Cihan University-Sulaimany

College of-Administration and Financial Sciences

Department -Business Administration &Accounting

Quantitative Methods

2and Stage

Prof.Dr.Obaid Mahmood Mohsin

Academic Year: 2023-2024

Department of - *Business Administration*

College of - *Administration and Financial Sciences*

University of Cihan- Sulaimaniya

Subject: *Quantitative Methods*

Course Book – Year -*2023-2024*

Lecturer's name *Prof.Dr.Obaid Mahmmood Mohsin*

Academic Year: 2023-2024

Course Book

1. Course name	**Quantitative Methods**
2. Lecturer in charge	**Prof.Dr.Obaid Mahmmood Mohsin**
3.Department/College	**Business Administration-** *Administration and Financial Sciences College*
4. Time (in hours) per week	**2hours per week**
5. Office hours	**4 hours morning**
6. Course code	BA 32116
7. Teacher's academic profile	
8. Keywords	

9. Course overview:

Concepts covered in this course include Linear Programming, Artificial Variable Technique, Graphical method , Simplex Method . At the end of this course the student will:-

1-understand what is meant of problem solving and decision making and quantitative methods.

2. understand what is meant by quantitative analysis methods and how to formulate the linear programming   problems.

3. learn how to constructing a model to represent the systems.
4 learn how to deriving a solution from the model and put the solution to work.
5. learn how use a graphical method to solve two-variable L.P. Problems.
6. learn how use a simplex method to solve two-variable L.P. Problems and more than two-variable problems.
7. understand the artificial variable technique, Big-M, and setting up the dual problems.

10. Course objective:

The main objective of this course is to give student a good theoretical and practical knowledge of quantitative analysis methods. The student will take courses from a variety of technique that focus extensively on statistical methodology, mathematical modeling, and computer implementation issues.
The student will be able to solve and interpret correctly the solutions of a problems and recognize the situations where QM techniques can be used as decision making tools and to interpret correctly the conclusions which can be derived using these techniques.

11. Student's obligation

 Mid examination 30% +10% for homework –final examination 60%

12. Forms of teaching

Teaching and Learning Strategy Contact hours: 3 theoretical weekly hours with discussions +Assignments

13. Assessment scheme

Midterm Examination	**30 %**	**Paper, Quiz, Project**	**10%**
Lab exam	****%**	**Final Practical Examination**	****%**
Final theory exam	**60%**		

14. Student learning outcome:
The main objective of this course is to give student a good theoretical and practical knowledge of quantitative analysis methods. The student will take courses from a variety of technique that focus extensively on statistical methodology, mathematical modeling, and computer implementation issues. The student will be able to solve and interpret correctly the solutions of a problems and recognize the situations where QM techniques can be used as decision making tools and to interpret correctly the conclusions which can be derived using these techniques.

15. Course Reading List and References:
Text Books and References—

-Text books: Hamdy, A.Taha; Operations Research an Introduction, 8th Edition, Pearson Education Inc., 2007.

Reference books: Gupta, P. K. & D. S. Hira; Operations Research, 2nd Edition,S. Chand & Company (Pvt) Ltd, Ram Nagar, New Delhi, 1987

16. The Topics:

Lecture No	Topic
Week/1+2 +3	**Chapter 1:** Basics of Quantitative Methods
Week/ 4+5 +6+7	**Chapter 2:** Linear Programming:
Week/ 8+11	**Chapter 3 :**Graphical Solution of Two-Variable L.P. Problems
Week/9+10	Mid Examination
Week/12+13+14	**Chapter 4:** Simplex Method**:**
Week/ 15+16	Final Examination

17. Peer review

    Main Lecturer   **Head of The Department**

Department of - *Accounting*

College of - *Administration and Financial Sciences*

University of Cihan- Sulaimaniya

Subject: *Quantitative Methods*

Course Book – Year   *-2023-2024*

Lecturer's name *Prof.Dr.Obaid Mahmmood Mohsin*

Academic Year: 2023-2024

Course Book

1. Course name	Quantitative Methods
2. Lecturer in charge	**Prof.Dr.Obaid Mahmmood Mohsin**
3. Department/ College	**Accounting**- *Administration and Financial Sciences College*
4. Time (in hours) per week	**3 hours per week**
5. Office hours	**4 hours morning**
6. Course code	ACC32116
7. Teacher's academic profile	
8. Keywords	

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Technique, Graphical method , Simplex Method . At the end of this course the student
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+Assignments

13. Assessment scheme

Midterm Examination 30 %   Paper, Quiz, Project   10%

Lab exam   **%   Final Practical Examination   **%

Final theory exam   60%

14. Student learning outcome:

 The main objective of this course is to give student a good theoretical and practical knowledge of quantitative analysis methods. The student will take courses from a variety of technique that focus extensively on statistical methodology, mathematical modeling, and computer implementation issues. The student will be able to solve and interpret correctly the solutions of a problems and recognize the situations where QM techniques can be used as decision making tools and to interpret correctly the conclusions which can be derived using these techniques.

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Week/4+_5 +6 +7	**Chapter 2:** Linear Programming:
Week/8+11	**Chapter 3 :** Graphical Solution of Two-Variable L.P. Problems
Week/12 +13 +14	**Chapter 4:** Simplex Method**:**
Week/9+10	Mid Examination
Week- 15+16	Final Examination

17. Peer review

 Main Lecturer   **Head of The Department**

Quantitative Methods

CHAPTER ONE

Basics of Quantitative Analysis Methods

1-Problem solving

2- Quantitative Analysis and Decision making

- Quantitative Analysis
- Model Development
  2023-2024

  Prof.Dr. Obaid Mahmmood Mohsin

Problem Solving

The problem -solving process involves the following seven steps:

1- Identify and define the problem.
2- Determine the set of alternative solutions.
3- Determine the criterion or criteria that will be used to evaluate the alternatives.
4- Evaluate the alternatives.
5- Choose an alternatives.
6- Implement the selected alternative.
7- Evaluate the results to determine whether a satisfactory solution has been obtained.

Decision Making

Is the term generally associated with the first **five steps** of the problem-solving process. Thus , the first step of decision making is to identify and define the problem . Decision making ends with the choosing of an alternative ,which is the act of making the decision.

Let us consider the following example of the decision- making process.
For the moment assume that you are currently unemployed and that you would like a position that will lead to a satisfying career.
Suppose that your job search results in offers from companies in A,B,C and D.Thus, the alternatives for your decision problem can be stated as follows :

1-Accept the position in A

2- Accept the position in B

3- Accept the position in C

4- Accept the position in D

The next step of the problem-solving process involves determining the criteria that will be used to evaluate the **four** alternatives.
Obviously, the starting salary is a factor of some importance.
If salary were the only criterion important to you ,the alternative selected as **best** would be the one with the highest starting salary.

Problems in which the objective is to find the best solution with respect to one criterion only are referred to as :

Single-criterion decision problems

Suppose that you also conclude that the potential for advancement and the location of the job are two other criteria of major importance. Thus, the **three** criteria in your decision problem are starting salary ,potential for advancement and job location.

Problems that involve more than one criterion are referred to as:

Multi-criteria decision problems

The next step of the decision –making process is to evaluate each of the alternatives with respect to each criterion.

For example, evaluating each alternative relative to the starting.

Table (1) Data for the job evaluation decision-making problem

Alternative	Starting salary	Potential for advancement	Job location
A	48500	Average	Average
B	46000	Excellent	Good
C	46000	Good	Excellent
D	47000	Average	Good

Salary criterion is done simple by recording the starting salary for each job alternative . Evaluating each alternative with respect to the potential for advancement and the location of the job is more difficult to do, however, because these evaluations are based primarily on subjective factors that are often difficult to subjective factors that are often difficult to quantify. Suppose for now that you decide to measure potential for advancement and job location by rating each of these criteria as poor ,fair, average , good, or excellent.

The data you compile are shown in table (1)you are now ready to make choice from the available alternatives .

What makes this choice phase so difficult is that the criteria are probably not all equally important , and no one alternative is **best** with regard to all

criteria. Although we will present a method for dealing with situations like this one later in the next , for now let us suppose that after a careful evaluation of the data in table (1), you decide to select **alternative (3),** Alternative (3) is thus referred to as **the decision**

At this point in time, the decision-making process is complete.

Example:

Let you search for a job , and you received four over letters from four companies A , B , C ,D .The following data for the job evaluation decision- making problem .

Alternative	salary	Potential for advancement	Job locations
A	60000	Fair	excellent
B	55000	good	good
C	57000	excellent	average
D	62000	good	excellent

Required:

1- Identify and define the problem.
2- Determine the set of alternative solutions
3- Determine the criterion or criteria that will be used to evaluate the alternatives.

4-if you have one criterion ,what is your decision?

5- if you have two criteria ,what is your decision?

6- if you have three criteria ,what is your decision?

In summary
We see that this process involves **five steps**
1-Define the problem.
2-Identify the alternatives
3-Determine the criteria.
4-Evaluate the alternatives.
5-Choose an alternative.

Note that missing from list are the last **two steps** in the problem-solving process :implementing the selected alternative and evaluating the results to determine whether a satisfactory solution has been obtained.
This omission is not meant to diminish the importance of each of these activities , but to emphasize the more limited scope of the term decision – making as compared to the term problem solving. Figure (1) summarizes the relationship between these two concepts.
Figure (1) The relationship between problem solving and decision making

| Define the problem |
| Identify the Alternatives |
| Determine the criteria |

Decision Making

Problem solving

| Evaluate the Alternatives |
| Choose an Alternative |

Decision

| Implement the decision |

| Evaluate the results |
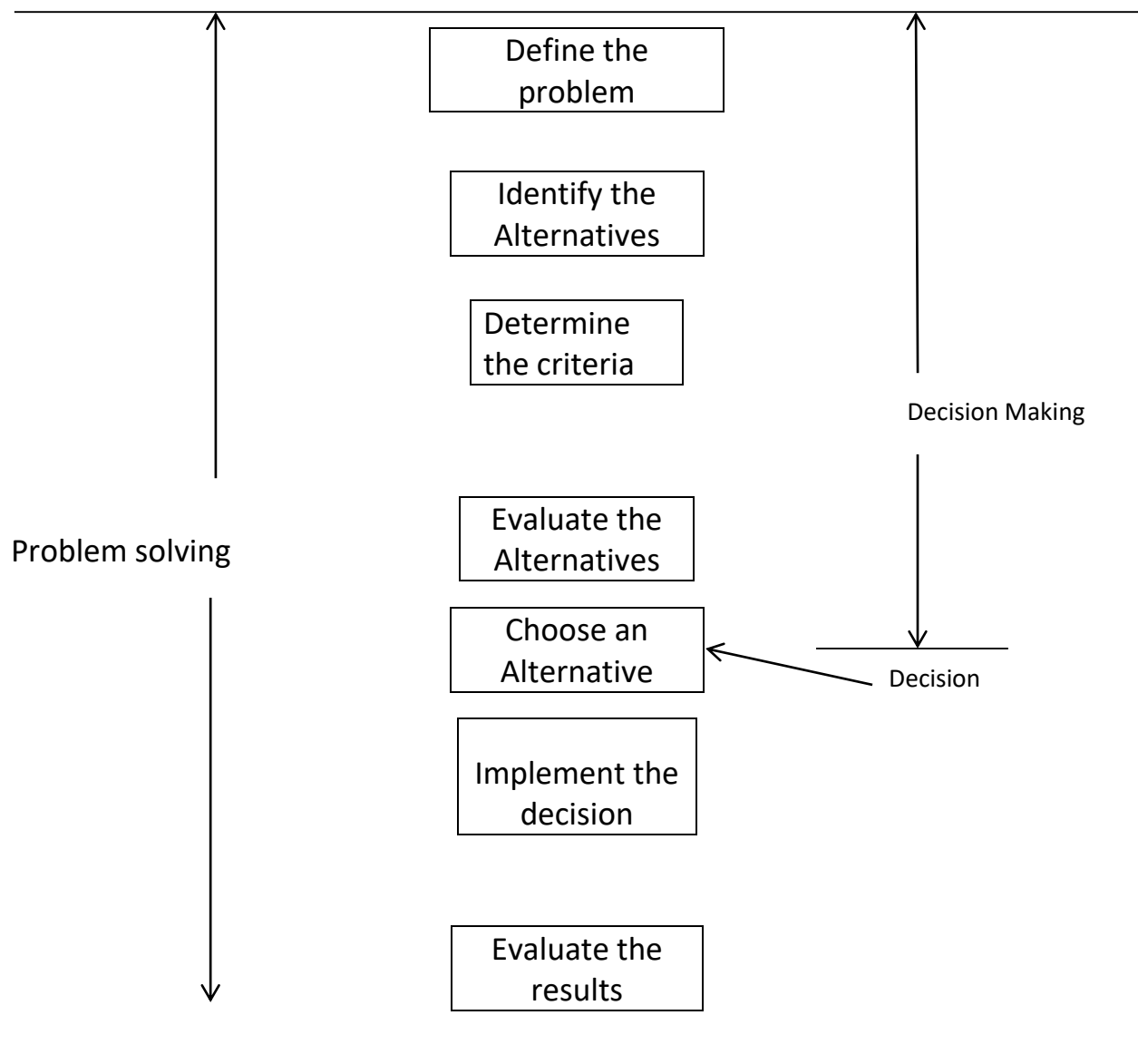
Quantitative Analysis And Decision making

Consider the flowchart in figure (2). Note that we combined the first **three steps** of the decision-making process under the heading of **structuring the problem** and the latter **two steps** under the heading **analyzing the problem**. Let us now consider in greater detail how to carry out the activities that make up the decision-making process.

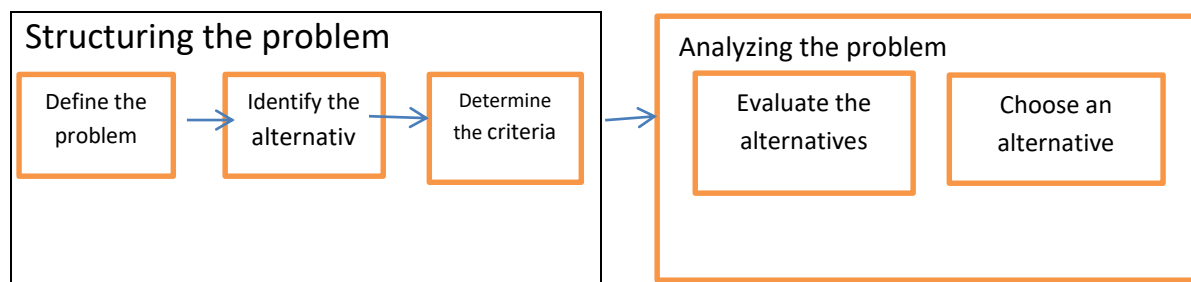Figure (2) A sub classification of the Decision –making process

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┌─────────────────────────────────────────┐      ┌──────────────────────────────────────────┐
│ Structuring the problem                   │      │ Analyzing the problem                      │
│  ┌──────────┐   ┌──────────┐  ┌──────────┐│      │  ┌──────────────┐    ┌──────────────┐     │
│  │ Define the│→ │Identify the│→│Determine ││  →   │  │ Evaluate the  │    │ Choose an     │     │
│  │ problem   │   │ alternativ │ │the criteria││     │  │ alternatives  │    │ alternative   │     │
│  └──────────┘   └──────────┘  └──────────┘│      │  └──────────────┘    └──────────────┘     │
└─────────────────────────────────────────┘      └──────────────────────────────────────────┘
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Figure (3) shows that the analysis phase of the decision-making process may take two basic forms:
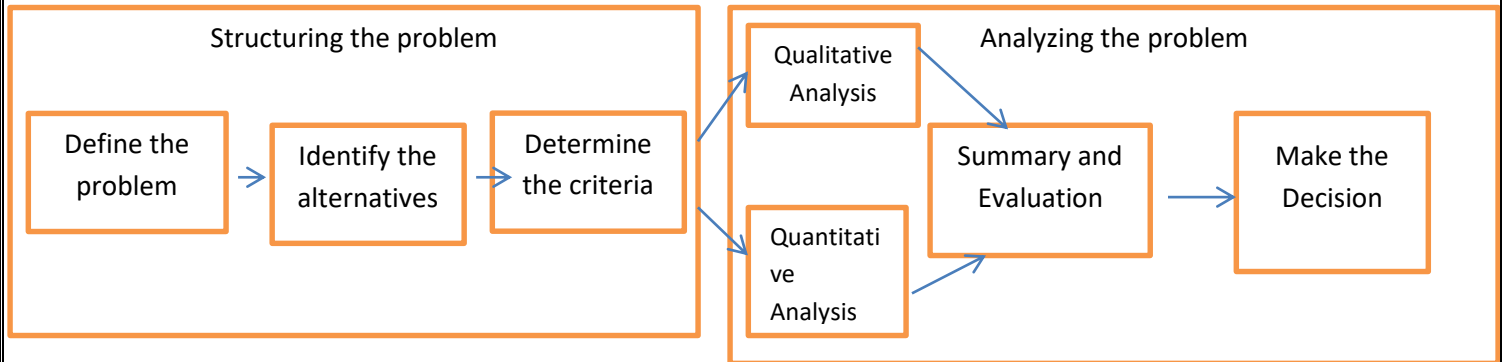
Qualitative

Quantitative

Qualitative analysis is based primarily on the manager's judgment and experience ,it includes the manager's intuitive feel for the problem and is more an art than a science ,if the manager has had experience with similar problems ,or if the problem is relatively simple, heavy emphasis may be placed upon a qualitative analysis.

