Strength of Materials

4 Trusses

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MSc in Structural Engineering

Trusses

Truss

Truss is structure composed of a number of members joined together at their end points in such a manner as to form a rigid body; each member only takes axial forces. Different types of trusses are used in bridges, roofs and covering different structures.

Trusses

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Trusses

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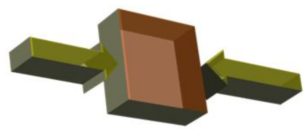
Trusses

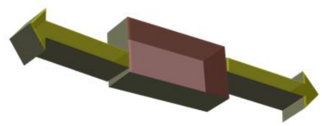
Forces

Compression



A body being squeezed

Tension



A body being stretched

Trusses

Simple Trusses

- A truss composed of slender members joined together at their end points

Planar Trusses

- Planar trusses used to support roofs and bridges
- Roof load is transmitted to the truss at joints by means of a series of purlins

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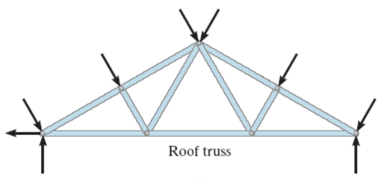
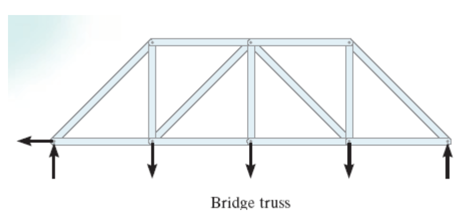
Trusses

Simple Trusses

Planar Trusses

- The analysis of the forces developed in the truss members is 2D
- Similar to roof truss, the bridge truss loading is also coplanar





Trusses

Simple Trusses

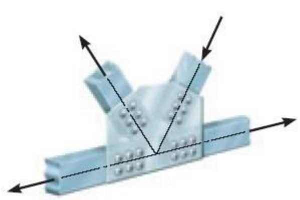
Assumptions for Design

1. "All loadings are applied at the joint"

   - Weight of the members neglected

2. "The members are joined together by smooth pins"

   - Assume connections provided the center lines of the joining members are *concurrent*

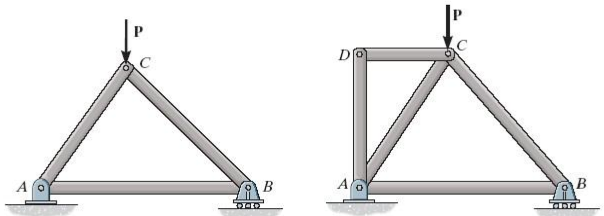


Trusses

Simple Trusses

Simple Truss

- Form of a truss must be rigid to prevent collapse
- The simplest form that is rigid or stable is a triangle



Trusses

Solving Truss Forces

Assumptions:

All members are perfectly straight.

All loads are applied at the joints.

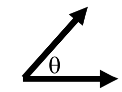
All joints are pinned and frictionless.

Each member has no weight.

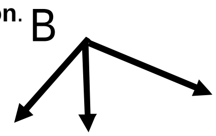
Members can only experience tension or compression forces.

Method of Joints

Use cosine and sine to determine x and y vector components.



Assume all members to be in tension. A **positive** answer will mean the member is in **tension**, and a **negative** number will mean the member is in **compression**.

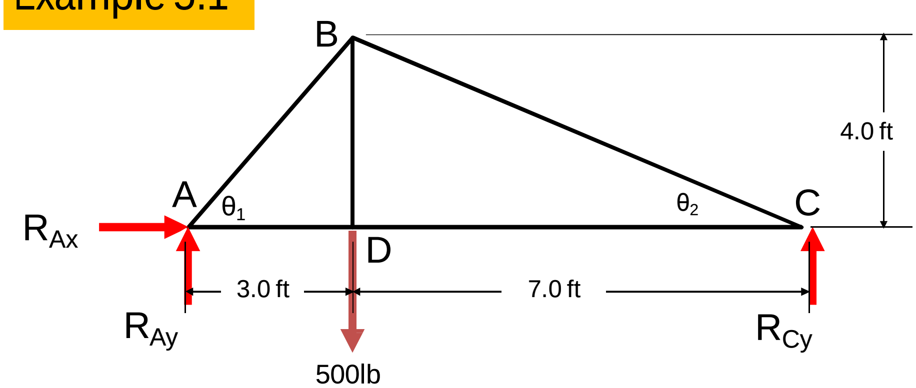


As forces are solved, update free body diagrams. Use correct magnitude and sense for subsequent joint free body diagrams.

