors**

A quantity that has both magnitude and direction



Source: <http://www.shodor.org>

**Position** of the plane is **25 miles** from **west southwest**

↓                      ↓                      ↓

**quantity**                      **magnitude**                      **direction**

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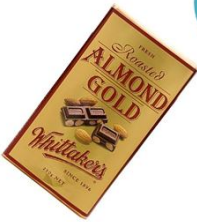

## Identify scalars and vectors

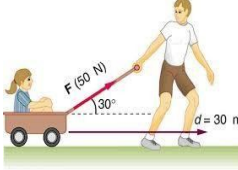


**Scalars**



Identify example of scalars?

**Vectors**

Identify example of vectors?

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## Comparison of Scalars and Vectors

**Scalars**

A quantity that has only a magnitude

- Mass, Length,
- Time,
- Temperature,
- Volume, Density

**Vectors**

A quantity that has both magnitude and direction

- Position,
- Displacement,
- Velocity,
- Acceleration,
- Momentum, Force

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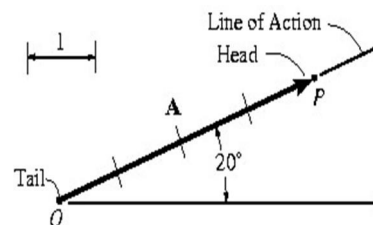
## Vectors

- Represent by a letter with an arrow over it such as  $\vec{A}$  or **A**
- Magnitude is designated as  $|\vec{A}|$  or simply  $A$
- Commonly, vector is presented as **A** and its magnitude (positive quantity) as  $A$

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## Characteristics of Vectors

- Represented graphically as an arrow
- Length of arrow = **Magnitude of Vector**
- Angle between the reference axis and arrow's line of action = **Direction of Vector**
- Arrowhead = **Sense of Vector**



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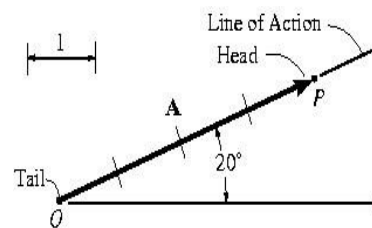
## Example of Vectors

Magnitude of Vector = 4 units

Direction of Vector =  $20^\circ$  measured counterclockwise from the horizontal axis

Sense of Vector = Upward and to the right

The point O is called **tail** of the vector and the point P is called the **tip** or **head**

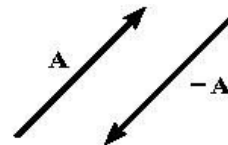


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## 2.2 Vector Operations

### ■ Multiplication and Division of a Vector by a Scalar

- Product of vector  $\mathbf{A}$  and scalar  $a = a\mathbf{A}$
- Magnitude =  $|aA|$
- If  $a$  is positive, sense of  $a\mathbf{A}$  is the same as sense of  $\mathbf{A}$
- If  $a$  is negative sense of  $a\mathbf{A}$ , it is opposite to the sense of  $\mathbf{A}$

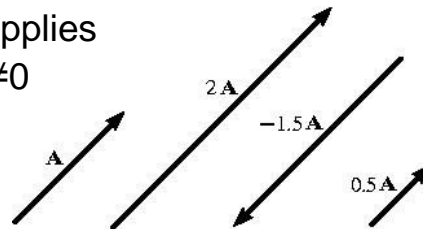


Vector  $\mathbf{A}$  and its negative counterpart

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## 2.2 Vector Operations

- **Multiplication and Division of a Vector by a Scalar**
  - Negative of a vector is found by multiplying the vector by ( -1 )
  - Law of multiplication applies
  - Eg:  $\mathbf{A}/a = ( 1/a ) \mathbf{A}$ ,  $a \neq 0$