If the manager has had little experience with similar problems , or if the problem is sufficiently complex, then a quantitative analysis of the problem can be an especially important consideration in the manager's final decision. When using the quantitative approach, an analyst will concentrate on the quantitative facts or data associated with the problem and develop mathematical expressions. Then , by using one or more quantitative

methods ,the analyst will make are commendation based on the quantitative aspects of the problem.

Figure (3) The role of qualitative and quantitative analysis

Structuring the problem			Analyzing the problem
Define the problem → Identify the alternatives → Determine the criteria		Qualitative Analysis / Quantitative Analysis	Summary and Evaluation → Make the Decision

Some of the reasons why a quantitative approach might be used in the decision-making process include the following :

1-The problem is complex ,and the manager cannot develop a good solution without the aid of quantitative analysis.

2-The problem is especially important (e.g.a great deal of money is involved),and the manager desires a thorough (or a complete) analysis before attempting to make a decision.

3-The problem is new, and the manager has no previous experience from which to draw.

4-The problem is repetitive, and the manager saves time and effort by relying on quantitative procedures to make routine decision recommendations.

Quantitative Analysis

To successfully apply quantitative analysis to decision making , the analyst must work closely with the manager or user of the results.

When both the quantitative analyst and the manager agree that the problem has been   adequately structured ,work can begin on developing a model to represent the problem mathematically .Solution procedures can

then be employed to find the best solution for the model . This best solution for the model then becomes a recommendation to the decision maker.

The process of developing and solving models is the essence of the quantitative analysis process .

<u>**Model Development**</u>

The model which includes representations of a problem by a system of symbols and mathematical relationships or expressions .

Such models are referred to as: **Mathematical models.**

And are a critical part of any quantitative approach to decision making.

For example :

The total profit from the sale of a product can be determined by multiplying the profit per unit by the quantity sold.

Let X=No. of units produced and sold.

   P=The total profit—(with a profit of 10$ per unit)

The mathematical model which defines the total profit be:

$$P=10X \dots\dots\dots\dots\dots(1)$$

If we expect selling three units

Then   P=(10)(3)=30$

We usually find that the problem definition phase leads to a specific objective such as :

          Maximization of profit   or

           Minimization of cost

And

Possibly a set of **restrictions or constraints** (such as production capacities)

The success of the mathematical model and quantitative approach will depend heavily on how accurately the objective and constraints can be expressed in mathematical equations or relationships.

The mathematical expression that defines the quantity to be maximized or minimized is referred to as the   **(objective function) .**

For example;

Let X=No, of units produced and sold each week.

Objective-----------maximize total weekly profit with profit 10$ per unit.

-------the objective function is **10X**

A production capacity constraint would be necessary-let 5 hours are required to produce each unit and 40 hours available per week.

The production capacity constraint is given by :

$5X \leq 40$ ……………………………..(2)

5X=total time required to produce X units.

The decision problem or question is the following :

How many units of the product should be produced each week to maximize profit.

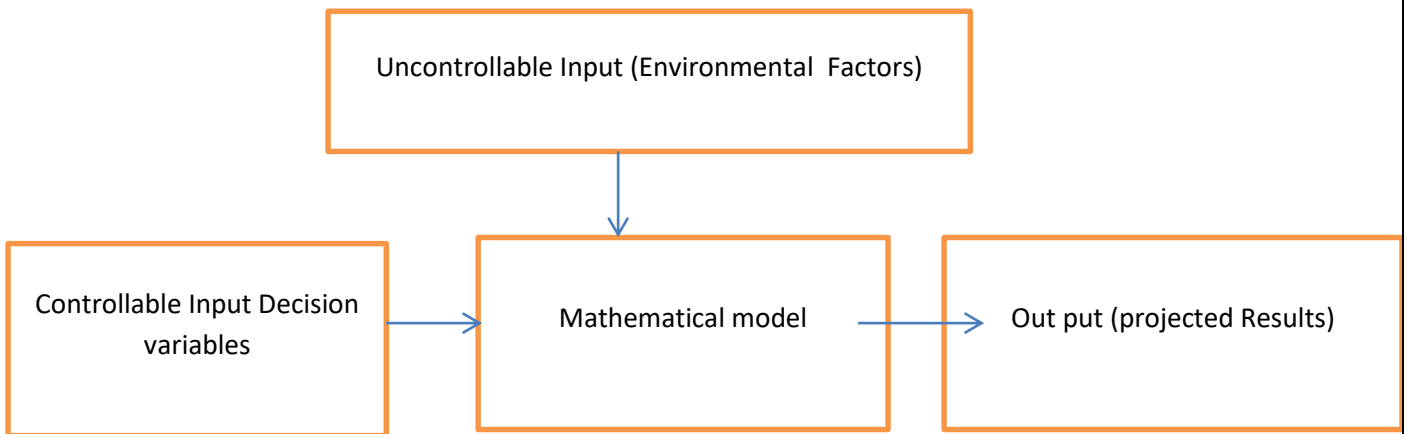A complete mathematical model for this simple production problem is :

Maximize   10X   objective function

Subject to (s.t)

$5X \leq 40$ (constraints)

$X \geq 0$

The optimal solution of this model can be easily calculated and is given by :

X=8

With an associated profit of   80$

This model is an example of a linear programming model.

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                    ┌─────────────────────────────────────┐
                    │ Uncontrollable Input (Environmental  │
                    │            Factors)                  │
                    └─────────────────────────────────────┘
                                    │
                                    ▼
┌──────────────────────┐  ┌──────────────────────┐  ┌──────────────────────┐
│ Controllable Input   │→ │  Mathematical model  │→ │ Out put (projected    │
│ Decision variables   │  │                      │  │      Results)         │
└──────────────────────┘  └──────────────────────┘  └──────────────────────┘
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┌──────────────────────────────────────────┐
│            Uncontrollable inputs           │
└──────────────────────────────────────────┘
                      │
                      ▼
┌──────────────────────────────────────────┐
│   Profit per unit 10$ -labor-hours per unit =5   │
│                                            │
│          -labor-hours capacity=40          │
└──────────────────────────────────────────┘
                      │
                      ▼
┌────────────────────────┐   ┌──────────────────────────────┐   ┌────────────────────────────────────┐
│ Value for the production │   │ Max 10(8)--s.t 5(8)≤40--8≥0— │   │ Profit=80—time used =40---out put │
│   quantity (x=8)—        │──▶│      mathematical model       │──▶│                                      │
│   controllable input     │   │                                │   │                                      │
└────────────────────────┘   └──────────────────────────────┘   └────────────────────────────────────┘
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17

Problems

1- Write the problem- solving steps & decision- making steps.
2- Draw the flowchart of the relationship between problem- solving and decision- making.
3- Draw the flowchart of a sub classification of the decision making process.
4- Write the reasons that lead to the use of quantitative analysis of the problem of access to the decision -making process.
5- Explain :   a-single-criterion decision problems.
                b-multi-criteria decision problems .

   6-The following data for the job evaluation decision- making problem .

Alternative	salary	Potential for advancement	Job locations
A	50000	Fair	excellent
B	45000	good	good
C	47000	excellent	average
D	52000	good	excellent

Required:

1-if you have one criterion ,what is your decision?

2- if you have two criteria ,what is your decision?

3- if you have three criteria ,what is your decision?

7-Explain: 1- quantitative analysis.

                2- mathematical model.

Solutions

a-**The problem -solving process or steps are:**

1-Identify and define the problem.

2-Determine the set of alternative solutions.

3-Determine the criterion or criteria that will be used to evaluate the alternatives.

4   -Evaluate the alternatives.

5-Choose an alternatives.

6-Implement the selected alternative.

7-Evaluate the results to determine whether a satisfactory solution has been obtained.

b- The Decision Making steps are :
1-   Identify and define the problem.
2-   Determine the set of alternative solutions.
3-   Determine the criterion or criteria that will be used to evaluate the alternatives.
4-   Evaluate the alternatives.
5-   Choose an alternatives

2- See figure (1)
3- See figure (2)
4- Some of the reasons why a quantitative approach might be used in the decision-making process include the following :
1-The problem is complex ,and the manager cannot develop a good solution without the aid of quantitative analysis.
2-The problem is especially important (e.g.a great deal of money is involved),and the manager desires a thorough (or a complete) analysis before attempting to make a decision.

3-The problem is new, and the manager has no previous experience from which to draw.

4-The problem is repetitive, and the manager saves time and effort by relying on quantitative procedures to make routine decision recommendations.

5- Problems in which the objective is to find the best solution with respect to one criterion are referred to as :

                    Single-criterion decision problems

Problems that involve more than one criterion are referred to as:

                    Multi-criteria decision problems

6- (i)for salary—D   -for potential for advancement—C-for job location-A or D

(ii) for salary and potential for advancement –C or D

   for salary and job location ---D

  For job location and potential for advancement ---D

(iii) D

7- Quantitative Analysis :   The process of developing and solving models is the essence of the quantitative analysis process .

Mathematical models. The model which includes representations of a problem by a system of symbols and mathematical relationships or expressions .Such models are referred to as: Mathematical models.

Quantitative Methods

CHAPTER TWO

Linear Programming

-Define Linear programming

-     =   linear

-     =   programming

-Uses linear programming

- Linear programming assumptions

-Formulation of linear programming model

-General mathematical formulation of linear programming model

-The standard form of the linear programming model.

-Examples of applications of linear programming

-Problems

Prof.Dr. Obaid Mahmmood Mohsin

2023-2024

Linear Programming

Def:

Linear programming: is a problem solving approach developed to help managers make decisions.

Linear programming: Mathematical method used to determine the best allocation of rare or limited resources. Such as workers, raw materials, and crude and hours of work at the facility that would help the administration achieve its goal either reduce the cost or profit maximization, that there are some constraint.

Programming : Choosing and applying the best possible solutions to the various problems from a range of possible solutions in accordance with the possibilities of the limited resources available in the facility and the existence of restrictions that restrict the use of these resources for the purpose of achieving the administration's goal.

Linear : Means that the constraints of the problem and the objective function are linear functions as well as the relationships that bind variables are first degree.

Uses Linear Programming:

Linear Programming Assumptions:

1- The linear programming approach is deterministic technique and that means :

-Amounts of available resources $b_1, b_2, \ldots, b_m$

and

- Contribution per unit of activity in the objective function

and :

--Requirements per unit of resource activities $a_{1n}, a_{2n}, \ldots\ldots, a_{mn}$

All be known and constant.

2-The validity and accuracy of the data used by the administration to build

a linear programming model .

3-Decision variables Xj .(j=1,2,3,……,n) be ≥ zero .

Decision variables : unknowns that we want to determine their values in the solution phase model and thus be relied upon in decision making.

4- The presence of more than one alternative (decision variable) to solve the problem .

Elements of linear programming model :

1- **Decision variables** and parameters .
   Parameters : fixed values in the model ; such as requirements per unit
      a particular activity of a particular resource.

2-**Constraints** : Represent mathematical expression of constraints of the problem under study. Constraints be as a linear equations or linear inequalities.

 For example:

If   A=available amount of raw materials , it represents the upper limit that can be used .

If X_1 ,X_2 Decision variables

Then

$a_{11}X_1$= what you need per unit of the first product $X_1$of the raw material A .

$a_{12} X_2$= what you need per unit of the second product X_2 of the raw material A .

so we can write the constraint as follow:

$a_{11}X_1 + a_{12} X_2 \leq A$

and:

If the model includes another constraint s.t. the number of hours available for the production X_1 and X_2

per unit of product X_1 , we need to $a_{21}X_1$ (the number of hours weekly)

per unit of product X_2 , we need to $a_{22}X_2$ (the number of hours weekly)

we can write this constraint for X_1 and X_2 as:

$a_{21} X_1 + a_{22} X_2 \leq B$

3-Objective function

If C_1, C_2, the profit per unit of X_1 and X_2

Then the objective function to maximize profit be

Maximize $(Z) = C_1 X_1 + C_2 X_2$

4-The decision variables X_1 , X_2 , be ≥ 0

Formulation of Linear Programming model :

1- Determination decision variables.
2- = objective function which be linear form and include be maximize profit or minimize cost .
3- = the constraints as a questions or inequalities.

General Mathematical Formulation of Linear Programming Model:

Let

X_j, (j= 1,2,......,n) decision variables.

a_{ij} =Technical coefficients

=The amount of resources i needed per unit of activity j .

C_j = profit per unit of activity   j .

b_i = available amount of supplier (i) .

The mathematical Formulation of linear programming:

Maximize or minimize

   $$Z = C_1 X_1 + C_2 X_2 + \ldots\ldots\ldots + C_n X_n \ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

   Subject to the constraints (s.t)

   $$a_{11} X_1 + a_{12} X_2 + \ldots\ldots\ldots\ldots + a_{1n} X_n \quad \leq\ =\ \geq\ b_1$$

   $$a_{21} X_1 + a_{22} X_2 + \ldots\ldots\ldots\ldots + a_{2n} X_n \quad \leq\ =\ \geq\ b_2$$

   .

   $$a_{m1} X_1 + a_{m2} X_2 + \ldots\ldots\ldots\ldots + a_{mn} X_n \quad \leq\ =\ \geq\ b_m$$

and:

   $$X_1 \geq 0 , \quad X_2 \geq 0 , \quad \ldots\ldots, X_n \geq 0$$

X_j The variables required find values .

a_{ij} , b_i ,c_j , known constants .

from eq. (1)

$C_1 X_1$ = first contribution activity .

we can write the general mathematical formulation of linear programming model as follow:

maximize (or minimize) $Z = \sum C_j X_j$

        s.t.

        $$\sum a_{ij} X_j \leq\ =\ \geq\ b_i \quad , i = 1, 2, \ldots.., m$$

        $$X_j \geq 0 \quad , j = 1, 2, \ldots., n$$

The Standard Form of the Linear Programming Model :

Linear programming problem can be converted into the standard form ,its characteristics:

1- All constraints which be ≥ or ≤ must be equation marking equality.
2- The values of right side be no negative.
3- Objective function be maximize or minimize .
4- All the variables (decision variables) ≥ 0 .

Note: To write the linear programming model in standard form :

(i)   In the case that the constraint of type (≤) can be transferred to the equation by **adding** slack variable to left side .

(ii)   In the case that the constraint of the type (≥) can be transferred to the equation by **subtracting** surplus variable from left side .

We adding artificial variable to left side for the purpose of starting basic solution .

Examples of Applications of Linear Programming :

Ex;1-

A factory produce two type of products (P_1 ,P2) using (3) types of machines (M_1 ,M_2 ,M_3).

The production (P_1) must pass through machines (M_1 ,M_2).

The production (P_2) must pass through machines (M_1 ,M_2, M_3).

The following table represents the time it takes to produce one unit of each product per hour.

	M_1	M_2	M_3
P_1	0.25	0.4	0
P_2	0.5	0.2	0.8

The every machine from the machines must operate **at the most** (40) hours per week.

If you know that the factory gets a profit (2) $ per unit of (P$_1$) , (3)$ per unit of (P$_2$).

Required:

The formulation of a linear programming model to calculate the number of units to be produced from each product in the week to maximize profit factory.

Solution: The example include determining the number of units produced from both productions P$_1$,P$_2$ a week **(decision variables)** using (3) types of machines M$_1$,M$_2$,M$_3$ **(constraints)** for the purpose of profit maximization **(objective function)**

1- Determine decision variables (X_1 ,X_2)
2- Determine the constraints:
   (i) $0.25X_1 + 0.5 X_2 \leq 40$
   (ii) $0.4X_1 + 0.2X_2 \leq 40$
   (iii) $0.8 \ X_2 \leq 40$

And

$X_1 \geq 0$,$X_2 \geq 0$

3-Determine objective function Z

  Maximize $Z = 2X_1 + 3X_2$

Where

$2X_1$ =The contribution of the first product in profit.

$3X_2$ = The contribution of the second product in profit.

The solution can be rearranged personal decision variables first ,then the objective function and finally constraints.

X_1 =The number of units of the product (P_1) produced in week.

X_2 =The number of units of the product (P_2) produced in week.

Maximize $Z=2X_1 +3X_2$

Subject to : s.t.:

$$0.25X1 +0.5 X2 \leq 40$$

$$0.4X1 + 0.2X2 \leq 40$$

$$0.8 \ X2 \leq 40$$

$$X1 \geq 0 , X2 \geq 0$$

Example 2-

One of the companies planning production of household items that need to be workers and raw materials for their production , that this company is making (3)of these type of household items A,B, and C. The marketing department at this company provide us with the following statements:

Production household items			
A	B	C	
3	4	2	Works(h/unit)
6	4	3	Raw materials(kg/unit)
10	8	12	Profit($/unit)

Available from working hours to produce three items not exceed (20)hours aday , as well as a mount of raw materials that go into the production of items should not exceed (140) kg/day.

<u>Required:</u>

The formulation of a linear programming model to determine the daily production of the three household items , note that the company's goal is to get the greatest possible profit.

<u>Solution:</u>

1- Define the decision variables X_A , X_B , X_C
   s.t.
   X_A=Daily production amount of item A.
   X_B = Daily production amount of item B .
   X_C= Daily production amount of item C .
   Where these variables represent the decision variables to be determined.

   2-Determine the objective function
        Maximize $Z=10X_A +8X_B +12X_C$

   3-Determine the constraints.
   Note the presence of two constraints are **workers** and **raw materials** .
   $3X_A +4X_B +2X_C \leq 20$ constraint of the working hours .
   $6X_A +4X_B +3X_C \leq 140$ constraint of the raw materials .