Method of Joints

Truss Dimensions

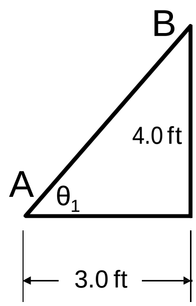
Example 5.1



Trusses

Method of Joints

Using Truss Dimensions to Find Angles



$$\tan\theta_1 = \frac{opp}{adj}$$

$$\tan\theta_1 = \frac{4.0\ \text{ft}}{3.0\ \text{ft}}$$

$$\theta_1 = \tan^{-1}\frac{4.0}{3.0}$$

$$\theta_1 = 53.130°$$

Trusses

Method of Joints

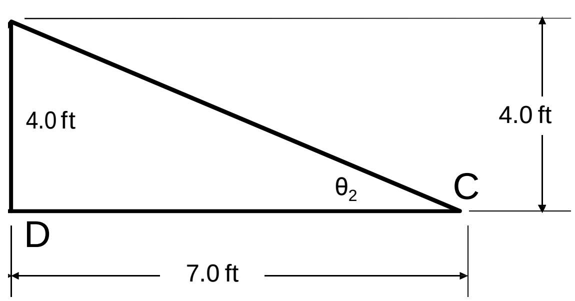
Using Truss Dimensions to Find Angles

$$\tan\theta_1 = \frac{opp}{adj}$$

$$\tan\theta_1 = \frac{4.0\ \text{ft}}{7.0\ \text{ft}}$$

$$\theta_1 = \tan^{-1}\frac{4.0}{7.0}$$

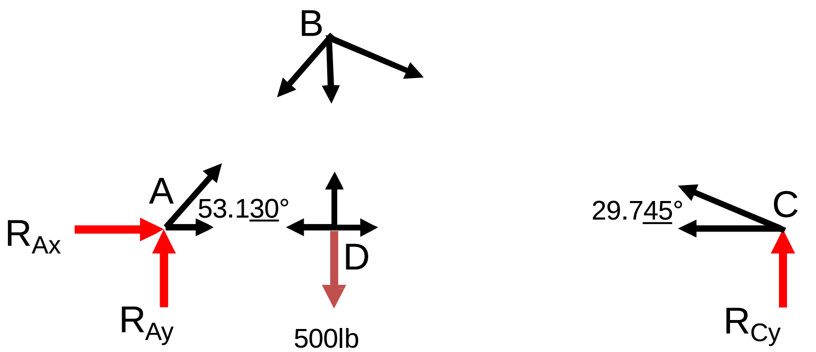
$$\theta_1 = 29.745°$$



Trusses

Method of Joints

Draw a free body diagram of each pin.



Every member is assumed to be in tension. A positive answer indicates the member is in tension, and a negative answer indicates the member is in compression.

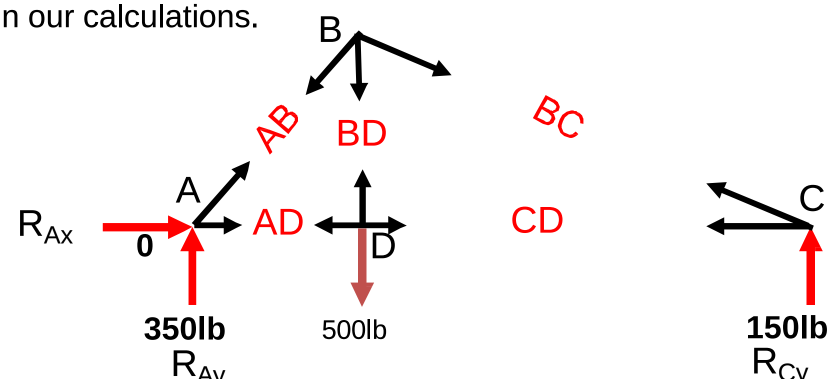
Trusses

Method of Joints

Where to Begin

Choose the joint that has the least number of unknowns.