Scalar Multiplication and Division

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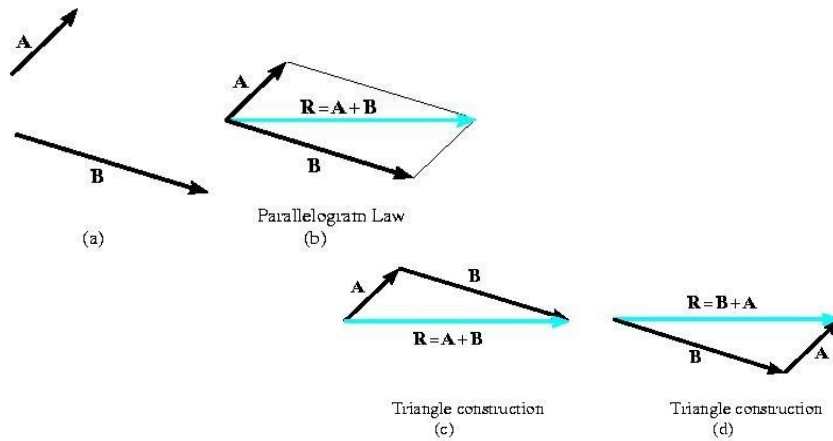
## 2.2 Vector Operations

- **Vector Addition**
  - Addition of two vectors **A** and **B** gives a resultant vector **R** by the **parallelogram law**
  - Result **R** can be found by **triangle construction**
    - Commutative
    - Eg:  $\mathbf{R} = \mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$

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## 2.2 Vector Operations

### ■ Vector Addition

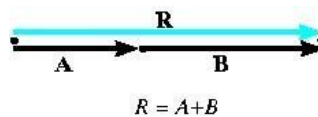


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## 2.2 Vector Operations

### ■ Vector Addition

- Special case: Vectors  $A$  and  $B$  are *collinear* (both have the same line of action)



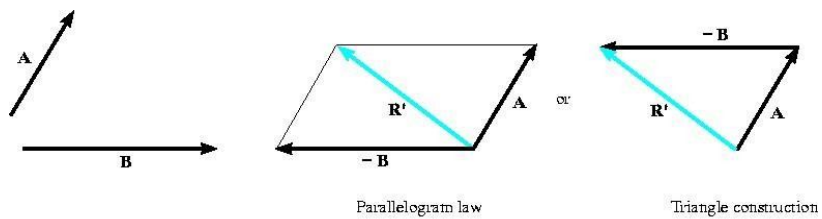
Addition of collinear vectors

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## 2.2 Vector Operations

### ■ Vector Subtraction

- Special case of addition
- Eg:  $\mathbf{R}' = \mathbf{A} - \mathbf{B} = \mathbf{A} + (-\mathbf{B})$
- Rules of Vector Addition Applies

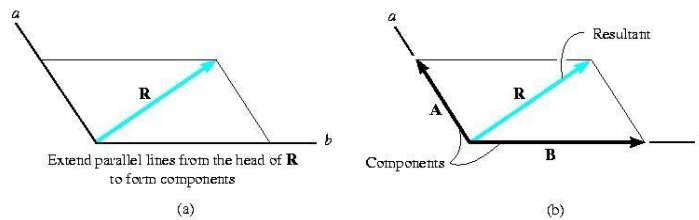


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## 2.2 Vector Operations

### ■ Resolution of Vector

- Any vector can be resolved into two components by the **parallelogram law**
- The two components  $\mathbf{A}$  and  $\mathbf{B}$  are drawn such that they extend from the tail or  $\mathbf{R}$  to points of intersection

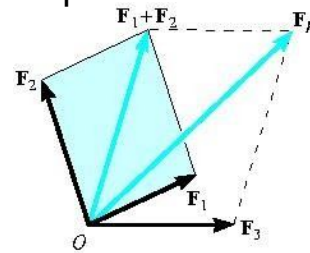


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## 2.3 Vector Addition of Forces

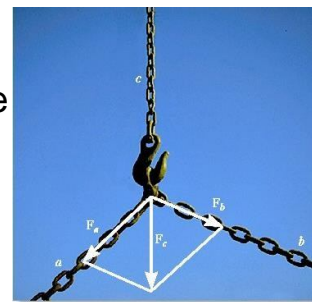
- When two or more forces are added, successive applications of the **parallelogram law** is carried out to find the resultant
- Eg: Forces  $\mathbf{F}_1$ ,  $\mathbf{F}_2$  and  $\mathbf{F}_3$  acts at a point O
- First, find resultant of
  - $\mathbf{F}_1 + \mathbf{F}_2$
  - Resultant,
  - $\mathbf{F}_R = (\mathbf{F}_1 + \mathbf{F}_2) + \mathbf{F}_3$