   $X_A \geq 0$, $X_B \geq 0$, $X_C \geq 0$ constraint of the variables which must
                                        be   ≥zero

   <u>Example 3-</u>
   A person follow a specific diet for the purpose of obtaining at least (2000) calories per day and (100) units per day of different food (carbohydrates ,proteins ,fats)needed to do various day –to day events. If you know that the person can buy (5) of food items (A ,B ,C , D , E)and assigned each of them (500 ,300 ,700 ,150 ,800) ID/kg respectively.
   The following table shows what every type of food calories and various food,

Food items	A	B	C	D	E
calories	300	250	400	200	50
Units of various food	8	3	4	7	6

<u>Required:</u>

The formulation of a linear programming model to determine the quantity bought from the five food items which cost less possible to satisfy the needs of person of calories.

<u>Solution:</u>

Decision variables (types of food)-A ,B ,C ,D , E ,

Constraints –calories (quantities of various items)

Objective function (less the cost of calories required) that are of type minimize .

X_j j= 1 ,2 ,3 ,4 ,5   (A ,B ,C ,D ,E)   Decision variables, where:

X_1=number of units from food A.

X_2=number of units from food B

X_3= =   = =   =   = C

X_4= =   = =   =   = D

X_5 = =   = =   =   = E

Objective function

     minimize $Z = 500X_1 + 300X_2 + 700X_3 + 150X_4 + 800X_5$

where:

$500 X_1$ first class food contribution in cost

$300X_2$ second class food contribution in cost.

s.t.:

$300X_1 + 250X_2 + 400X_3 + 200X_4 + 50X_5 \geq 2000$

$8X_1 + 3X_2 + 4X_3 + 7X_4 + 6X_5 \geq 100$

 $X_j \geq 0$,j=1 ,2 ,3 ,4 ,5.

Example 4-:

A farmer has (300) acres of farmland to exploit agriculture (4) types of crops (tomato , corn ,potatoes ,beans) . If you know :

Each acre planted tomato needs (1) workday and (1000) ID to cover various expenses.

Each acre planted corn needs (2) workday and (1500) ID to cover various expenses.

Each acre planted potatoes needs (3) workday and (2000) ID to cover various expenses.

Each acre planted beans needs (4) workday and (2500) ID to cover various expenses.

 Profit derived from farming of each acre of land under crops four (40 ,20 ,70 ,50)ID/kg

The total number of working days ,is (340) days a year .

Total capital available for farmer is (100000)ID.

Required :

Formulation a linear programming model to determine how to distribute the four crops on area to achieve the greatest possible profit .

Solution :

Decision variables :   number of acres planted for crops

$$X_j \quad , j= 1 ,2 .3 ,4 ,\text{where:}$$

X1 =number of acres planted for tomato.

 X2   =number of acres planted for corn.

X3 =number of acres planted for potatoes.

 X4 = number of acres planted for beans .

Constraints :

working days

various expenses

number of acres of farmland

objective function :

achieve the greatest possible profit .

maximize $Z = 40X_1 + 20X_2 + 70X_3 + 50X_4$

s.t.

$1000X_1 + 1500 X_2 + 2000 X_3 + 2500 X_4 \leq 100000$

$X_1 + X2 + X_3 + X_4 = 300$

$X_1 + 2X_2 + 3X_3 + 4X_4 \leq 340$

$X_1, X_2, X_3, X_4, \geq 0$

Example 5:

A computer center manager employment a number of programmers and systems analysts. He wants to determine the daily working hours by both programmers and systems analysts in order to maximize profit.

If you know:

Each programmer receive (5)$ per hour.

Each systems analyst receive (10)$ per hour .

And:

The weekly hours of programmer no more than (30) hours , while the number of hours of programmers and systems analysts do not exceed (80) hours per week .

Required :

Formulating a linear programming model to determine the number of programmers and system analysts needed by the computer center for the purpose of profit maximization.

Solution :

Z   objective function (profit)

X_1 number of programmers

X_2 number of system analysts

X_1 ,X_2 ,   decision variables

Maximize   Z= $5X_1$ +$10X_2$

s.t.

$X_1 \leq 30$

X_1 +$X_2 \leq 80$

X_1 ,X_2 ≥ 0 **and integers**

Example -6:

A hospital is running a number of nurses to carry out nursing. The following table shows the nurses time during the day and time to engage each period during the day and the number of nurses employed in each period .

period	Time to engage	The number of nurses(minimum)
1	8 a.m-12 p.m	80
2	12 p.m -4 p.m	60
3	4 p.m-8 p.m	40
4	8 p.m -12 mid night	30
5	12 mid night-4 a.m	20
6	4 am -8 am	10

Number of working hours of the nurses in each period must be at least (8) continuous hours per day. Hospital administration want to run the lowest number of nurses during the day so that fill the need for nursing services.

Required :

Formulating a linear programming model to determine the number of nurses

Solution:

X_j , j = 1 ,2 ,3 ,4 ,5 ,6 ----- number of nurses employed in different periods

(decision variables)

Minimize $Z = X_1 + X_2 + X_3 + X_4 + X_5 + X_6$

s.t.

$$X_6 + X_1 \geq 80$$

$$X_1 + X_2 \geq 60$$

$$X_2 + X_3 \geq 40$$

$$X_3 + X_4 \geq 30$$

$$X_4 + X_5 \geq 20$$

$$X_5 + X_6 \geq 10$$

$X_j \geq 0$,j =1 ,2 ,3, 4 ,5 ,6---and integers

Example-7:

A factory produces coal with certain specifications under which fit no more than (0.05%) of the phosphorus and (4.25 %) of ash. The table below shows the grades of (3) types of coal and phosphorus ratios and ash for each grade ,as well as costs for each grade .

grade	Phosphorus ratio %	Ash ratio %	Cost \$/ton
A	0.08	3	20
B	0.05	5	15
C	0.03	4	10

Required:

Find the best max ratio of three grades of coal , which costs the least we can .

Solution :

X_1 ,X_2 ,X_3 decision variables - (grades of three types of coal)

Minimize $Z = 20X_1 + 15X_2 + 10X_3$

s.t.

$$0.08X_1 + 0.05\ X_2 + 0.03\ X_3 \leq 0.05$$

$$3X_1 + 5X_2 + 4X_3 \quad \leq\ 4.25$$

$$X_1 + X_2 + X_3 = 1$$

$$X1\ , X_2\ , X_3\ \geq\ 0$$

Problems

1- A manufacturing company is engaged in producing three types of products: A ,B ,and C. The production department produce, each day, components sufficient to make 50 units of A,25 units of B , and 30 units of C . The management is confronted with the problem of optimizing the daily production of products in assembly department where only 100 man-hours are available daily to assemble the products. The additional information is given in the table .The company has a daily order commitment for 20 units of product A and a total of 15 units of product B and C.

Type of product	Profit contribution per unit of product	Assembly time per product (hrs)
A	12	0.8
B	20	1.7
C	45	2.5

Formulate this problem as an LP model so as to maximize the total profit.

2- A firm produces three products .These products are processed on three different machines .The time required to manufacture one unit of each of the three products and the daily capacity of the three machines are given in the table

Machine	Time per unit (minutes)			Machine capacity(minutes /day)
	Product 1	Product 2	Product 3	
M1	2	3	2	440
M2	4	-	3	470
M3	2	5	-	430

It is required to determine the daily number of units to be manufactured for each product. The profit per unit for product 1 ,2 and 3 is Rs. 4 , Rs. 3 andRs.6 respectively. It is assumed that all the amounts produced are consumed in the market . Formulate the mathematical model of linear programming that will maximize the daily profit.

3- A firm produces an alloy having the following specifications :

(i)   Specific gravity ≤ 0.98 ,

(ii)   Chromium $\geq 8\%$

(iii)   Melting point $\geq 450^0C$

Raw materials A,B ,and ,C having the properties shown in the table can be used to make the alloy.

property	Properties of raw material		
	A	B	C
Specific gravity	0.92	0.97	1.04
chromium	7%	13%	16%
Melting point	440^0C	490^0C	480^0C

Cost of the various raw materials per ton are : Rs.90 for A,Rs. 280 for B and Rs. 40 for C . Formulate the LP model to find the proportion in which A,B, and C be used to obtain an alloy of desired properties while the cost of raw materials is minimum.

4- A firm manufactures three products A,B ,and C . Time to manufacture product A is twice that for B and thrice that for C and if the entire labor is engaged in making product A, 1600 units of this product can be produced . These products are to be produced in ratio 3: 4 :5. There is demand for at least 300 , 250 ,and 200 units of products A, B ,and C and the profit earned per unit is Rs. 90 , Rs. 40 ,and RS. 30 respectively .

Raw material	Requirement per unit of product (kg)			Total availability (kg)
	A	B	C	
P	6	5	2	5000
Q	4	7	3	6000

Formulate the problem as a linear programming problem.

5-Acompany has two plants , each of which produces and supplies two products , A ,and , B . The plants can each work up to 16 hours a day .In plant ,1,it takes three hours to prepare and pack 1000 gallons of A and one hour to prepare and

pack one quintal of B . In plant 2, it takes two hours to prepare and pack 1000 gallons of A ,and Rs.28000 to prepare and pack a quintal of B , Whereas these costs are Rs. 18000 and Rs. 26000 respectively in plant 2 . The company is obelized to produce daily at least 10 thousand gallons of A and 8 quintals of B .

Formulate this problem as an LP model to find out as to how the company should organize its production so that the required amounts of the two products be obtained at minimum cost .

6-An electronic company is engaged in the production of two components C_1 and C_2 used in radio set . Each unit of C_1 costs the company Rs. 5 in wages and Rs. 5 in material ,while each of C_2 costs the company Rs. 25 in wages and Rs. 15 in material . The company sells both products on one-period credit terms , but the company's labor and material expenses must be paid in cash . The selling price of C_1 is Rs. 30 per unit and of C_2 it is Rs.70 per unit. Because of the strong monopoly of the company for these components , it is assumed that the company can sell at the prevailing prices as many units as it produces . The company's production capacity is ,however , limited by two consideration. First ,at the beginning of period 1   , the company has an initial balance of Rs.4000 (cash plus bank credit plus collections from past credit sales) . Second , the company has available in each period 2000 hours of machine time and 1400 hours of assembly time .The production of each C_1 requires 3 hours of machine time and 2 hours of assembly time , whereas the production of each C_2 requires 2 hours of machine time and 3 hours of assembly time .

Formulate this problem as an LP model so as to maximize the total profit to the company.

7-An oil company produces two grades of gasoline P and Q which it sells at Rs. 30 and Rs. 40 per liter .The company can buy four different crude oils with the following constraints and costs :

Crude oil	constraints			Price/litre(Rs.)
	A	B	C	
1	0.75	0.15	0.10	20.00
2	0.20	0.30	0.50	22.50
3	0.70	0.10	0.20	25.00
4	0.40	0.10	0.50	27.50

Gasoline P must have at least 55 per cent of constituent A and not more than 40 per cent of C . Gasoline Q must not have more than 25 per cent of C . Determine how the crudes should be used to maximize the profit.

8-

A company sells two different products A and B ,making a profit of Rs. 40 and RS . 30 per unit on them ,respectively . They are produced in a common production process and are sold in two different markets . The production process has a total capacity of 30000 man-hours. It takes three hours to produce a unit of A and one hour to produce a unit of B . The market has been surveyed and company official feel that the maximum number of units of A that can be sold is 8000 units and that of B is 12000 units .Subject to these limitations , products can be sold in any combination . Formulate this problem as an LP model to maximize profit .

9-

The manager of an oil refinery must decide on the optimal mix of two possible blending processes of which the input and output per production run are given as follow :

Process(units)	Input (units)		Out put	
	Crude A	Crude B	Gasoline X	Gasoline Y
1	5	3	5	8
2	4	5	4	4

The maximum amount available of crude A and B are 200 units and 150 units , respectively . Market requirements show that at least 100 units of gasoline X and

80 units of gasoline Y must be produced . The profit per production run from process 1 and process 2 are Rs . 300 and Rs . 400 , respectively .

Formulate this problem as an LP model to maximize profit .

10-

Vitamins A and B are found in food F_1 and F_2 .One unit of food F_1 contains three units of vitamin A and four units of vitamin B .One unit of food F_2 contains six units of vitamin A and three units of vitamin B . One unit of food F_1 and F_2 cost Rs. 4 and 5, respectively. The minimum daily requirement (for a person)of vitamins A and B is 80 and 100 units ,respectively . Assuming that anything in excess of the daily minimum requirement of A and B is not harmful.

Formulate this problem as an LP model to find out the optimum mixture of food F_1 and F_2 at the minimum cost which meet the daily minimum requirement of vitamin A and B .

CH-2　　　　　　　SOLUTIONS

1-

The data of the problem is summarized as follow:

Resources /constraints	Product type			total
	A	B	C	
Production capacity (units)	50	25	30	
Man-hours per unit	0.8	1.7	2.5	100
Order commitment (units)	20	15	15	
Profit contribution (Rs/units)	12	20	45	

Decision variables : let X_1 ,X_2 ,and X_3,= number of units of products A,B, and C to be produced respectively.

The LP model:

Maximize(total profit) $Z=12X_1 +20X_2 +45X_3$

s.t.

(i) labor and materials constraints

$0.8X_1 +1.7X_2 +2.5X_3 \leq 100$

$X_1 \leq 50$

$X_2 \leq 25$

$X_3 \leq 30$

$X_1 ,X_2 , X_3 ,\geq 0$

(ii) order commitment constraints

$X_1 \geq 20$

$X_2 +X_3 \geq 15$

2-

Decision variables : let X_1, X_2 and X_3 = Amount (quantity) of product 1 ,2 ,and 3 to be manufactured ,respectively.

The LP model

Maximize (total profit) $Z=4X_1 + 3X_2 + 6X_3$

S.t.

(i) $2X_1 + 3X_2 +2X_3 \leq 440$

(ii) $4X_1 +0X_2 +3X_3 \leq 470$

(iii) $2X_1 +5X_2 +0X_3 \leq 430$

And $X_1 ,X_2 ,X_3 \geq 0$

3-

decision variables   : let X_1 ,X_2 and X_3 = percentage contents of raw materials A ,B ,and ,C , to be used for making the alloy respectively .

The LP model

Minimize (total cost) $Z =90X_1 + 280 X_2 + 40 X_3$

s.t.

$0.92X_1 +0.97 X_2 +1.04 X_3 \leq 0.98$ (specific gravity)

$440 X_1 + 490 X_2 +480 X_3 \geq 450$ (melting point)

$7X_1 +13 X_2 +16 X_3 \geq 8$ (chromium percentage)

$X_1 +X_2 +X_3 =100$ (percentage contents)

And

$X_1 ,X_2 ,X3 \geq 0$

4-

Decision variables : let X_1, X_2, and, X_3 =number of units of products A, B,and C, to be manufactured respectively.

Maximize (total profit) $Z = 90X_1 + 40X_2 + 30X_3$

s.t.

(i) raw material P and Q required:

   P: $6X_1 + 5X_2 + 2X_3 \leq 5000$

   Q : $4X_1 + 7X_2 + 3X_3 \leq 6000$

(ii) manufacturing time required for product A ,B ,and C: $X_1 + X_2/2 + X_3/3 \leq 1600$

(iii) market demand requires : $X_1 \geq 300$, $X_2 \geq 250$, $X_3 \geq 200$.

(iv) production ratio : $X_1 : X_2 : X_3 : : 3 : 4 : 5$, i.e. $X_1/3 = X_2/4$ and $X_2/4 = X_3/5$

   or $4X_1 - 3X_2 = 0$, and $5X_2 - 4X_3 = 0$

and $X_1, X_2, X_3 \geq 0$

5-   The data of the problem is summarized as follows:

Resources /constraints	product		total
	A	B	Availability(hrs)
Preparation time (hrs)	Plant 1:3 hrs/thousand gollons	1 hr /quintal	16
	Plant 2:2 hrs /thousand gallons	1.5 hr/quintal	16
Minimum daily production	10 thousand gallons	8 quintals	
Cost of production (Rs.)	Plant 1: 15000/thousand gallons	28000/ quintals	
	Plant 2 :18000 / thousand gallons	26000 / quintals	

Decision variables let

X_1 ,X_2 ,=quantity of product A (in 000 gallons)to be produced in plant 1 and 2 , respectively.

X_3 ,X_4 = quantity of product B (in 000 gallons)to be produced in plant 1 and 2 , respectively.

The LP model

Minimize (total cost) $Z= 15000 X_1 + 18000 X_2 + 28000 X_3 + 26000 X_4$

s.t.

(i) preparation time

$$3X_1 + X_2 \leq 16$$

$$2X_3 + 1.5 X_4 \leq 16$$

(ii) minimum daily production

$$X_1 + X_2 \geq 10$$

$$X3 + X4 \geq 8$$

And X_1 ,X_2 ,X_3 ,X_4 .≥ 0

6-

The data of the problem is summarized as follow :

Resources /constraints	components		
	C1	C2	Total availability
Budget(Rs.)	10/unit	40/unit	Rs. 4000
Machine time	3 hrs/unit	2 hrs/ unit	2000 hours
Assembly time	2 hrs/unit	3 hrs/unit	1400 hours
Selling price	Rs. 30	Rs. 70	
Cost (wages +material)price	Rs. 10	Rs. 40	

Decision variables   -let X_1 and X_2 = number of units of components C_1 and C_2 to be produced , respectively.

The LP model :

Maximize (total profit) Z= Selling price –Cost price

$$=(30-10) X_1 + (70-40) X_2$$

$$=20X_1 +30X_2$$

s.t.