Reaction forces at joints **A** and **C** are both good choices to begin our calculations.

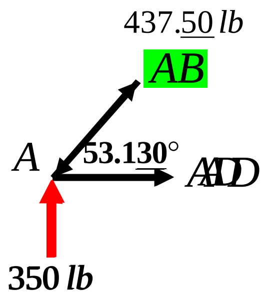


Trusses

Method of Joints

$$\Sigma F_Y = 0$$

$$R_{Ay} + AB_y = 0$$



$$350lb + AB\sin 53.130° = 0$$

$$AB\sin 53.130° = -350lb$$

$$AB = \frac{-350\ lb}{\sin 53.13}$$

$$AB = -438\ lb$$ **COMPRESSION**

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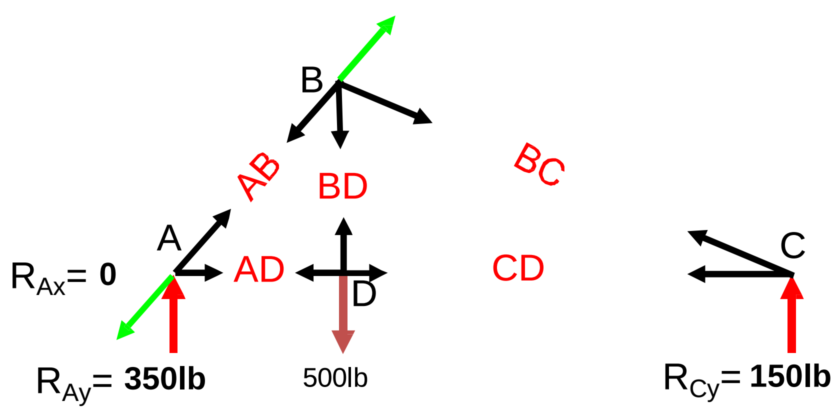
Trusses

Method of Joints

Update the all force diagrams based on AB being under compression.



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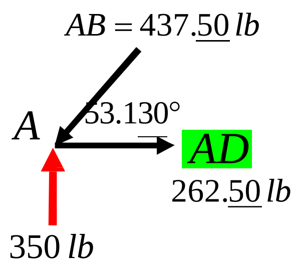
Trusses

Method of Joints

$$\Sigma F_X = 0$$

$$-AB_x + AD = 0$$

$$-437.\underline{50}\ lb \cos 53.1\underline{30}° + AD = 0$$



$$AD = 437.\underline{50}\ lb \cos 53.1\underline{30}°$$

$$AD = 262.50\ lb$$ **TENSION**

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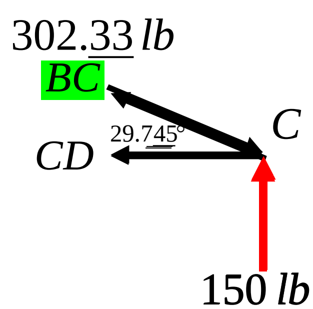
Trusses