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## Example of Vector Addition of Forces

- $\mathbf{F}_a$  and  $\mathbf{F}_b$  are forces exerting on the hook.
- Resultant,  $\mathbf{F}_c$  can be found using the *parallelogram law*
  - Lines parallel to a and b
  - from the heads of  $\mathbf{F}_a$  and  $\mathbf{F}_b$  are
  - drawn to form a parallelogram
  - Similarly, given  $\mathbf{F}_c$ ,  $\mathbf{F}_a$  and  $\mathbf{F}_b$
  - can be found



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## Steps to Solve the Vectors Operations

- **Parallelogram Law**
  - Make a sketch using the *parallelogram law*
  - Two components forces add to form the resultant force
  - Resultant force is shown by the diagonal of the parallelogram
  - The components is shown by the sides of the parallelogram

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## Steps to Solve the Vectors Operations

- **Parallelogram Law**
  - To resolve a force into components along two axes directed from the tail of the force
  - Start at the head, constructing lines parallel to the axes
  - Label all the known and unknown force magnitudes and angles
  - Identify the two unknown components

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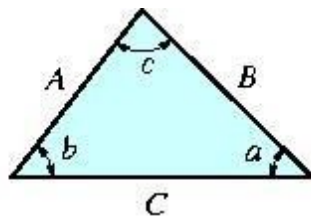
## Steps to Solve the Vectors Operations

- **Trigonometry**
- Redraw half portion of the parallelogram
- Magnitude of the resultant force can be determined by the **law of cosines**
- Direction if the resultant force can be determined by the **law of sines**

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## Steps to Solve the Vectors Operations

- **Trigonometry**
- Magnitude of the two components can be determined by the **law of sines**



Sine law:

$$\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}$$

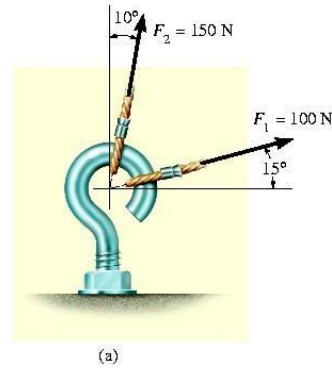
Cosine law:

$$C = \sqrt{A^2 + B^2 - 2AB \cos c}$$

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### Example 2.1

The screw eye is subjected to two forces  $F_1$  and  $F_2$ . Determine the magnitude and direction of the resultant force.

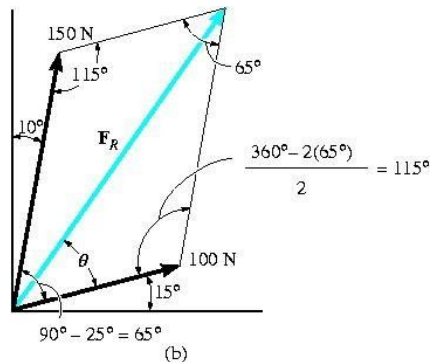


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### Solution Example 2.1

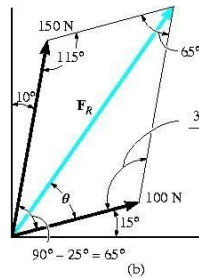
From Parallelogram Law

Unknown: magnitude of  $F_R$  and angle  $\theta$

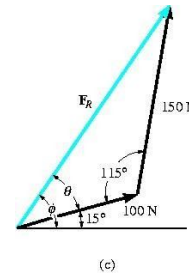


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## Solution Example 2.1



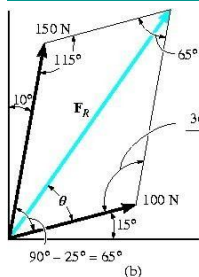
### Trigonometry Law of Cosines



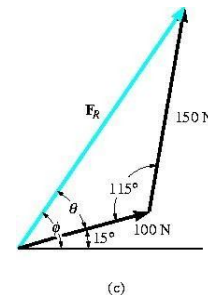
$$\begin{aligned}
 F_R &= \sqrt{(100\text{ N})^2 + (150\text{ N})^2 - 2(100\text{ N})(150\text{ N})\cos 115^\circ} \\
 &= \sqrt{10000 + 22500 - 30000(-0.4226)} \\
 &= 212.6\text{ N} \\
 &= 213\text{ N}
 \end{aligned}$$