(i) the total budget available

$$10X_1 +40X_2 \leq 4000$$

(ii) production time

$$3X_1 +2X_2 \leq 2000$$

$$2X_1 +3X_2 \leq 1400$$

And , $X_1 ,X_2 ,\geq 0$

7-

LP Model Formulation

Decision variables   let X_{ij} =Amount of crude oil (in liters) i (= 1 ,2 ,3 ,and ,4) to be used in making gasoline j(=grades P and Q)

The LP Model :

Maximize (net profit) $Z = 30 (X_{1P} +X_{2P} +X_{3P} +X_{4P}) + 40 (X_{1q} +X_{2q} +X_{3q} +X_{4q})$

$-20 (X_{1P} +x_{1q}) - 22.50 (x_{2p} +x_{2q}) - 25 (x_{3p} +x_{3q}) - 27.50 (x_{4p} +x_{4q})$

$=10x_{1p} +7.50x_{2p} +5 x_{3p} +2.50x_{4p} +20x_{1q} +17.50 x_{2q} +15 x_{3q} +12.50 x_{4q}$

s.t.

$0.75x_{1p} +0.20x_{2p} +0.70 x_{3p} +0.40 x_{4p} \geq 0.55 (x_{1p} + x_{2p} +x_{3p} +x_{4p})$

45

$$0.10x_{1p} + 0.50x_{2p} + 0.20x_{3p} + 0.50x_{4p} \leq 0.40 \, (\, x_{1p} + x_{2p} + x_{3p} + x_{4p} \,)$$

$$0.10x_{1q} + 0.50x_{2q} + 0.20x_{3q} + 0.50x_{4q} \leq 0.25 \, (\, x_{1q} + x_{2q} + x_{3q} + x_{4q} \,)$$

And

$$X_{1p}, x_{2p}, x_{3p}, x_{4p}, x_{1q}, x_{2q}, x_{3q}, x_{4q} \geq 0$$

8-

Let X_1, X_2 = number of units of product A and B to be produced, respectively.

Max $Z = 40X_1 + 30X_2$

s.t.

$$3X_1 + X_2 \leq 30000 \qquad \text{(Man-hours)}$$

$$X_1 \leq 8000 \, ; \qquad X_2 \leq 1200 \quad \text{(marketing)}$$

And

$$X_1, X_2 \geq 0$$

9-   Let X_1, X_2 = number of productive runs of process 1 and 2, respectively

Max $Z = 300X_1 + 400X_2$

s.t.

$$5X_1 + 4X_2 \geq 200 \, ; \quad 3X_1 + 5X_2 \geq 150 \qquad \text{(max amount of crude A and B)}$$

$$5x_1 + 4x_2 \geq 100 \, ; \, 8X_1 + 4X_2 \geq 80 \quad \text{(market requirement of gasoline X and Y)}$$

And

$$X_1, X_2 \geq 0$$

10-

Let X_1 and X_2 = number of vitamin units purchased of food F_1 and F_2 respectively.

Min $Z = 4 X_1 + 5 X_2$

s.t

$3X_1 + 6X_2 \geq 80$

$4X1 + 3 X_2 \geq 100$

And

$X_1, X2 \geq 0$

Quantitative Methods

CHAPTER THREE
Solving Linear Programming model
Graphical method

-Graphical method.

-Steps solution using the graphical method .

-Types of solutions in graphical method.

-Examples

-problems

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Solving linear programming model

Graphical Method: Suitable method to resolve the problems that contain **only two variables .**

Steps solution using the graphical method :

1- Write the objective function and the constraints after converted to linear equations.
2- Draw each of the constraints of chart (on the basis that the constraints are equal relationship) and define a set of points that achieve each constraint through the points of intersection with axes .
   Note that each of the points mentioned represents values of (X_1 ,X_2) .
3- Determine the solutions area:

   Constraints ≥ represents the area **far** from (0 ,0)
   Constraints ≤ represents the area **near** from (0 ,0)
   Constraints = represents the **points along it .**

   Then highlight the common area of the possible solutions for all points .
   And will be at least one point of these points (consisting of the solutions) represent the optimal solution to the problem .
4- Draw objective function by selecting a certain value and move it parallel to its location within the possible solutions to the region , while access to the optimal solution to the problem .
5- Choose the optimal solution for the problem by compensation the values of each of the points that formed the area of possible solutions .
   - Select the point that gives the **greatest** value if the objective function **maximize .**
   - Select the point that gives the **lowest** value if the objective function **minimize .**

Types of solutions in Graphical Method :

1- Unique optimal solution .
2- Alternative optimal solution .

3- Unbounded solution .

4- Infeasible solution .

Example -1-

Use the graphical method to solve the following linear programming problem .And what is the type of solution ?

Max Z =15 X_1 +10 X_2

      s.t.

          $4X_1$ +$6X_2$ ≤ 360

          $3X_1$ +$0X_2$ ≤ 180

        $0X_1$ +$5X_2$ ≤ 200

      X_1 ,X_2 ≥ 0

Solution:

1-The given LP problem is already in mathematical form.

2- we shall treat X_1 as the horizontal axis and X_2 as the vertical axis. Each constraint will be used to mark the area of feasible solutions .

Consider the first constraint $4X_1$ +$6X_2$ ≤ 360 . treat it as the equation ,

$4X_1$ +$6X_2$ = 360 . the easiest way to plot this line is to find any two points that satisfy the equation , then drawing a straight line through them . the two points are generally the points at which the line intersects the X_1 and X_2 axes . for example , when X_1 =0 we get 6 X_2 =360 or X_2 =60 .similarly , when X_2 =0 ,$4X_1$=360, X1=90 .

These two points are then connected by a straight line as shown in fig. . but the question is : where are the points satisfying $4X_1$ +$6X_2$ ≤ 360 . Any point above the constraint line violates the inequality condition . but any point below the line

does not violate the constraint . thus , the inequality and non-negativity condition can only be satisfied by the shaded area (feasible region or solution space) as shown in fig .

Similarly, the constraints $3 X_1 \le 180$ and $5X_2 \le 200$ are also plotted on the graph and are indicated by the shaded area as shown in fig.
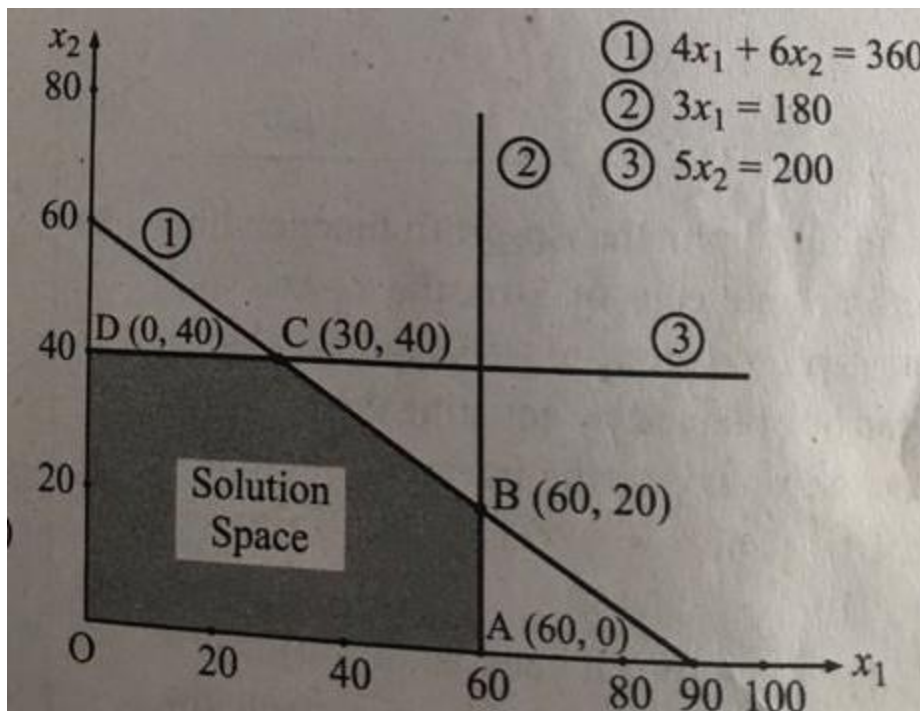
Since all constraints have been graphed , the area which is bounded by all the constraints lines including all the boundary points is called the feasible region

(or solution space) . the feasible region is shown in fig . by the shaded area OABCD .

3-(a)- since the optimal value of the objective function occurs at one of the extreme points of the feasible region . it is necessary to determine their coordinates . the coordinates of extreme points of the feasible region are :

O (0 ,0). A (60 ,0) , B (60 ,20) , C (30 ,40) ,D (0 ,40) .

(b)- Evaluate objective function value at each extreme point of the feasible region as shown below :

Extreme point	Coordinates (x_1, x_2)	Objective function value $Z = 15 X_1 + 10 X_2$
O	(0 ,0)	15 (0) +10 (0) =0
A	(60 ,0)	15 (60) +10 (0) =900
B	(60 ,20)	15 (60) +10 (20) =1100
C	(30 ,40)	15 (30) +10 (40) =850
D	(0 , 40)	15 (0) +10 (40) =400

© - since maximum value of   Z=1100 is achieved at the point B (60 ,20),the optimal solution to the given LP problem is :

X$_1$ =60 , X$_2$ =20 and max Z= 1100

Remark : to determine which side of a constraint equation is in the feasible region , examine whether the origin (0 ,0)satisfies the constraints . if it does , then all points on and below the constraint equation towards the origin are feasible points . if it does not, then all points on and above the constraint equation away from the origin are feasible points .

The type of solution is unique optimal solution .

Example   2-   *(unique optimal solution)*

Resolve the following linear programming problem by graphical method . And what is the type of solution ?

$$Max\ Z = 12 X_1 + 15 X_2$$

s.t.

$$4X_1 + 3X_2 \leq 12$$

$$2X_1 + 5X_2 \leq 10$$

$$X_1 , X_2 \geq 0$$

Solution:

Max $Z = 12X_1 + 15 X_2$

s.t,

$4 X_1 + 3 X_2 = 12$ (0 ,4) , (3 , 0)

$2 X_1 + 5 X_2 = 10$ (0 , 2) , (5 , 0)
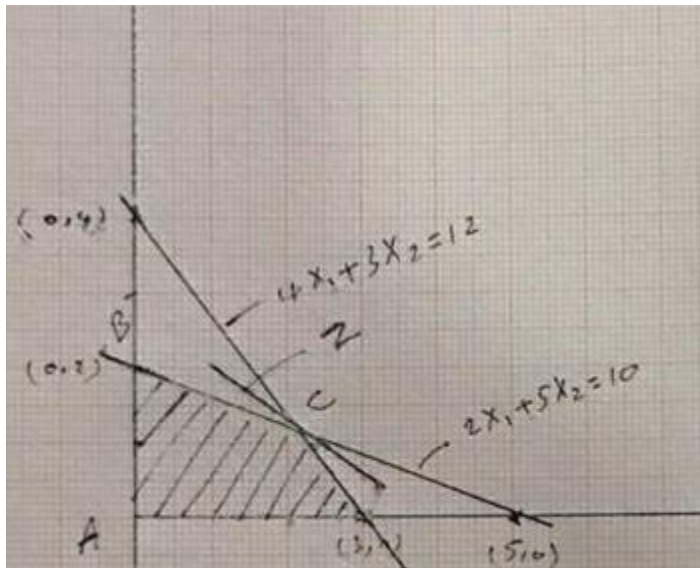
To find C

$4X_1 + 3 X_2 = 12$ ----------------$4 X_1 = 12 - 3X_2$ -----$X_1 = (12 - 3X_2)/4$

$2X_1 + 5 X_2 = 10$ ----------- $2 (12 - 3 X_2)/4 + 5 X_2 = 10$

     $X_2 = 8/7$

  $X_1 = 15/7$

C = (15/7 , 8/7)

A : (0 , 0) --------------- $Z = 0$

B : (0 , 2)------------ $Z = 30$

C : (15/7 , 8/7) ------ $Z = 300/7$

D : (3 , 0)--------- $Z = 36$

Extreme point	Coordinates (x_1, x_2)	Objective function value $Z = 12x_1 + 15x_2$
A	(0 ,0)	0
B	(0 ,2)	30
C	(15/7 ,8/7)	300/7
D	(3 ,0)	36

We choose the point (C) because it gives greater value when compensated in the objective function .

For the purpose of drawing the objective function compensate optimal value (300/7) in the objective function as follows :

$12 X_1 + 15 X_2 = 300/7$

And we get the two points and draw objective function that passes from the point (C) because it represent to unique optimal solution .

The type of solution is unique optimal solution .

Example -3 :   (**Alternative optimal solution**)

Resolve the following linear programming problem by graphical method . And what is the type of solution ?

Max $Z = 5 X_1 + 10 X_2$

s.t.

$2X_1 + 4 X_2 \leq 8$

$5 X_1 + 2 X_2 \leq 10$

$X_1, X_2 \geq 0$

Solution :

Max $Z = 5 X_1 + 10 X_2$

s.t.

$$2X_1 + 4X_2 = 8 \quad \text{----------}(0,2), (4,0)$$

$$5X_1 + 2X_2 = 10 \quad \text{--------}(0,5), (2,0)$$
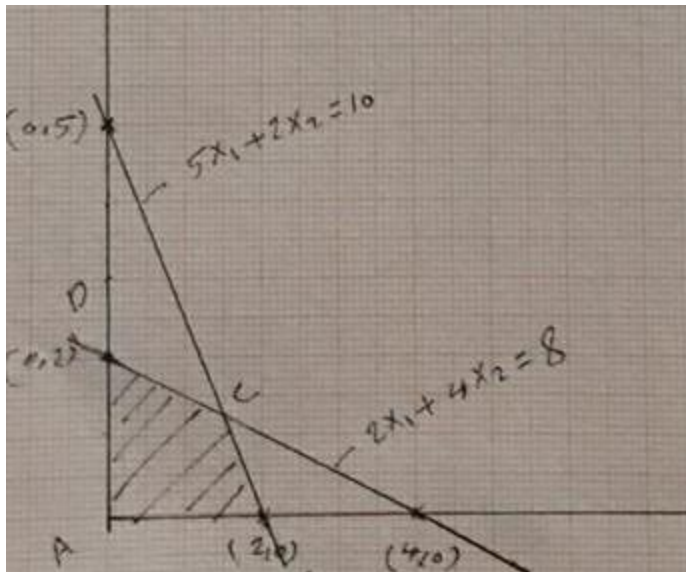
A : (0,0) -------   Z=0
B: (2,0) ------   Z=10
C: (1.5 ,1.25) ----   **Z=20**
D: (0,2)-----------   **Z=20**

$$5X_1 + 10X_2 = 20$$

Extreme point	Coordinates (x_1, x_2)	Objective function value $Z = 5x_1 + 10x_2$
A	(0,0)	0
B	(2,0)	10
C	(1.5,1.25)	20
D	(0,2)	20

We get the two points (0 ,2) , (4 ,0) and draw objective function , and we note that the objective function applies to the first equation because the solution is alternative optimal solution .

The type of solution is alternative optimal solution

Example 4 :   **(unbounded solution)**

Resolve the following linear programming problem by graphical method . . And what is the type of solution ?

$$\min \quad Z = 10 X_1 + 4 X_2$$

s.t.

$$8X_1 + 5 X_2 \geq 20$$

$$5 X_1 + 10 X_2 \geq 30$$

$$X_1 , X_2 \geq 0$$

Solution:

$$\min \quad Z = 10 X_1 + 4 X_2$$

s.t.

$8X_1 + 5 X_2 = 20$ (0 ,4) , (20/8 ,0)

$5 X_1 + 10 X_2 = 30$ (0 ,3   ) , (6 ,0)
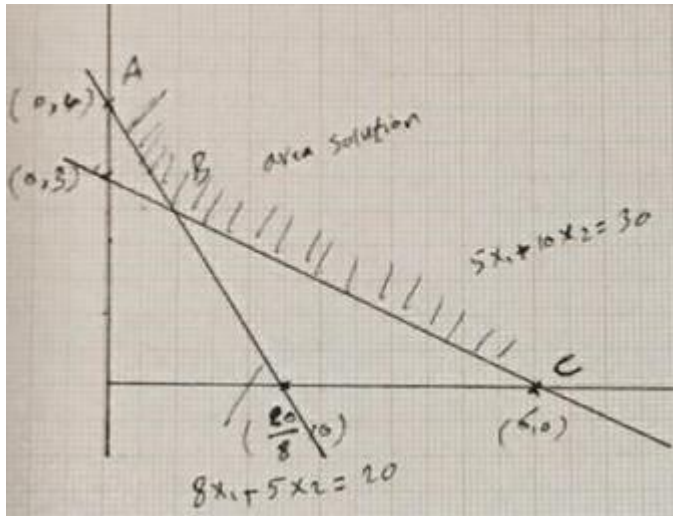
Area solutions is highlight area and determined by points A ,B ,C.

The resulting solution is unbounded solution because there is no bounded area solution .