Method of Joints

$$\Sigma F_Y = 0$$

$$R_{Cy} + BC_y = 0$$



$$150\ lb + BC \sin 29.7\underline{45}° = 0$$

$$BC \sin 29.7\underline{45}° = -150\ lb$$

$$BC = \frac{-150 lb}{\sin 29.7\underline{45}°}$$

$$BC = -302\ lb$$ **COMPRESSION**

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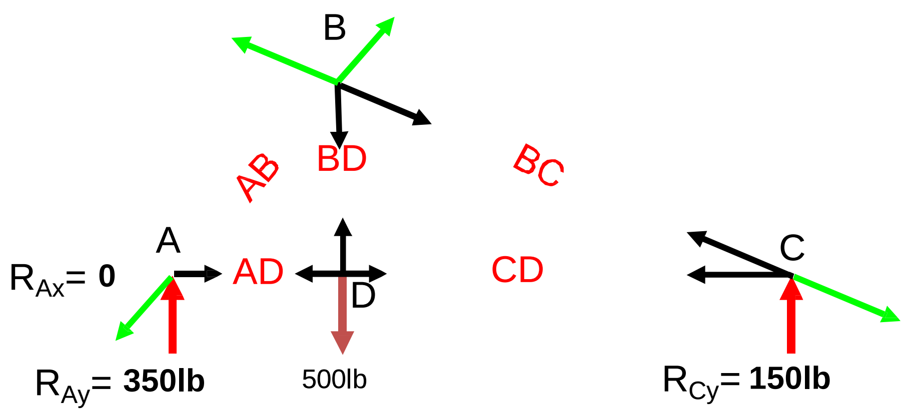
Trusses

Method of Joints

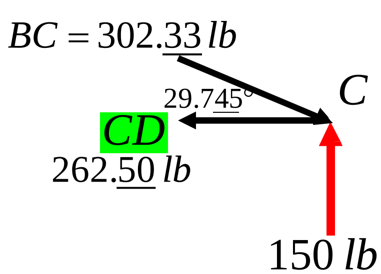
Update the all force diagrams based on BC being under compression.



Mr. Diyari Burhan | 21

Trusses

Method of Joints



$$\Sigma F_X = 0$$

$$BC_x - CD = 0$$

$$302.33\ lb \cos 29.745° - CD = 0$$

$$CD = 302.33\ lb \cos 29.745°$$

$$CD = 262.50\ lb$$ **TENSION**

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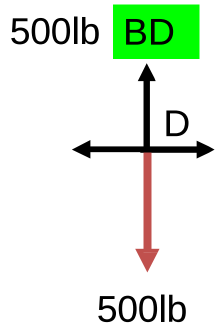
Trusses

Method of Joints



$$\Sigma F_Y = 0$$

$$BD - F_D = 0$$

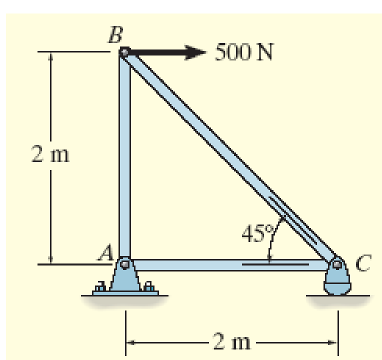
$$BD - 500lb = 0$$

$$BD = 500lb$$ **TENSION**

Trusses

Example 5.2

Determine the force in each member of the truss and indicate whether the members are in tension or compression.



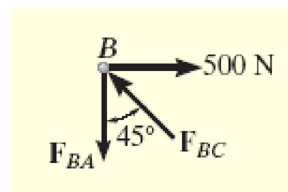
Trusses

Solution

- 2 unknown member forces at joint B
- 1 unknown reaction force at joint C
- 2 unknown member forces and 2 unknown reaction forces at point A

For Joint B,

$$+\rightarrow \sum F_x = 0;$$
$$500N - F_{BC} \sin 45^\circ N = 0 \Rightarrow F_{BC} = 707.1N(C)$$
$$+\uparrow \sum F_y = 0;$$
$$F_{BC} \cos 45^\circ N - F_{BA} = 0 \Rightarrow F_{BA} = 500N(T)$$



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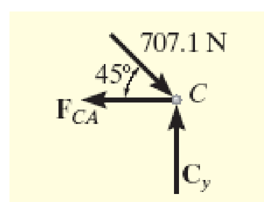
25