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## Solution Example 2.1



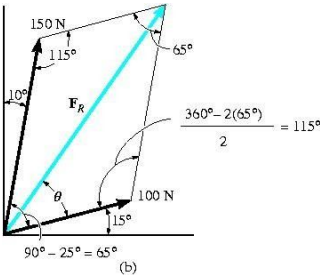
### Trigonometry Law of Sines



$$\begin{aligned}
 \frac{150\text{ N}}{\sin \theta} &= \frac{212.6\text{ N}}{\sin 115^\circ} \\
 \sin \theta &= \frac{150\text{ N}}{212.6\text{ N}} (0.9063) \\
 \theta &= 39.8^\circ
 \end{aligned}$$

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### Solution Example 2.1



$$\frac{360^\circ - 2(65^\circ)}{2} = 115^\circ$$

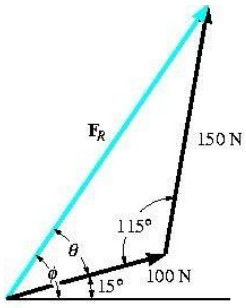
$$90^\circ - 25^\circ = 65^\circ$$

(b)

**Trigonometry**  
 Direction  $\phi$  of  $F_R$  measured from the horizontal

$$\phi = 39.8^\circ + 15^\circ$$

$$= 54.8^\circ \angle \phi$$



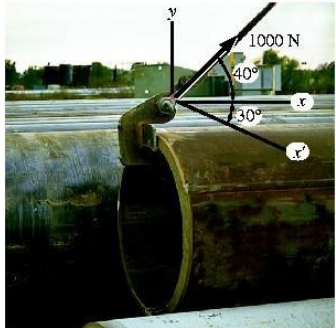
(c)

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### Example 2.2

Resolve the 1000 N ( $\approx 100\text{kg}$ ) force acting on the pipe into the components in the

- (a) x and y directions,
- (b) x' and y' directions.



(a)