Example 5:   (infeasible solution)

Resolve the following linear programming problem by graphical method . . And what is the type of solution ?

$$\max \quad Z = 10 X_1 + 8 X_2$$

s.t.

$$2X_1 + 3 X_2 \geq 24$$

$$5 X_1 + 2 X_2 \leq 10$$

$$X_1 , X_2 \geq 0$$

Solution:

$$\max \quad Z = 10 X_1 + 8 X_2$$

s.t.

$$2X_1 + 3 X_2 = 24 \qquad\qquad (0 , 8) ,(12 , 0)$$

$$5 X_1 + 2 X_2 = 10 \qquad\qquad (0 ,5) , (2 ,0)$$
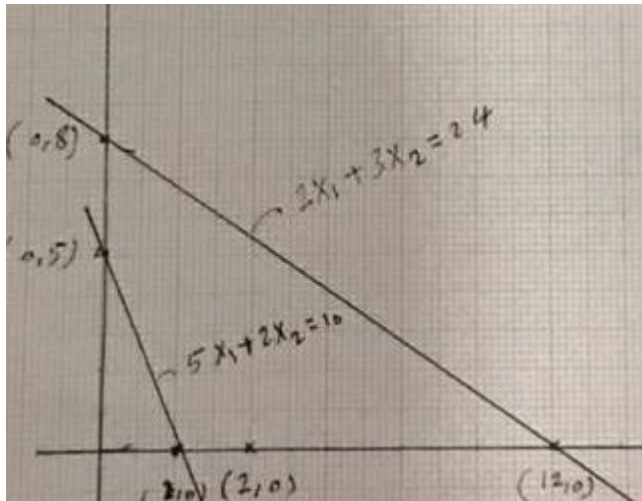
We note from the graph the absence of a common area between the equations .

So the resulting is infeasible solution because there is no common area solution .

Problems

1-

Use the graphical method to solve the following LP problem . And what is the type of solution ?

$$Max\ Z = 2X_1 + X_2$$
s.t.
$$X_1 + 2X_2 \leq 10$$
$$X_1 + X_2 \leq 6$$
$$X_1 - X_2 \leq 2$$
$$X_1 - 2X_2 \leq 1$$

$$X_1, X_2 \geq 0$$

2-

Use the graphical method to solve the following LP problem . And what is the type of solution ?

$$Max\ Z = 10X_1 + 15\ X_2$$
s.t.
$$2X_1 + X_2 \leq 26$$
$$2X_1 + 4X_2 \leq 56$$
$$-X_1 + X_2 \leq 5$$

And $X_1, X_2 \geq 0$

3-

Use the graphical method to solve the following LP problem. And what is the type of solution ?

$$Max\ Z = 40X_1 + 80\ X_2$$
s.t.

$2X_1 + 3 X_2 \leq 48$

$X_1 \leq 15$

$X_2 \leq 10$

And $X_1, X_2 \geq 0$

4-

Use the graphical method to solve the following LP problem. And what is the type of solution ?

Max Z = $60X_1 + 40 X_2$

s.t.

$X_1 \leq 25$

$X_2 \leq 35$

$2X_1 + X_2 \leq 60$

And $X_1, X_2 \geq 0$

5-

Use the graphical method to solve the following LP problem . And what is the type of solution ?

Max Z = $3X_1 + 1.5 X_2$

s.t.

$X_1 + X_2 \leq 4$

$X_1 + 2 X_2 \leq 6$

$3X_1 + 1.8 X_2 \leq 6$

And $X_1, X_2 \geq 0$

6-

Use the graphical method to solve the following LP problem . And what is the type of solution ?

Max Z = $X_1 + 2 X_2$

s.t.

$X_1 + 2X_2 \leq 10$

$X_1 + X_2 \geq 1$

$X_2 \leq 4$

And $X_1, X_2 \geq 0$

7-

Use the graphical method to solve the following LP problem. And what is the type of solution ?

Max Z = $12 X_1 + 4 X_2$

s.t.

$4X_1 + 2X_2 \leq 4$

$2X_1 \leq 8$

And $X_1, X_2 \geq 0$

8-

Use the graphical method to solve the following LP problem . And what is the type of solution ?

Max Z = $9 X_1 + 6 X_2$

s.t.

$4X_1 + 2X_2 \leq 4$

$6X_1 + 8X_2 \geq 8$

And $X_1, X_2 \geq 0$

9-

Use the graphical method to solve the following LP problem . And what is the type of solution ?

Max Z = $4X_1 - X_2$

s.t.

$X_1 + 2X_2 \leq 4$

$2X_1 + 3X_2 \leq 12$

$X_1 - X_2 \leq 3$

And $X_1, X_2 \geq 0$

Solutions:

1-

Plot on a graph each constraint by first treating them as a linear equation in the same way as discussed earlier. Use the inequality condition of each constraint to mark the feasible region (solution space) as shown in fig. . the feasible region is shown by the shaded area. Here it may be noted that we have not considered the area below the lines $X_1 - X_2 = 2$ and $X_1 - 2X_2 = 1$ for the negative values of X_2 . this is because of the non-negativity condition ,$X_2 \geq 0$, which implies that negative values of X_2 are not desirable .
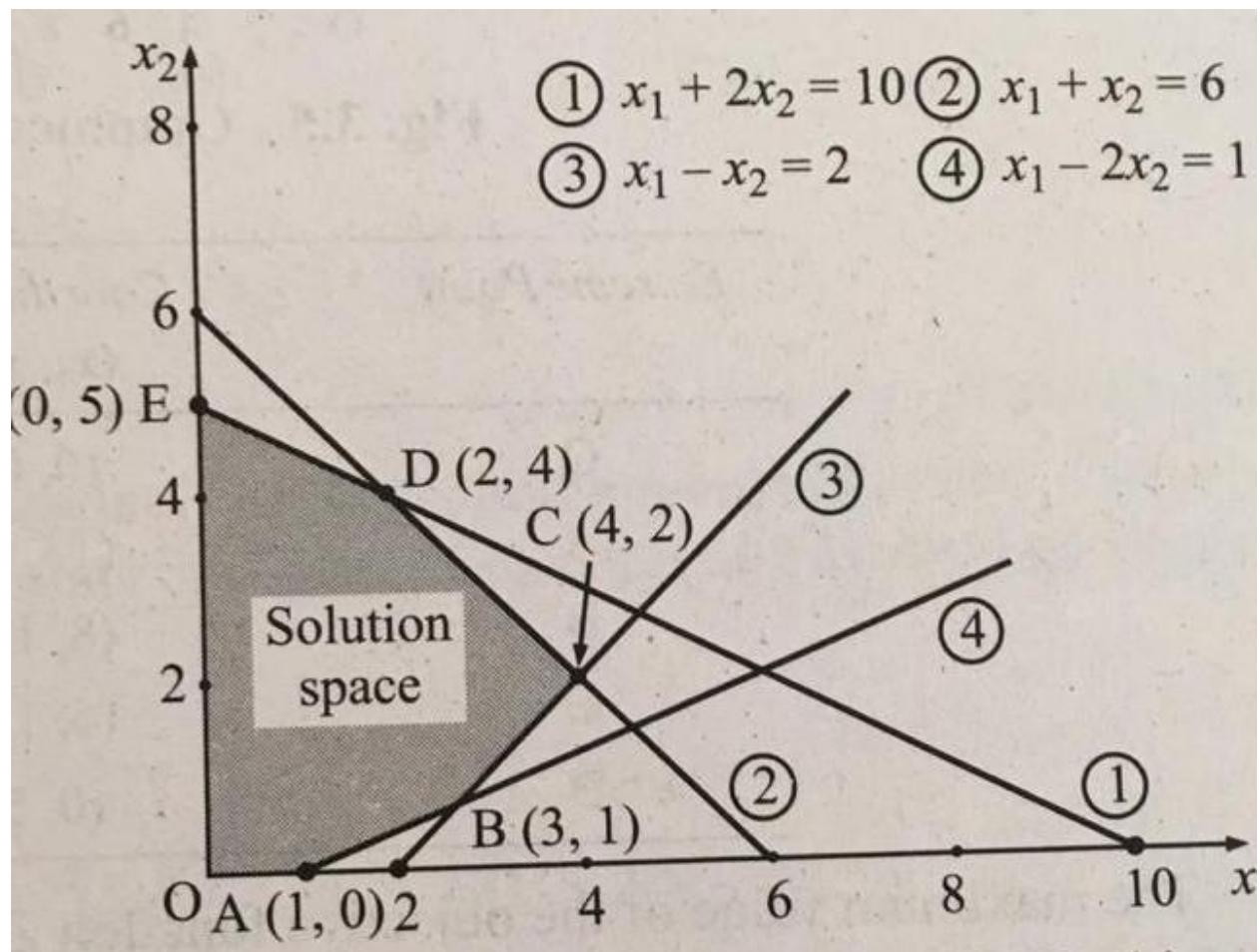
The coordinates of extreme points of the feasible region are :

O (0 ,0) ,A (1 ,0) , B(3,1), C (4 ,2) ,D (2 ,4) and ,E (0 ,5). The value of objective function at each of these extreme points is as follows :

Extreme point	Coordinates (X_1, X_2)	Objective function value $Z = 2X_1 + X_2$
O	(0 ,0)	2 (0) + 1 (0) =0
A	(1 ,0)	2 (1) + 1 (0) =2
B	(3 ,1)	2 (3) + 1 (1) =7
C	(4 ,2)	**2 (4) + 1 (2) = 10**
D	(2 ,4)	2 (2) + 1 (4) =8
E	(0 ,5)	2 (0) + 1 (5) =5

The maximum value of the objective function Z =10 occurs at the extreme point (4 ,2) . hence , the optimal solution to the given LP problem is :

X_1 =4 , X_2 =2 and max Z =10

The type of solution is unique optimal solution .

2-

This problem is depicted graphically in fig.. the feasible region is shown by shaded area . it may be noted here that we have not considered the area below the line

 -X_1 +X_2 =5 for the negative values of X_1 .

This is because of the non- negativity condition $X_1 \geq 0$, which implies that negative values of X_1 should not be considered.

$$① \quad 2x_1 + x_2 = 26$$
$$② \quad 2x_1 + 4x_2 = 56$$
$$③ \quad -x_1 + x_2 = 5$$
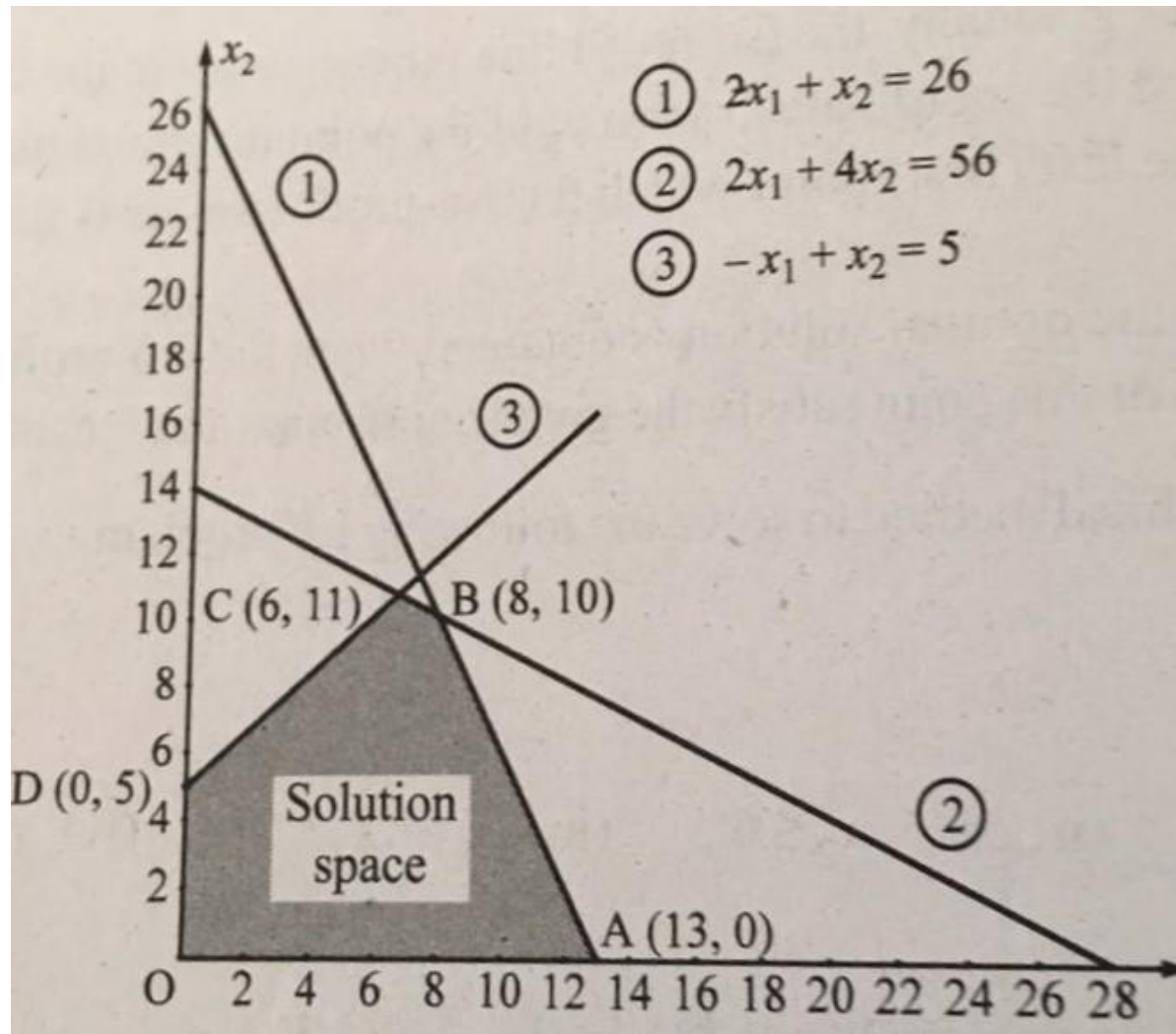
The coordinates of extreme points of the feasible region are :

O (0 ,0) ,A (13 ,0) ,B (8 ,10) ,C (6 ,11) ,and ,D (0 ,5).The value of the objective function at each of these extreme points is as follows :

Extreme point	Coordinates (X_1 , X_2)	Objective function value $Z = 10 X_1 + 15 X_2$
O	(0 ,0)	10 (0) + 15 (0) =0
A	(13 ,0)	10 (13) + 15 (0) = 130
B	(8 ,10)	**10 (8) + 15 (10) =230**
C	(6 ,11)	10 (6) + 15 (11) =225
D	(0 ,5)	10 (0) + 15 (5) =75

The maximum value of the objective function Z=230 occurs at extreme point

B (8 ,10). Hence ,optimal solution to the given LP problem is :

X₁ =8 , X₂= 10 and max Z =230 .

The type of solution is unique optimal solution .

3-

Graph each constraint by first treating them as a linear equation in the same way as discussed in the previous problems . use the inequality condition of each constraint to make the feasible region as shown in fig .. the feasible solution space or region is shown by shaded area .
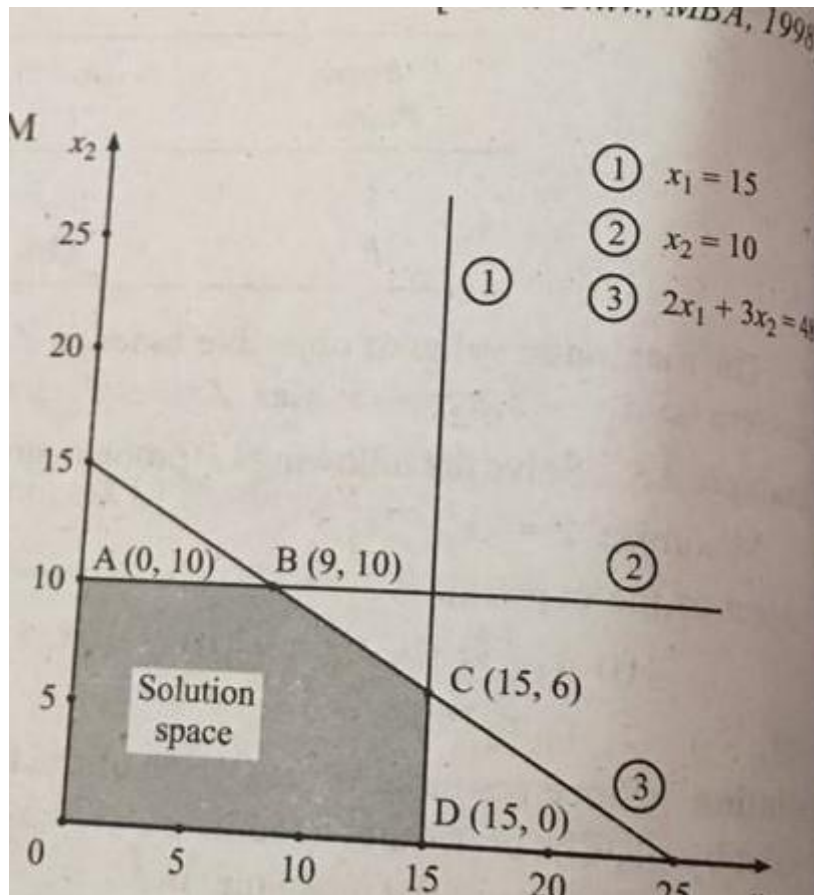
The coordinates of extreme point of the feasible region are :

O(0 ,0) , A (0 ,10) , B(9 ,10), C (15 ,6) ,and D(15 ,0) . The value of objective function at each of corner or extreme points is as follows :

Extreme point	Coordinates (X_1, X_2)	Objective function $Z = 40X_1 + 80 X_2$
O	(0 ,0)	40 (0) + 80 (0) =0
A	(0 ,10)	40 (0) + 80 (10) =800
B	(9 ,10)	**40 (9) + 80 (10) =1160**
C	(15 ,6)	40 (15) + 80 (6) =1080
D	(15 ,0)	40 (15) + 80 (0) =600

Since the maximum value of the objective function Z=1160 occurs at the extreme point (9 ,10), the optimum solution to the given LP problem is :

$X_1 =9$, $X_2 =10$ and maximum Z =Rs. 1160 .

The type of solution is unique optimal solution .