Trusses

Solution

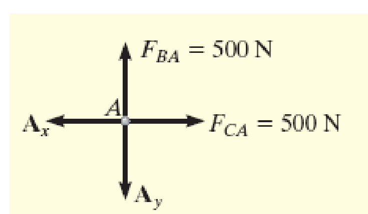
For Joint C,

$$+\rightarrow \sum F_x = 0;$$
$$-F_{CA} + 707.1\cos 45^\circ N = 0 \Rightarrow F_{CA} = 500N(T)$$
$$+\uparrow \sum F_y = 0;$$
$$C_y - 707.1\sin 45^\circ N = 0 \Rightarrow C_y = 500N$$



For Joint A,

$$+\rightarrow \sum F_x = 0;$$
$$500N - A_x = 0 \Rightarrow A_x = 500N$$
$$+\uparrow \sum F_y = 0;$$
$$500N - A_y = 0 \Rightarrow A_y = 500N$$



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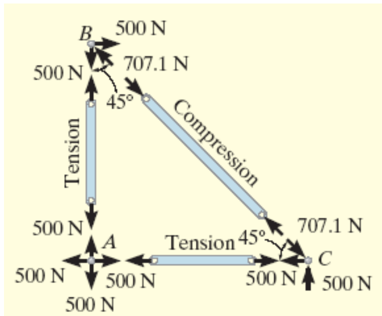
26

Trusses

Solution

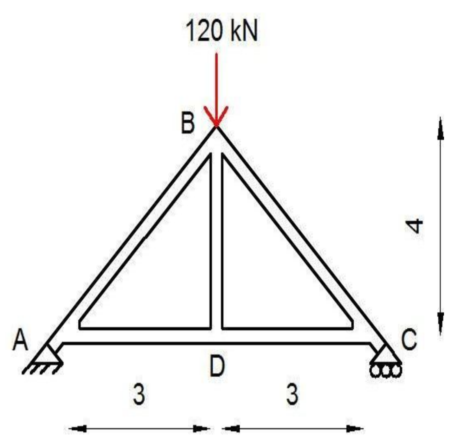
- FBD of each pin shows the effect of all the connected members and external forces applied to the pin
- FBD of each member shows only the effect of the end pins on the member



Trusses

Example 5.3

For the truss shown in the figure below, determine the axial forces in each member.



Trusses

Solution:

Firstly: find reactions:

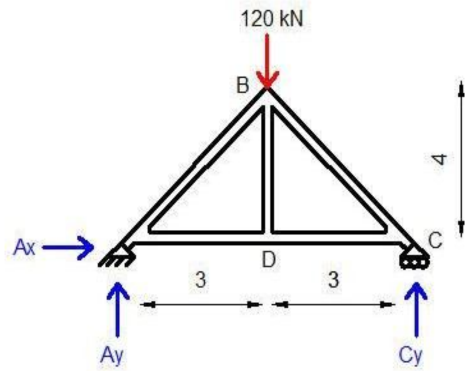
$\sum fx = 0 \Rightarrow Ax = 0$

$\sum M_c = 0 \Rightarrow +(Ay * 6) - (120 * 3) = 0$

$\Rightarrow Ay = 60\ kN$ t

$\sum M_A = 0 \Rightarrow -(Cy * 6) + (120 * 3) = 0$

$\Rightarrow Cy = 60\ kN$ t



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Trusses

Joint A:

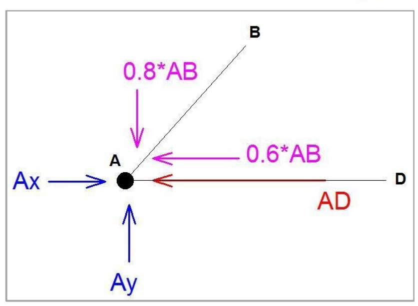
$\sum fy = 0 \Rightarrow Ay - 0.8\,AB = 0$

$AB = \frac{Ay}{0.8} = \frac{60}{0.8} = 75\ kN$ (compression)

$\sum fx = 0 \Rightarrow Ax - 0.6\,AB - AD = 0$

$AD = 0 - (0.6 * 75) = 0$

$AD = -45\ kN = 45\ kN \rightarrow$ (Tension)