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## Solution Example 2.2

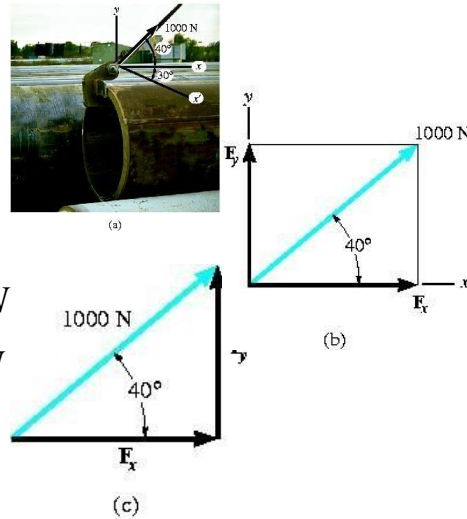
### (a) Parallelogram Law

From the vector diagram,

$$F = F_x + F_y$$

$$F_x = 1000 \cos 40^\circ = 766 \text{ N}$$

$$F_y = 1000 \sin 40^\circ = 643 \text{ N}$$

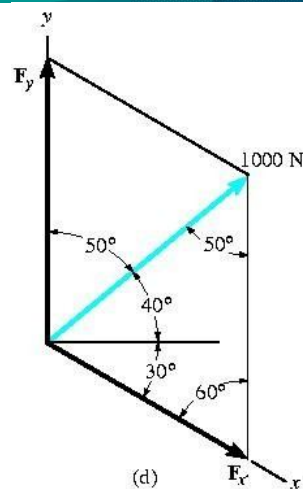


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## Solution Example 2.2

### (b) Parallelogram Law

$$F = F_x + F_{y'}$$



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## Solution Example 2.2

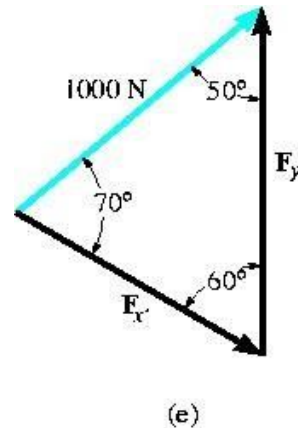
### (b) Law of Sines

$$\frac{F_x}{\sin 50^\circ} = \frac{1000N}{\sin 60^\circ}$$

$$F_x = 1000N \left( \frac{\sin 50^\circ}{\sin 60^\circ} \right) = 884.6N$$

$$\frac{F_y}{\sin 70^\circ} = \frac{1000N}{\sin 60^\circ}$$

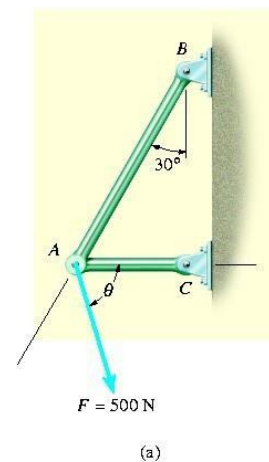
$$F_y = 1000N \left( \frac{\sin 70^\circ}{\sin 60^\circ} \right) = 1085N$$



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## Example 2.3

The force  $\mathbf{F}$  acting on the frame has a magnitude of 500N and is to be resolved into two components acting along the members AB and AC. Determine the angle  $\theta$ , measured below the horizontal, so that components  $\mathbf{F}_{AC}$  is directed from A towards C and has a magnitude of 400N.



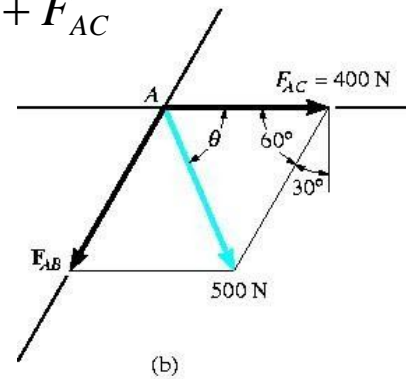
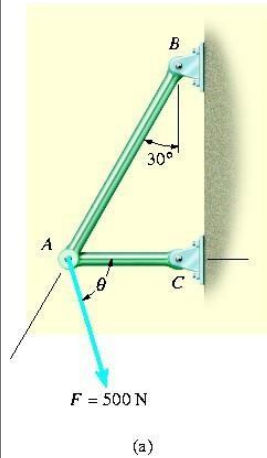
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## Solution Example 2.3

### Parallelogram Law

$$500\text{ N} = F_{AB} + F_{AC}$$



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## Solution Example 2.3

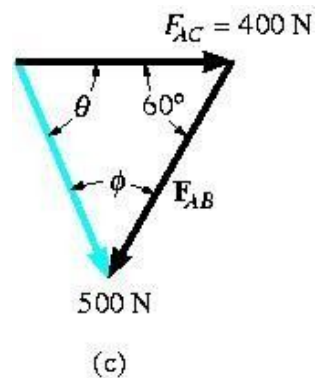
### Law of Sines

$$\frac{400\text{ N}}{\sin \phi} = \frac{500\text{ N}}{\sin 60^\circ}$$

$$\sin \phi = \left( \frac{400\text{ N}}{500\text{ N}} \right) \sin 60^\circ$$

$$\sin \phi = 0.6928$$

$$\phi = 43.9^\circ$$



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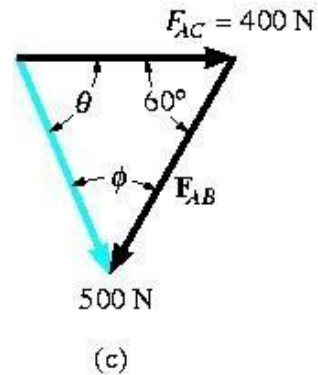
## Solution Example 2.3

Hence,

$$\theta = 180^\circ - 60^\circ - 43.9^\circ = 76.1^\circ \angle \theta$$

By Law of Cosines or  
Law of Sines

Hence, show that  $\mathbf{F}_{AB}$   
has a magnitude of 561N



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## Conclusion of The Chapter 2 part I

- Conclusions
  - The scalars and vectors have been identified and implemented in the mechanics
  - The vector operations have been identified and implemented in the mechanics



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# Thank you