4-

Graph each constraint by first treating them as linear equation in the usual manner . the coordinates of the extreme points of the feasible solution space or region are :

A (0 ,35) , B (12.5 ,35) , C (25 ,10) and D (25 ,0) as shown in fig . . The value of the objective function at each of these extreme points is as follows :

Extreme point	Coordinates (X_1, X_2)	Objective function $Z = 60 X_1 + 40X_2$
A	(0 ,35)	60 (0) + 40 (35) =1400
B	(12.5 ,35)	**60 (12.5) + 40 (35) =2150**
C	(25 ,10)	60 (25) + 40 (10) =1900
D	(25 ,0)	60 (25) + 40 (0) =1500
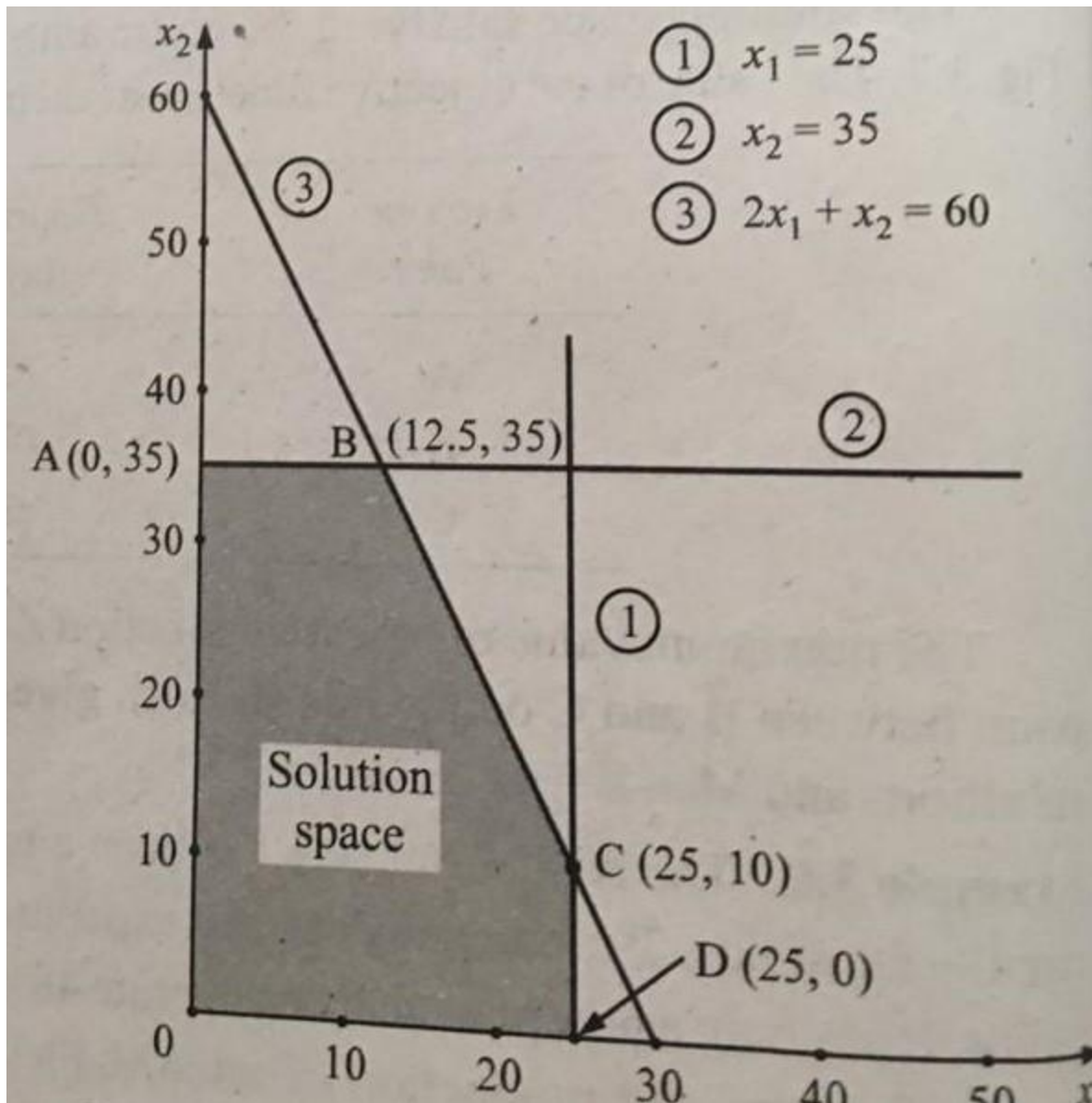
The type of solution is unique optimal solution .
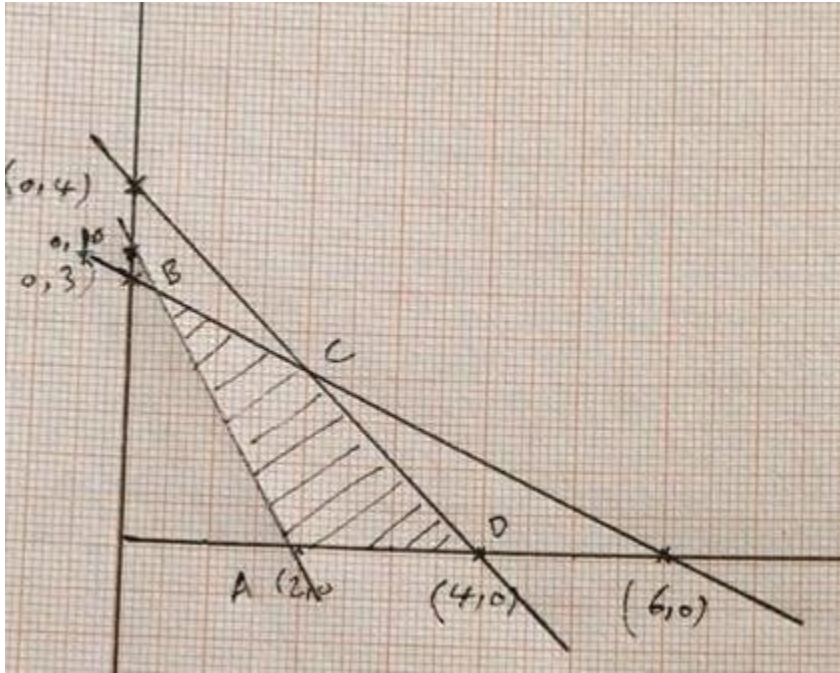
5 –

$$\text{Max } Z = 3X_1 + 1.5 \ X_2$$

s.t.

$$X_1 + X_2 \leq 4$$

$$X_1 + 2 X_2 \leq 6$$

$$3X_1 + 1.8 \ X_2 \geq 6$$

And $\quad X_1, X_2 \geq 0$

$X_1 + X_2 = 4$ ------ $(0,4)$, $(4,0)$

$X_1 + 2 X_2 = 6$ —-- $(0,3)$, $(6,0)$

$3X_1 + 1.8 X_2 = 6$ ---- $(0,10/3)$, $(2,0)$

the coordinates of the extreme points of the feasible solution space or region are :

A $(2,0)$, B $(2/7,20/7)$, C $(2,2)$ and D $(4,0)$ as shown in fig . . The value of the objective function at each of these extreme points is as follows :

Extreme point	Coordinates (X_1,X_2)	Objective function $Z = 3 X_1 + 1.5X_2$
A	$(2,0)$	$3(2) + 1.5(0) = 6$
B	$(2/7,20/7)$	$3(2/7) + 1.5(20/70) = 36/7$
C	$(2,2)$	$3(2) + 1.5(2) = 9$
D	**$(4,0)$**	**$3(4) + 1.5(0) = 12$**

The type of solution is unique optimal solution .

6 –

Max Z = $X_1 + 2 X_2$
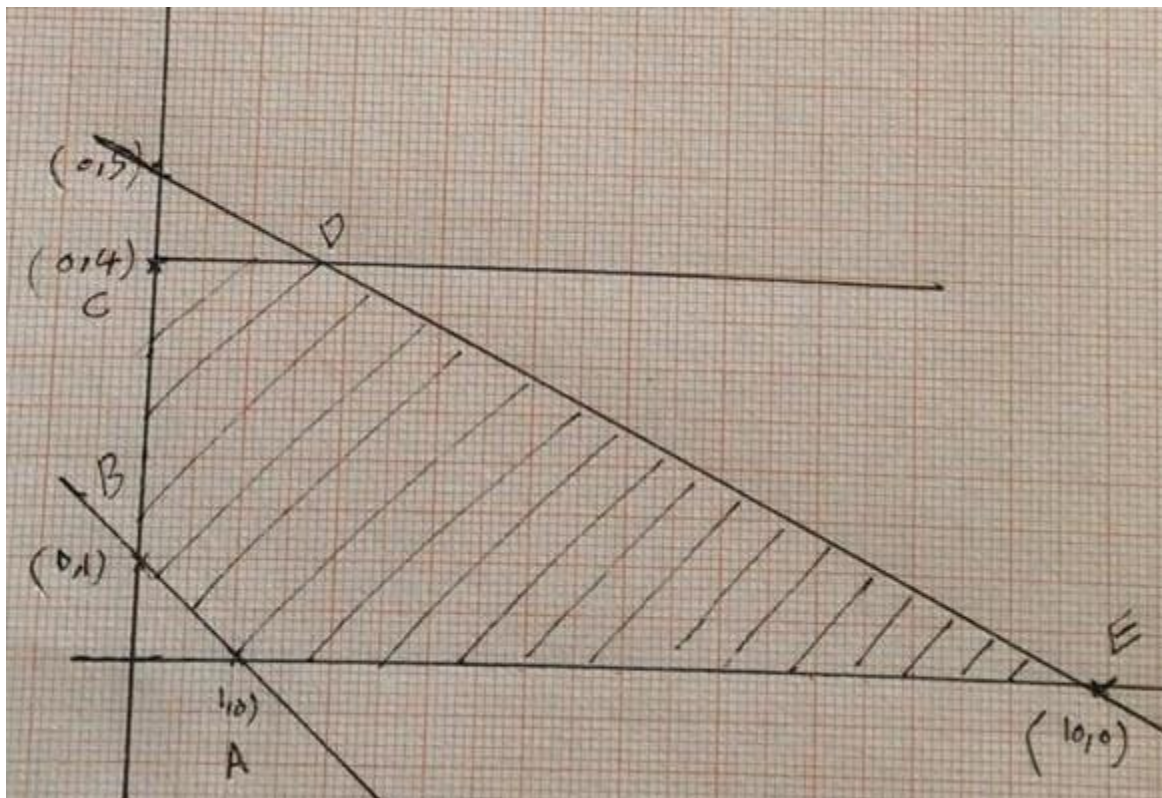
s.t.

$X_1 + 2 X_2 \leq 10$

$X_1 + X_2 \geq 1$

$X_2 \leq 4$

And $X_1, X_2 \geq 0$

$X_1 + 2 X_2 = 10$ ------ $(0,5)$, $(10,0)$

$X_1 + X_2 = 1$ —-- $(0,1)$, $(1,0)$

$X_2 = 4$

the coordinates of the extreme points of the feasible solution space or region are :

A (1 ,0) , B (0 ,1) , C (0 ,4) , D (2 ,4) and E (10 ,0) ,as shown in fig .   . The value of the objective function at each of these extreme points is as follows :

Extreme point	Coordinates (X_1 ,X_2)	Objective function $Z= X_1 + 2 X_2$
A	(1 ,0)	(1) + 2 (0) = 1
B	(0 ,1)	(0) + 2 (1) = 2
C	(0 ,4)	(0) + 2 (4) = 8
D	**(2 4)**	**(2) + 2 (4) = 10**
E	**(10 , 0)**	**(10) + 2 (0) = 10**

The type of solution is alternative optimal solution .

7-

Max $Z = 12 X_1 + 4 X_2$

s.t.

$4X_1 +2 X_2   \leq 4$

$2 X_2   \leq 8$

And $X_1 ,X_2 \geq 0$

$4 X_1 +2 X_2   = 4$ ------   (0 ,2) , (1 ,0)

$2 X_2=8$ ----   (0 ,4)
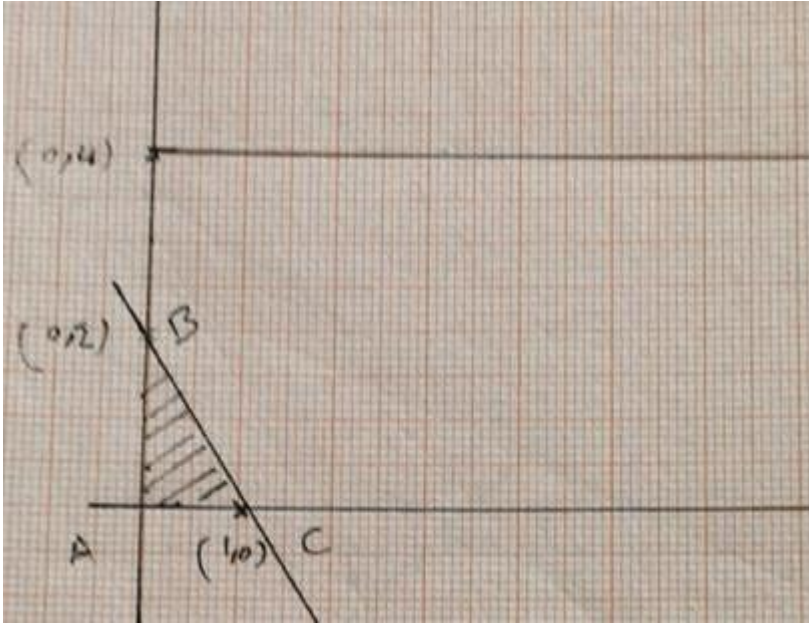
the coordinates of the extreme points of the feasible solution space or region are :

A (0 ,0) , B (0 ,2) , and C (1 ,0) ,as shown in fig . . The value of the objective function at each of these extreme points is as follows :

Extreme point	Coordinates (X_1, X_2)	Objective function $Z = 12 X_1 + 4 X_2$
A	(0,0)	12 (0) + 4 (0) = 0
B	(0 ,2)	(12 (0) + 4 (2) = 8
C	(1 ,0)	12 (1) + 4 (0) = 12

The type of solution is unique optimal solution .
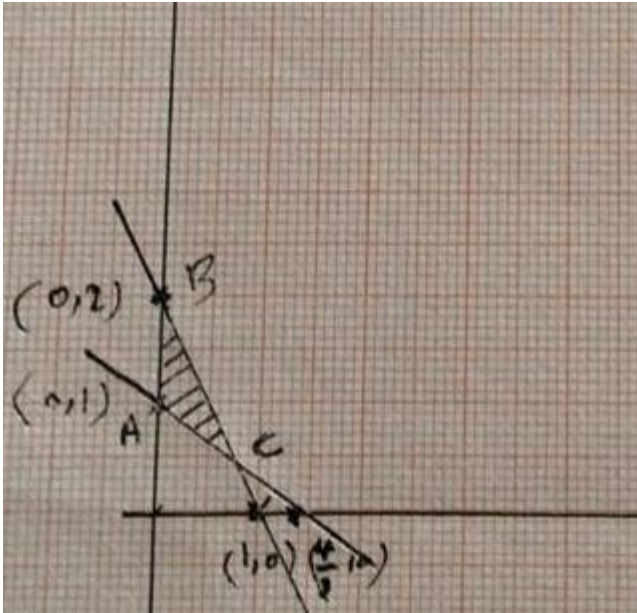
8-

Max Z = 9 X_1 + 6 X_2

s.t.

4X_1 +2 X_2 ≤ 4

6X_1 +8 X_2 ≥ 8

And X_1 ,X_2 ≥ 0

$$4X_1 + 2X_2 = 4 \text{ ------} \quad (0,2) , (1,0)$$
$$6X_1 + 8X_2 = 8 \quad\quad (0,1) , (4/3,0)$$

the coordinates of the extreme points of the feasible solution space or region are :

A $(0,0)$, B $(0,2)$, and C $(1,0)$,as shown in fig . . The value of the objective function at each of these extreme points is as follows :

Extreme point	Coordinates (X_1, X_2)	Objective function $Z = 9X_1 + 6X_2$
A	$(0,1)$	$9(0) + 6(1) = 6$
B	**$(0,2)$**	**$9(0) + 6(2) = 12$**
C	$(4/5, 2/5)$	$9(4/5) + 6(2/5) = 48/5$

The type of solution is unique optimal solution .

9-

Max Z = $4 X_1 - X_2$

s.t.