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Trusses

Joint B:

compression

$\sum fx = 0 \Rightarrow 0.6AB - 0.6BC = 0$

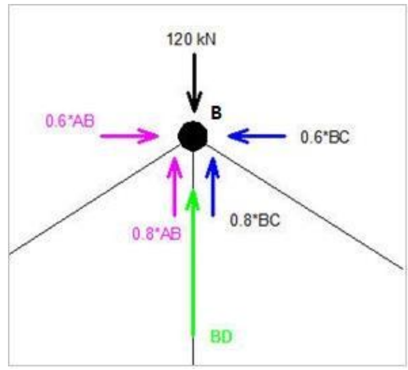
$\therefore BC = AB = 75\, kN\ (tension)$

$\sum fy = 0 \Rightarrow BD + 0.8BC + 0.8AB - 120 = 0$

$BD = 120 - (0.8 * 75) - (0.8 * 75) = 0$

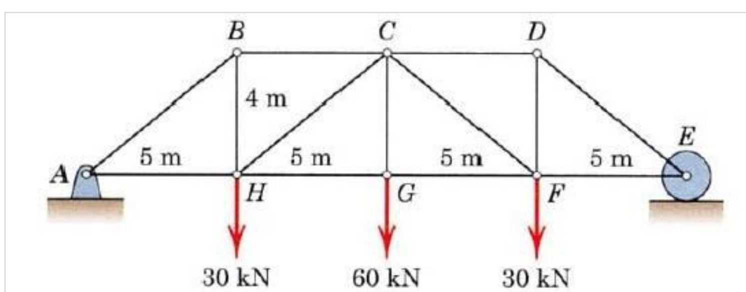
$\therefore BD = 0$ Tension

$\therefore DC = AD = 45\, kN\ (compresion)$



Trusses

Example 5.4



Trusses

Solution:

Step 1: Find reactions:

$\sum M_E = 0 \Rightarrow Ay * 20 - (30 * 15) - (60 * 10) - (30 * 5) = 0$

$\boldsymbol{Ay = 60\ kN}$ ⇧

$\sum M_A = 0 \Rightarrow (30 * 15) + (60 * 10) + (30 * 5) - 20Ey = 0$

$\boldsymbol{Ay = 60\ kN}$ ⇧

Check:

$\sum fy = 0 \Rightarrow 60 + 60 - 30 - 60 - 30 = 0 \qquad \therefore OK$

Trusses

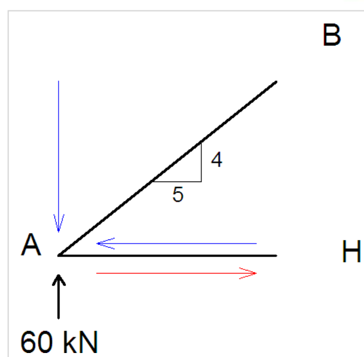
Joint A:

$\boldsymbol{\sum fy = 0 \Rightarrow \quad 60 - (\frac{4}{6.4} * AB) = 0}$

$\boldsymbol{\therefore AB = 96\ kN\ Comp.}$

$\boldsymbol{\sum fx = 0 \Rightarrow \quad AH - (\frac{5}{6.4} * AB) = 0}$

$\boldsymbol{\therefore AH = 75\ kN\ Ten.}$



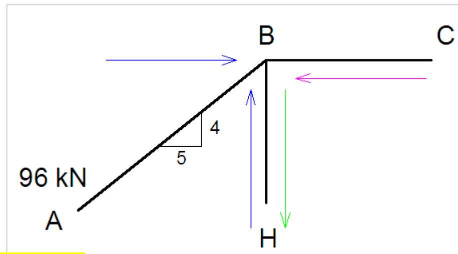
Trusses

Joint B:

$\sum fy = 0 \Rightarrow \quad (\frac{4}{6.4} * AB) - BH = 0$

$\therefore BH = 60\ kN\ Ten.$

$\sum fx = 0 \Rightarrow \quad (\frac{5}{6.4} * AB) - BC = 0$



$\therefore BC = 75\ kN\ Comp.$

Trusses

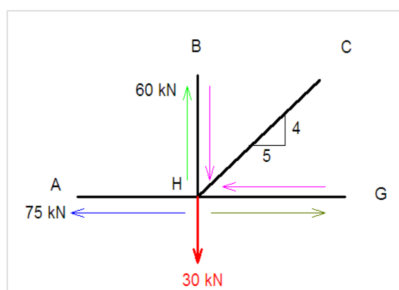
Joint H:

$\sum fy = 0 \Rightarrow 60 - 30 - (HC * \frac{4}{6.4}) = 0$

$HC = 48\ kN\ Comp.$

$\sum fx = 0 \Rightarrow HG - 75 - (HC * \frac{5}{6.4}) = 0$

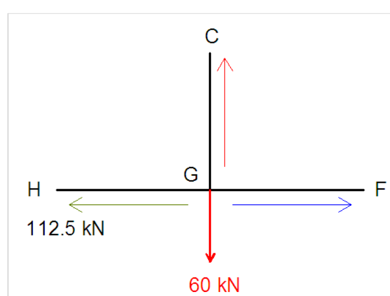
$HG = 112.5\ kN\ Ten.$



Joint G:

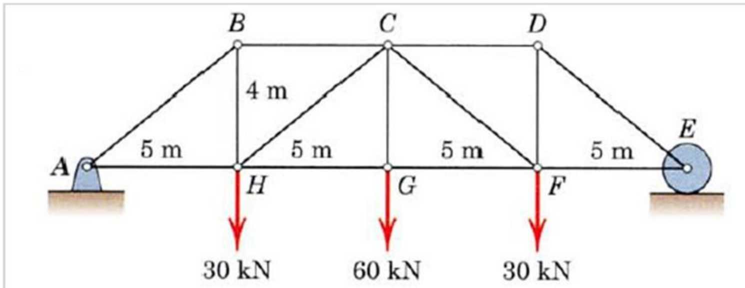
$\sum fx = 0 \quad \Rightarrow GH = GF = 112.5\ kN\ Ten.$

$\sum fy = 0 \quad \Rightarrow GC = 60\ kN\ Ten.$



Trusses

Example 5.4



Final answers are as the following:

$$\text{Ans. } AB = DE = 96.0 \text{ kN } C$$
$$AH = EF = 75 \text{ kN } T,\ BC = CD = 75 \text{ kN } C$$
$$BH = CG = DF = 60 \text{ kN } T$$
$$CH = CF = 48.0 \text{ kN } C,\ GH = FG = 112.5 \text{ kN } T$$

Trusses

Method of Sections

- If the forces in only a few members of a truss are to be determined, the ***method of sections*** is generally the most appropriate analysis procedure.
- The method of sections consists of passing an *imaginary line* through the truss, cutting it into sections.
- Each imaginary section must be in equilibrium if the entire truss is in equilibrium.

$$\sum F_x = 0 \quad \sum F_y = 0 \quad \sum M_z = 0$$

Trusses

Method of Sections

Procedure for analysis - the following is a procedure for analyzing a truss using the method of sections:

1. First, if necessary, determine the support reactions for the entire truss.
2. Next, make a decision on how the truss should be "cut" into sections and draw the corresponding free-body diagrams.
3. Try to apply the three equations of equilibrium such that simultaneous solution **is not** required.

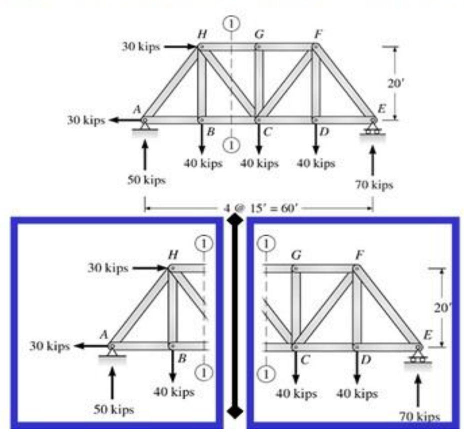
   Moments should be summed about points that lie at the intersection of the lines of action of two unknown forces, so that the remaining force may be determined.