$X_1 + 2 X_2 \leq 4$

$2X_1 + 3 X_2 \leq 12$

$X_1 - X_2 \leq 3$

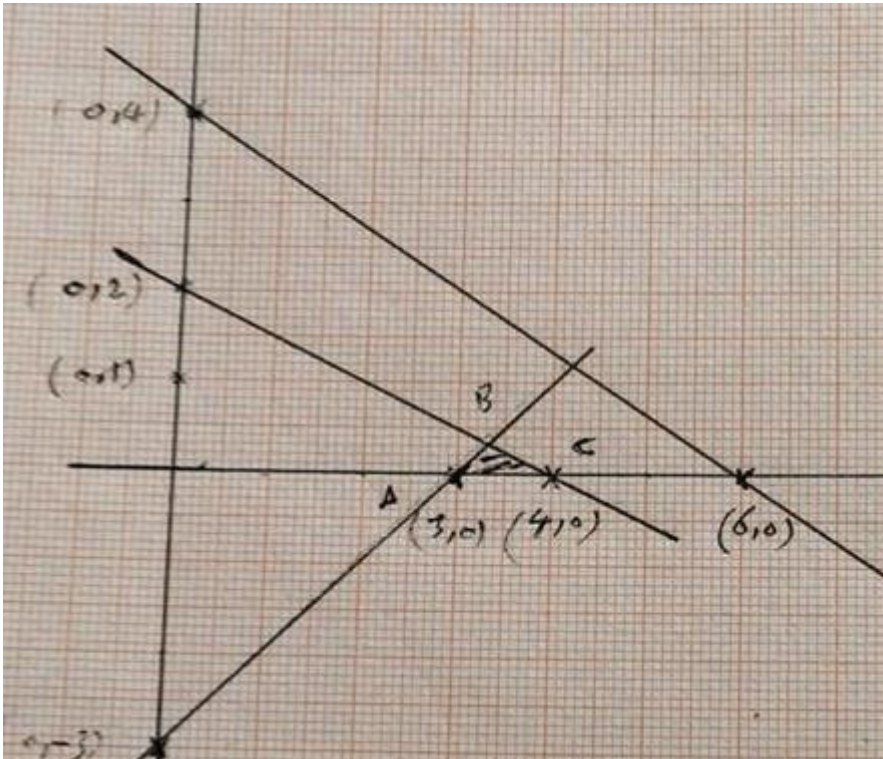And $X_1, X_2 \geq 0$

$X_1 + 2 X_2 = 4$ ------ $(4,0)$, $(0,2)$

$2X_1 + 3 X_2 = 12$ $(6,0)$, $(0,4)$

$X_1 - X_2 = 3$ --- $(3,0)$, $(0, -3)$

the coordinates of the extreme points of the feasible solution space or region are :

A (3 ,0) , B (10/3 ,1/3) , and C (4 ,0)   ,as shown in fig .   . The value of the objective function at each of these extreme points is as follows :

Extreme point	Coordinates (X_1, X_2)	Objective function $Z = 4X_1 - X_2$
A	(3,0)	4 (3) - (0) = 12
B	(10/3 ,1/3)	4 (10/3) - (1/3) = 13
C	**(4 ,0)**	**4 (4) - (0) = 16**

The type of solution is unique optimal solution .

Quantitative Methods

CHAPTER FOUR

Solving Linear Programming Model

Simplex method

-Simplex method.

-Basic variables .

-Non- Basic variables .

-Slack , surplus & Artificial variables .

The steps of the simplex method .

-Examples

-problems

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Simplex method

Simplex method: is the method to solve linear programming models which contain two or more than two , decision variables.

Basic variables:

Are the variables which coefficients <u>One</u> in the equations and <u>Zero</u> in the other equations.

Non-Basic variables:

Are the variables which coefficients are taking any of the values, whether positive or negative or zero.

Slack, surplus & artificial variables:

a) If the inequality be $\leq$ (less than or equal, then we add a **slack variable + S** to change $\leq$ to =.

b) If the inequality be $\geq$ (greater than or equal, then we subtract a **surplus variable - S** to change $\geq$ to =.

c) If we have = we use artificial variables.

The steps of the simplex method:

<u>Step 1:</u>

Determine a starting basic feasible solution.

<u>Step 2:</u>

Select an entering variable using the **optimality condition**. Stop if there is no entering variable.

<u>Step 3:</u>

Select a leaving variable using the **feasibility condition.**

Optimality condition:

The entering variable in a maximization (minimization) problem is the non-basic variable having the **most negative (positive) coefficient in the Z row**.

The optimum is reached at the iteration where all the Z row coefficient of the non-basic variables are **non-negative (non-positive).**

Feasibility condition:

For both maximization and minimization problems the leaving variable is the basic associated with the smallest non-negative ratio (with strictly positive denominator).

Pivot row:

      a) Replace the leaving variable in the basic column with the entering variable.

      b) New pivot row equal to current pivot row divided by pivot element.

      c) All other rows:

      New row=current row - (pivot column coefficient) *new pivot row.

Example 1:

Use the simplex method to solve the (LP) model:

$$max \ Z = 5x_1 + 4x_2$$

Subject to

$$6x_1 + 4x_2 \leq 24$$

$$x_1 + 2x_2 \ \leq 6$$

$$-x_1 + x_2 \ \leq 1$$

$$x_2 \ \leq 2$$

$$x_1, x_2 \ \geq 0$$

Solution:

$$max \ Z - 5x_1 - 4x_2 = 0$$

Subject to

$$6x_1 + 4x_2 + S_1 = 24$$

$$x_1 + 2x_2 + S_2 \ = 6$$

$$-x_1 + x_2 + S_3 = 1$$

$$x_2 + S_4 = 2$$

Table 1:

Basic	x_1	x_2	S_1	S_2	S_3	S_4	Sol.
S_1	6	4	1	0	0	0	24
S_2	1	2	0	1	0	0	6
S_3	-1	1	0	0	1	0	1
S_4	0	1	0	0	0	1	2
Max Z	-5	-4	0	0	0	0	0

$\frac{24}{6} = 4$

$\frac{6}{1} = 6$

$\frac{1}{-1} = -1$ (ignore)

$\frac{2}{0} = \infty$ (ignore)

The entering variable is x_1 and S_1 is a leaving variable.

Table 2:

Basic	x_1	x_2	S_1	S_2	S_3	S_4	Sol.
x_1	1	2/3	1/6	0	0	0	4
S_2	0	4/3	-1/6	1	0	0	2
S_3	0	5/3	1/6	0	1	0	5
S_4	0	1	0	0	0	1	2
Max Z	0	-2/3	5/6	0	0	0	20

■ Pivot row or new x_1 row=$\frac{1}{6}$ [current S_1 row]

$$= \frac{1}{6}[6 \quad 4 \quad 1 \quad 0 \quad 0 \quad 0 \quad 24]$$

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$$= [1 \quad \frac{2}{3} \quad \frac{1}{6} \quad 0 \quad 0 \quad 0 \quad 4]$$

- New S_2 row=[current S_2 row]-(1)[new x_1 row]

   =[1 2 0 1 0 0 6]- (1)[1 2/3 1/6 0 0 0 0 4]

   =[0 4/3 -1/6 1 0 0 2]

- New S_3 row=[current S_3 row]-(1)[new x_1 row]

   =[-1 1 0 0 1 0 1]- (-1)[1 2/3 1/6 0 0 0 0 4]

   =[0 5/3 1/6 0 1 0 5]

- New S_4 row=[current S_4row]-(0)[new x_1 row]

   =[0 1 0 0 0 1 2]- (0)[1 2/3 1/6 0 0 0 0 4]

   =[0 1 0 0 0 − 1 2]

- New Z row=[current Z row]-(-5)[new x_1 row]

   =[-5 -4 0 0 0 0 0]-(-5)[1 2/3 1/6 0 0 0 0 4]

   =[0 -2/3 5/6 0 0 0 20]

Now:

$$\frac{4}{\frac{2}{3}} = 6$$

2/(4/3) =3/2

$$\frac{5}{\frac{5}{3}} = 3$$

$$\frac{2}{1} = 2$$

The entering variable is x_2 and S_2 is a leaving variable.

Table 3: (optimal solution):

Basic	x_1	x_2	S_1	S_2	S_3	S_4	Sol.
x_1	1	0	1/4	-1/2	0	0	3
x_2	0	1	-1/8	3/4	0	0	3/2
S_3	0	0	3/8	-5/4	1	0	5/2
S_4	0	0	1/8	-3/4	0	1	1/2
Max Z	0	0	3/4	1/2	0	0	21

- Pivot row or new x_2 row=$\frac{1}{\frac{4}{3}}$ [current S_2 row]

$=\frac{1}{\frac{4}{3}}[0 \quad 4/3 \quad -1/6 \quad 1 \quad 0 \quad 0 \quad 2]$

$=[0 \quad 1 \quad -1/8 \quad ¾ \quad 0 \quad 0 \quad 3/2]$

- New x_1 row=[current x_1 row]-(2/3)[new x_2 row]

$=[1 \quad 2/3 \quad 1/6 \quad 0 \quad 0 \quad 0 \quad 4]- (2/3)[0 \quad 1 \quad -1/8 \quad ¾ \quad 0 \quad 0 \quad 3/2]$

$=[1 \quad 0 \quad ¼ \quad -1/2 \quad 0 \quad 0 \quad 3]$

- New S_3 row=[current S_3 row]-(5/2)[new x_2 row]

$=[0 \quad 5/3 \quad 1/6 \quad 0 \quad 1 \quad 0 \quad 5]-(5/3)[0 \quad 1 \quad -1/8 \quad ¾ \quad 0 \quad 0 \quad 3/2]$

$=[0 \quad 0 \quad 3/8 \quad -5/4 \quad 1 \quad 0 \quad 5/2]$

- New S_4 row=[current S_4 row]-(1)[new x_2 row]

$=[0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 1 \quad 2]-(1)[0 \quad 1 \quad -1/8 \quad ¾ \quad 0 \quad 0 \quad 3/2]$

$=[0 \quad 0 \quad 1/8 \quad -3/4 \quad 0 \quad 1 \quad ½]$

New Z row=[current Z row]-(-2/3)[new x_2 row]

$=[0 \quad -2/3 \quad 5/6 \quad 0 \quad 0 \quad 0 \quad 20]-(-2/3)[0 \quad 1 \quad -1/8 \quad ¾ \quad 0 \quad 0 \quad 3/2]$

$=[0 \quad 0 \quad ¾ \quad ½ \quad 0 \quad 0 \quad 21]$

Then the solution is:

$$x_1 = 3 \ \& \ x_2 = \frac{3}{2} \ \& \ S_3 = \frac{5}{2} \ \& \ S_4 = \frac{1}{2}$$

$S_1 = 0, \ S_2 = 0 //$max z= 21

Example 2:

Use the simplex method to solve the (LP) model:

$$max \ Z = 2x_1 + 3x_2$$

Subject to

$$0.25x_1 + 0.5x_2 \leq 40$$

$$0.4x_1 + 0.2x_2 \quad \leq 40$$

$$0.8x_2 \quad \leq 40$$

$$x_1, x_2 \quad \geq 0$$

Solution:

$$max \ Z - 2x_1 - 3x_2 = 0$$

Subject to

$$0.25x_1 + 0.5x_2 + S_1 = 40$$

$$0.4x_1 + 0.2x_2 + S_2 = 40$$

$$0.8x_2 + S_3 = 40 \qquad x_1, x_2, \ S_1, \ S_2,$$

$$S_3 \geq 0$$

Table 1:

Basic	x_1	x_2	S_1	S_2	S_3	Sol.
S_1	0.25	0.5	1	0	0	40
S_2	0.4	0.2	0	1	0	40
S_3	0	0.8	0	0	1	40
Max Z	-2	-3	0	0	0	0

$$\frac{40}{0.5} = 80$$

$$\frac{40}{0.2} = 200$$

$$\frac{40}{0.8} = \boxed{50}$$

- Pivot row or new S_3 row=$\frac{1}{0.8}$ [0   0.8   0   0   1   40]

$\qquad\qquad\qquad\qquad$ =[0   1   0   0   1.25   50]

New S_1 row=[current S_1 row]-(0.5)[new x_2 row]

=[0.25   0.5   1   0   0   40]-(0.5)[0   1   0   0   1.25   50]

=[0.25   0   1   0   -0.625   15]

New S_2row=[current S_2 row]-(0.2)[new x_2 row]

=[0.4   0.2   0   1   0   40]-(0.2)[0   1   0   0   1.25   50]

[0.4   0   0   1   -0.25   30]

New Zrow=[current Z row]-(-3)[new x_2 row]

=[-2   -3   0   0   0   0]-(-3)[0   1   0   0   1.25   50]

=[-2   0   0   0   3.75   150]

Table 2:

Basic	x_1	x_2	S_1	S_2	S_3	Sol.
S_1	0.25	0	1	0	-0.625	15
S_2	0.4	0	0	1	-0.25	30
x_2	0	1	0	0	1.25	50
Max Z	-2	0	0	0	3.75	150

$\frac{15}{0.25} = 60$

$\frac{30}{0.4} = 75$

$\frac{50}{0} = \infty$ (ignore)

Pivot row or new S_1row=$\frac{1}{0.25}$ [0.25   0   1   0   -0.625   15]

$\qquad\qquad\qquad\qquad$ =[1   0   4   0   -2.5   60]

New S_2row=[current S_2 row]-(0.4)[new x_1 row]

$\qquad$ =[0.4   0   0   1   -0.25   30]-(0.4)[1   0   4   0   -2.5   60]

$\qquad$ [0   0   -1.6   1   0.75   6]

New $x_2$row=[0   1   0   0   1.25   50]-(0)[1   0   4   0   -2.5   60]

$\qquad$ =[0   1   0   0   1.25   50]

New Zrow=[current Z row]-(-2)[1   0   4 0   -2.5   60]

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$$=[-2\ 0\ 0\ 0\ 3.75\ 150]-(-2)[1\ 0\ 4\ 0\ -2.5\ 60]$$
$$[0\ 0\ 8\ 0\ -1.25\ 270]$$

Table 3:

Basic	x_1	x_2	S_1	S_2	S_3	Sol.
x_1	1	0	4	0	-2.5	60
S_2	0	0	-1.6	1	0.75	6
x_2	0	1	0	0	1.25	50
Max Z	0	0	8	0	-1.25	270

$$\frac{60}{-2.5}=-24 \text{ (ignore)}$$

$$\frac{6}{0.75}=8$$

$$\frac{50}{1.25}=40$$

New S_2row=$\frac{1}{0.75}$ =[current S_2row] =$\frac{1}{0.75}$ [0 0 -1.6 1 0.75 6]
$$=[0\ \ 0\ \ -2.133\ \ 1.33\ \ 1\ \ 8]$$
New $x_1$row= [1 0 4 0 -2.5 60]-(-2.5)[0 0 -2.133 1.33 1   8]
$$=[1\ \ 0\ \ -1.333\ \ 3.325\ \ 0\ \ 80]$$
New $x_2$row= [0 1 0 0 1.25 50]-(-1.25)[0 0 -2.133 1.33 1   8]
$$=[0\ \ 1\ \ -2.666\ \ -1.663\ \ 0\ \ 40]$$
New Zrow= [0 0 8 0 -1.25 270]-(-1.25)[0 0 -2.133 1.33 1   8]
$$=[0\ \ 0\ \ 5.33\ \ 1.663\ \ 0\ \ 280]$$

Table 4: (optimal solution):

Basic	x_1	x_2	S_1	S_2	S_3	Sol.
x_1	1	0	-1.333	3.325	0	80
S_3	0	0	-2.133	0	1	8
x_2	0	1	2.666	-1.663	0	40
Max Z	0	0	5.33	1.663	0	280

The optimal solution :

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x_1=80   , $x_2 = 40$,S₃=8 $S_1 = 0$ & $S_2 = 0$ // max Z=280

Example 3:

Use the simplex method to solve the (LP) model:
$$min\ Z = -6x_1 - 10x_2 - 4x_3$$

Subject to

$$x_1 + x_2 + x_3 \leq 1000$$

$$x_1 + x_2 \qquad \leq 500$$

$$x_1 + 2x_2 \qquad \leq 700$$

$$x_1, x_2, x_3 \qquad \geq 0$$

Solution:

$$min\ Z + 6x_1 + 10x_2 + 4x_3 = 0$$

Subject to

$$x_1 + x_2 + x_3 + S_1 = 1000$$

$$x_1 + x_2 \ + S_2 \qquad = 500$$

$$x_1 + 2x_2 \ + S_3 \qquad = 700$$

$$x_1, x_2, x_3\ , S_1, S_2\ S_3 \ \geq 0$$

Table 1:

Basic	x_1	x_2	x_3	S_1	S_2	S_3	Sol.
S_1	1	1	1	1	0	0	1000
S_2	1	1	0	0	1	0	500
S_3	1	2	0	0	0	1	700
Min Z	6	10	4	0	0	0	0

$$\frac{1000}{1} = 1000$$

← $$\frac{500}{1} = 500$$

$$\frac{700}{2} = \boxed{350}$$

New S_3 row or x_2 row $= \frac{1}{2}[1 \ 2 \ 0 \ 0 \ 0 \ 1 \ 700]$

$$= [\frac{1}{2} \ 1 \ 0 \ 0 \ 0 \ \frac{1}{2} \ 350]$$

New S_1 row $= [1 \ 1 \ 1 \ 1 \ 0 \ 0 \ 1000]-(1)[\frac{1}{2} \ 1 \ 0 \ 0 \ 0 \ \frac{1}{2} \ 350]$

$$= [\frac{1}{2} \ 0 \ 1 \ 1 \ 0 \ -\frac{1}{2} \ 650]$$

New S_2 row $= [1 \ 1 \ 0 \ 0 \ 1 \ 0 \ 500]-(1)[\frac{1}{2} \ 1 \ 0 \ 0 \ 0 \ \frac{1}{2} \ 350]$

$$= [\frac{1}{2} \ 0 \ 0 \ 0 \ 1 \ -\frac{1}{2} \ 150]$$

New Z row $= [6 \ 10 \ 4 \ 0 \ 0 \ 0 \ 0]-(10)[\frac{1}{2} \ 1 \ 0 \ 0 \ 0 \ \frac{1}{2} \ 350]$

$$= [1 \ 0 \ 4 \ 0 \ 0 \ -5 \ -3500]$$

Table 2:

Basic	x_1	x_2	x_3	S_1	S_2	S_3	Sol.
S_1	1/2	0	1	1	0	-1/2	650
S_2	1/2	0	0	0	1	-1/2	150
x_2	1/2	1	0	0	0	1/2	350
Min Z	1	0	4	0	0	-5	-3500

$$\frac{650}{1} = \boxed{650}$$

$$\frac{150}{0} = \infty \qquad \text{(ignore)}$$

$$\frac{350}{0} = \infty \qquad \text{(ignore)}$$

New S_1 row or x_3 row $= 1[\frac{1}{2} \ 0 \ 1 \ 1 \ 0 \ -\frac{1}{2} \ 650]$

$$= [\frac{1}{2} \ 0 \ 1 \ 1 \ 0 \ -\frac{1}{2} \ 650]$$

New S_2 row = $[\frac{1}{2}\ 0\ 0\ 0\ 1\ -\frac{1}{2}\ \ 150]-(0)[\frac{1}{2}\ \ 0\ 1\ \ 1\ 0\ -\frac{1}{2}\ \ 650]$

$\qquad\qquad =[\frac{1}{2}\ 0\ 0\ 0\ 1\ -\frac{1}{2}\ \ 150]$

New x_2 row = $[\frac{1}{2}\ 1\ 0\ 0\ 0\ \ \frac{1}{2}\ \ 350]-(0)[\frac{1}{2}\ \ 0\ 1\ \ 1\ 0\ -\frac{1}{2}\ \ 650]$

$\qquad\qquad =[\frac{1}{2}\ 1\ 0\ 0\ 0\ \ \frac{1}{2}\ \ 350]$

New Z row = $[1\ 0\ 4\ 0\ 0\ -5\ \ -3500]-(4)[\frac{1}{2}\ \ 0\ 1\ \ 1\ 0\ -\frac{1}{2}\ \ 650]$

$\qquad\qquad =[-1\ 0\ 0\ -4\ 0\ \ -3\ \ -6100]$

Table 3: (optimal solution):

Basic	x_1	x_2	x_3	S_1	S_2	S_3	Sol.
x_3	1/2	0	1	1	0	-1/2	650
S_2	1/2	0	0	0	1	-1/2	150
x_2	1/2	1	0	0	0	1/2	350
Min Z	-1	0	0	-4	0	-3	-6100

The optimal solution :

x_3=650 , $x_2 = 350$, $S_2 = 150$ S_3 & $S_1 = 0$, $x_1 = 0$ // Min Z=6100

Example 4:

Use the simplex method to solve the (LP) model:

$$max\ Z = 4x_1 - x_2$$

Subject to

$$x_1 + 2x_2\ \le 4$$

$$2x_1 + 3x_2\ \le 12$$

$$x_1 - x_2\ \ \ \le 3$$

$$x_1, x_2\ \ \ \ \ \ge 0$$

Solution:

$$max\ Z - 4x_1 + x_2 = 0$$

Subject to

$$x_1 + 2x_2 + S_1 = 4$$

$$2x_1 + 3x_2 + S_2 = 12$$

$$x_1 - x_2 + S_3 = 3$$

$$x_1, x_2, S_1, S_2, S_3 \geq 0$$

Table 1:

Basic	x_1	x_2	S_1	S_2	S_3	Sol.
S_1	1	2	1	0	0	4
S_2	2	3	0	1	0	12
S_3	1	-1	0	0	1	3
Max Z	-4	1	0	0	0	0

$$\frac{4}{1} = 4$$

$$\frac{12}{2} = 6$$

$$\frac{3}{1} = 3$$

New S_3 row or x_1 row = 1[1 -1 0 0 1 3]

$\quad\quad\quad\quad\quad\quad\quad$ =[1 -1 0 0 1 3]

New S_1 row = [1 2 1 0 0 4]-(1)[1 -1 0 0 1 3]

$\quad\quad\quad\quad$ =[0 3 1 0 -1 1]

New S_2 row = [2 3 0 1 0 12]-(2)[1 -1 0 0 1 3]

$\quad\quad\quad\quad$ =[0 5 0 1 -2 6]

New Z row = [-4 1 0 0 0 0]-(-4)[1 -1 0 0 1 3]

$\quad\quad\quad\quad$ =[0 -3 0 0 4 12]

Table 2:

Basic	x_1	x_2	S_1	S_2	S_3	Sol.
S_1	0	3	1	0	-1	1
S_2	0	5	0	1	-2	6
x_1	1	-1	0	0	1	3
Max Z	0	-3	0	0	4	12

$$\frac{1}{3} = \frac{1}{3}$$

$$\frac{6}{5} = \frac{6}{5}$$

$$\frac{3}{-1} = -3 \quad \text{(ignore)}$$

New S_1 row or x_2 row $= (\frac{1}{3})[0\ \ 3\ 1\ 0\ \ -1\ \ 1]$

$\qquad\qquad\qquad\qquad =[0\ \ 1\ 1/3\ 0\ \ -1/3\ \ 1/3]$

New S_2 row $= [0\ 5\ 0\ 1\ -2\ \ 6]-(5)[\ 0\ \ 1\ 1/3\ \ 0-1/3\ \ 1/3]$

$\qquad\qquad\qquad =[0\ \ 0\ -5/3\ 1\ -1/3\ \ 13/3]$

New x_1 row $= [1\ -1\ 0\ 0\ 1\ \ 3]-(-1)[\ 0\ \ 1\ 1/3\ \ 0-1/3\ \ 1/3]$

$\qquad\qquad\qquad =[1\ \ 0\ 1/3\ 0\ 2/3\ \ 10/3]$

New Z row $= [0\ -3\ 0\ 0\ 4\ \ 12]-(-3)[\ 0\ \ 1\ 1/3\ \ 0-1/3\ \ 1/3]$

$\qquad\qquad\qquad =[0\ \ 0\ 1\ 0\ 3\ \ 13]$

Table 3: (optimal solution):

Basic	x_1	x_2	S_1	S_2	S_3	Sol.
x_2	0	1	1/3	0	-1/3	1/3
S_2	0	0	-5/3	1	-1/3	13/3
x_1	1	0	1/3	0	2/3	10/3
Max Z	0	0	1	0	3	13

The optimal solution:

$x_1=10/3$, $x_2 = 1/3$, $S_2 = 13/3$ S_1 & $S_3 = 0$ // Z=13

Example 5:

Use the simplex method to solve the (LP) model:

$$max\ Z = 16x_1 + 17x_2 + 10x_3$$

Subject to

$$x_1 + 2x_2 + 4x_3 \le 2000$$

$$2x_1 + x_2 + x_3 \le 3600$$

$$x_1 + 2x_2 + 2x_3 \le 2400$$

$$x_1 \le 30$$

$$x_1, x_2,\ x_3 \ge 0$$

Solution:

$$max\ Z - 16x_1 - 17x_2 - 10x_3 = 0$$

Subject to

$$x_1 + 2x_2 + 4x_3 + S_1 = 2000$$

$$2x_1 + x_2 + x_3 + S_2 = 3600$$

$$x_1 + 2x_2 + 2x_3 + S_3 = 2400$$

$$x_1 + S_4 = 30$$

$$x_1, x_2,\ x_3 \ge 0, \quad S_1, S_2,\ S_3, S_4 \ge 0$$

Table 1:

Basic	x_1	x_2	x_3	S_1	S_2	S_3	S_4	Sol.
S_1	1	2	4	1	0	0	0	2000
S_2	2	1	1	0	1	0	0	3600
S_3	1	2	2	0	0	1	0	2400
S_4	1	0	0	0	0	0	1	30
Max Z	-16	-17	-10	0	0	0	0	0

$$\frac{2000}{2} = 1000$$

$$\frac{3600}{1} = 3600$$

$$\frac{2400}{2} = 1200$$

$$\frac{30}{0} = \infty \quad \text{(ignore)}$$

New S_1 row or x_1 row = $(\frac{1}{2})[1 \quad 2 \quad 4 \quad 1 \quad 0 \quad 0 \quad 0 \quad 2000]$

$\qquad\qquad\qquad = [1/2 \quad 1 \, 2 \quad 1/2 \quad 0 \quad 0 \quad 0 \quad 1000]$

New S_2 row = $[2 \quad 1 \quad 1 \quad 0 \quad 1 \quad 0 \quad 0 \quad 3600]$

$\qquad\qquad -(1)[\, 1/2 \quad 1 \quad 2 \quad 1/2 \quad 0 \quad 0 \quad 0 \quad 1000]$

$= [3/2 \quad 0 \quad -1 \quad -1/2 \quad 1 \, 0 \quad 0 \quad 2600]$

New S_3 row = $[1 \quad 2 \quad 2 \quad 0 \quad 0 \quad 1 \quad 0 \quad 2400]$

$\qquad\qquad -(2)[\, 1/2 \quad 1 \quad 2 \quad 1/2 \quad 0 \quad 0 \quad 0 \quad 1000]$

$= [0 \quad 0 \quad -2 \quad -1 \quad 0 \, 1 \quad 0 \quad 400]$

New S_4 row = $[1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 30]$

$\qquad\qquad -(0)[\, 1/2 \quad 1 \quad 2 \quad 1/2 \quad 0 \quad 0 \quad 0 \quad 1000]$

$= [1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1 \, 30]$

New Z row = $[-16 \quad -17 \quad -10 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]$

$\qquad\qquad -(-17)[\, 1/2 \quad 1 \quad 2 \quad 1/2 \quad 0 \quad 0 \quad 0 \quad 1000]$

$= [--15/2 \quad 0 \quad 24 \quad 17/2 \quad 0 \quad 0 \quad 0 \quad 17000]$

Table 2:

Basic	x_1	x_2	x_3	S_1	S_2	S_3	S_4	Sol.
x_2	1/2	1	2	1/2	0	0	0	1000
S_2	3/2	0	-1	-1/2	1	0	0	2600
S_3	0	0	-2	-1	0	1	0	400
S_4	1	0	0	0	0	0	1	30
Max Z	-15/2	0	24	17/2	0	0	0	17000

The entering variable is x_1

1000/1/2=2000

2600/3/2=5200/2

400/0 ignore

30/1= 30

Pivot Row=[1 0 0 0 0 0 1 30]

Table 3: (optimal solution):

Basic	x_1	x_2	x_3	S_1	S_2	S_3	S_4	Sol.
x_2	0	1	2	1/2	0	0	-1/2	985
S_2	0	0	-1	-1/2	1	0	-3/2	2555
S_3	0	0	-2	-1	0	1	0	400
x_1	1	0	0	0	0	0	1	30
Max Z	0	0	24	17/2	0	0	15/2	17225

New x1 row =[1/2 1 2 ½ 0 0 0 1000]-(1/2)[1 0 0 0 0 0 1 30}= [1/2 1 2 ½ 0 0 0 1000]-[1/2 0 0 0 0 0 ½ 15]

=[0 1 2 ½ 0 0 -1/2 985]

New S2 row = [3/2 0 -1 -1/2 1 0 0 2600]-(3/2)[1 0 0 0 0 0 1 30]

= [3/2 0 -1 -1/2 1 0 0 2600]- [3/2 0 0 0 0 0 3/2 45]

=[0 0 -1 -1/2 1 0 -3/2 2555]

New S3 row = [0 0 -2 -1 0 1 0 400]-(0) [pivot row]

=[0 0 -2 -1 0 1 0 400]

New Z row = [-15/2 0 24 17/2 0 0 0 17000]-(-15/2) [1 0 0 0 0 0 1 30]

=[0 0 24 17/2 0 0 15/2 17225]

The optimal solution:

x_1=30 , $x_2 = 985$, $S_2 = 2555$, $S_3 = 400, S_4 = 30$, $x_3 , S_1, S_4 = 0$ /

Z=17225

Example 6:

Use the simplex method to solve the (LP) model:

$$max \ Z = 3x_1 + 5x_2 + 4x_3$$

Subject to

$$2x_1 + 3x_2 \leq 8$$

$$2x_1 + 5x_2 \leq 10$$

$$3x_1 + 2x_2 + 4x_3 \leq 15$$

$$x_1, x_2, \ x_3 \ \geq 0$$

Solution:

$$max \ Z - 3x_1 - 5x_2 - 4x_3 = 0$$

Subject to

$$2x_1 + 3x_2 + S_1 \qquad \leq 8$$

$$2x_1 + 5x_2 \ + S_2 \qquad \leq 10$$

$$3x_1 + 2x_2 + 4x_3 \ + S_3 \quad \leq 15$$

$$x_1, x_2, \ x_{3,} S_1, \ S_2, S_3 \geq 0$$

Table 1:

Basic	x_1	x_2	x_3	S_1	S_2	S_3	Sol.
S_1	2	3	0	1	0	0	8
S_2	2	5	0	0	1	0	10
S_3	3	2	4	0	0	1	15
Max Z	-3	-5	-4	0	0	0	0

$$\frac{8}{3} = 2.7$$

$$\frac{10}{5} = 2$$

$$\frac{15}{2} = 7.5$$

New S_2 row or x_2 row $=(\frac{1}{5})$[2 5 0 0 1 0 10]

$\qquad\qquad\qquad$ = [2/5 1 0 0 1/5 0 2]

New S_1 row = [2 3 0 1 0 0 8]

$\qquad\qquad$ -(3)[2/5 1 0 0 1/5 0 2]

$\qquad\qquad$ = [4/5 0 0 1 -3/5 0 2]

New S_3 row = [3 2 4 0 0 1 15]

$\qquad\qquad$ -(2)[2/5 1 0 0 1/5 0 2]

$\qquad\qquad$ = [11/5 0 4 0 -2/5 1 11]

New Z row = [-3 -5 -4 0 0 0 0]

$$-(-5)[\ 2/5\ \ 1\ \ 0\ \ 0\ \ 1/5\ \ 0\ \ 2]$$
$$= [-1\ \ 0\ \ -4\ \ 0\ \ 10\ \ 10]$$

Table 2:

Basic	x_1	x_2	x_3	S_1	S_2	S_3	Sol.
S_1	4/5	0	0	1	-3/5	0	2
x_2	2/5	1	0	0	1/5	0	2
S_3	11/5	1	4	0	-2/5	1	11
Max Z	-1	0	-4	0	1	0	10

New S_3 row or x_3 row $= (\frac{1}{4})[11/5\ \ 0\ \ 4\ \ 0\ \ -2/5\ \ 1\ \ 11]$

$\quad\quad\quad\quad\quad = [11/20\ \ 0\ 1\ \ 0\ \ -1/10\ \ 1/4\ \ 11/4]$

New S_1 row $= [4/5\ \ 0\ \ 0\ \ 1\ \ -3/5\ \ 0\ \ 2\ ]$

$\quad\quad\quad -(0)[\ 11/20\ \ 0\ \ 1\ \ 0\ \ -1/10\ \ 1/4\ \ 11/4]$

$\quad\quad\quad = [4/5\ \ 0\ \ 0\ \ 1\ \ -3/5\ 0\ \ 2]$

New x_2 row $= [2/5\ \ 1\ \ 0\ \ 0\ \ 1/5\ \ 0\ \ 2\ ]$

New Z row $= [-1\ \ 0\ \ -4\ \ 0\ \ 1\ \ 0\ \ 10\ ]$

$\quad\quad\quad -(-4)[\ 11/20\ \ 0\ \ 1\ \ 0\ \ -1/10\ \ 1/4\ \ 11/4]$

$\quad\quad\quad = [6/5\ \ 0\ \ 0\ \ 0\ \ 3/5\ 1\ \ 21]$

Table 3: (optimal solution):

Basic	x_1	x_2	x_3	S_1	S_2	S_3	Sol.
S_1	4/5	0	0	1	-3/5	0	2
x_2	2/5	1	0	0	1/5	0	2
x_3	11/20	0	1	0	-1/10	1/4	11/4
Max Z	6/5	0	0	0	3/5	1	21

The optimal solution:

$x_2 = 2,$

$x_3 = 11/4$,

$S_1 = 2,$

Z=21

$x_1 = 0 ,\ \ S_2 = 0,\ \ S_3 = 0,$

Problems

Use the simplex method to solve the (LP) model:

1-

$$max \ Z = 3x_1 + 5x_2 + 4x_3$$

Subject to

$$2x_1 + 3x_2 \leq 8$$

$$2x_2 + 5x_3 \leq 10$$

$$3x_1 + 2x_2 + 4x_3 \leq 15$$

$$x_1, x_2, x_3 \geq 0$$

2-

$$max \ Z = 4x_1 + 3x_2$$

Subject to

$$2x_1 + x_2 \leq 1000$$

$$x_1 \leq 400$$

$$x_2 \leq 700$$

$$x_1, x_2, \geq 0$$

3-

$$max \ Z = 270x_1 + 144x_2 + 225x_3$$

Subject to

$$8x_1 + 4x_2 + 5x_3 \leq 1000$$

$$5x_1 + 3x_2 + 3x_3 \leq 650$$

$$9x_1 + 6x_2 + 9x_3 \leq 1260$$

$$x_1, x_2, x_3 \geq 0$$

4-

$$max\ Z = 10x_1 + 15x_2 + 20x_3$$

Subject to

$$10.7x_1 + 5x_2 + 2x_3 \leq 2705$$

$$5.4x_1 + 10x_2 + 4x_3 \leq 2210$$

$$0.7\ x_1 + x_2 + x_3 \leq 445$$

$$x_1, x_2, x_3 \geq 0$$

5-

$$max\ Z = 90x_1 + 60x_2$$

Subject to

$$5x_1 + 8\ x_2 \leq 2000$$

$$x_1 \leq 175$$

$$x_2 \leq 2255$$

$$7x_1 + 4\ x_2 \leq 1400$$

$$x_1, x_2, \geq 0$$

6-

$$min\ Z = x_1 - 3x_2 + 3x_3$$

Subject to

$$3x_1 - x_2 + 2x_3 \leq 7$$

$$2x_1 + 4x_2 \geq -12$$

$$-4-4x_1 + 3\ x_2 + 8x_3 \leq 10$$

$$x_1, x_2, x_3 \geq 0$$