Trusses

Method of Sections

Imagine cutting a structure into two sections about line 11

Left side

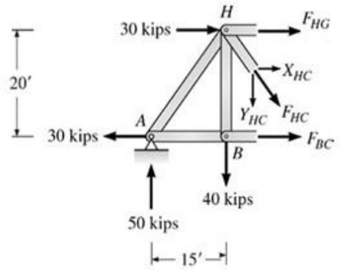


Right side

Trusses

Method of Sections

- Typically the section with the fewest forces or with section with the most convenient geometry is selected.
- In this example the left-hand side.



- Apply the three equations of equilibrium to the section.
- If possible, attempt to develop an equation in just one unknown.
- Look for points where the lines of action of several forces are concurrent.

Trusses

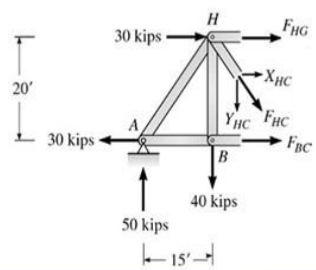
Method of Sections

$$\sum M_H = 0 \quad = F_{BC}(20\,\text{ft.}) - 30k(20\,\text{ft.}) - 50k(15\,\text{ft.})$$

$F_{BC} = 67.5\ k$

$$\sum M_C = 0 \quad = -F_{HG}(20\,\text{ft.}) - 30k(20\,\text{ft.}) - 50k(30\,\text{ft.}) + 40k(15\,\text{ft.})$$

$F_{HG} = -75\ k$



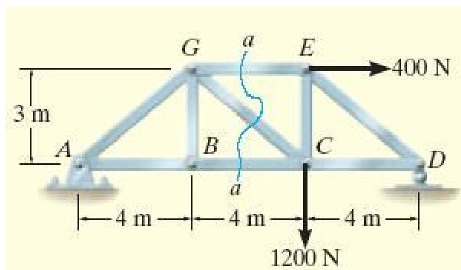
$$\sum F_y = 0 \quad = -\frac{4}{5}F_{HC} - 40k + 50k$$

$F_{HC} = 12.5\ k$

$F_{BC} = 67.5\ k$ (T) $F_{HG} = 75\ k$ (C) $F_{HC} = 12.5\ k$ (T)

Example 5.5

Determine the force in members GE, GC, and BC of the truss. Indicate whether the members are in tension or compression.



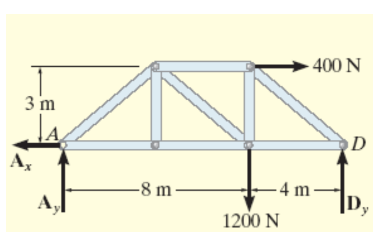
Solution

- Choose section a-a since it cuts through the three members
- Draw FBD of the entire truss

$$+\rightarrow \sum F_x = 0; \quad 400N - A_x = 0 \Rightarrow A_x = 400N$$

$$\sum M_A = 0; \quad -1200N(8m) - 400N(3m) + D_y(12m) = 0 \Rightarrow D_y = 900N$$

$$+\uparrow \sum F_y = 0; \quad A_y - 1200N + 900N = 0 \Rightarrow A_y = 300N$$



Trusses

Solution

- Draw FBD for the section portion

$$\sum M_G = 0; \quad -300N(4m) - 400N(3m) + F_{BC}(3m) = 0 \Rightarrow F_{BC} = 800N(T)$$

$$\sum M_C = 0; \quad -300N(8m) + F_{GE}(3m) = 0 \Rightarrow F_{GE} = 800N(C)$$

$$+\uparrow \sum F_y = 0; \quad 300N - \frac{3}{5}F_{GC} = 0 \Rightarrow F_{GC} = 500N(T)$$