Cihan University Sulaimani
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CIHAN UNIVERSITY SULAIMANI

Chapter Three:
Simple Strain

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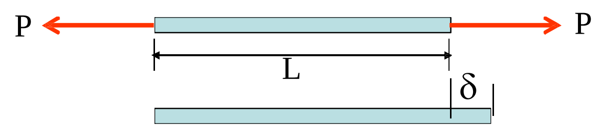
Normal (or Axial or Direct) Strain:

- When loads are applied to a body, some deformation will occur resulting to a change in dimension.
- Consider a bar, subjected to axial tensile loading force, P.
- If the bar extension is (δ) and its original length (before loading) is (L), then tensile strain is:



Strain (ϵ) = Change in Length / Original Length

i.e. $\epsilon = \delta / L$ Where:

ϵ : Axial or Direct Strain

δ : Change in length

L : Original length

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- As strain is a ratio of lengths, so it is **dimensionless**.
- Similarly, for compression by amount, δ : Compressive strain = $-\delta / L$
- **Note:** Strain is positive for an increase in dimension and negative for a reduction in dimension.

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Stress-Strain Diagram

-Suppose that a metal specimen be placed in tension – compression testing machine.

-As the axial load is gradually increased in increments, the total elongation over the gage length is measured at each load increment and this is continued until failure of the specimen takes place .

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* Knowing the original cross-sectional area and length of the specimen, the normal stress (σ) and the strain (ϵ) can be obtained.

* The graph of these quantities with the stress (σ) along the y-axis and the strain (ϵ) along the x-axis is called the stress-strain diagram.

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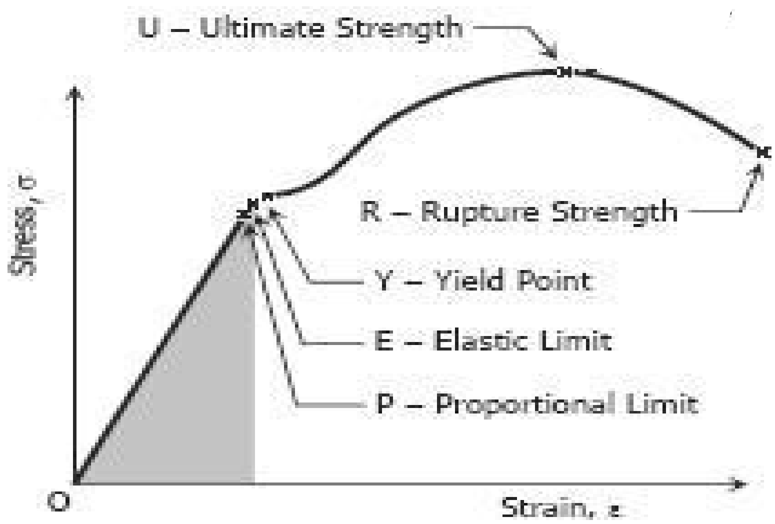

The stress-strain diagram for a medium carbon structural steel

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- From the origin O to the point called proportional limit, the stress – strain curve is a straight line.
- This linear relation between elongation and the axial force was first noticed by Sir Robert Hooke in 1678 and is called Hooke's Law that within the proportional limit, the stress is directly proportional to strain.
- $\sigma \propto \epsilon$ or $\sigma = \text{constant} \cdot \epsilon$

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- the constant = σ / ϵ
- $E = \sigma / \epsilon$ ⟶ Modulus of Elasticity

- **ELASTIC LIMIT:** The elastic limit is the limit beyond which the material will no longer go back to its original shape when the load is removed.

NOTE:

The region in stress-strain diagram from O to P is called the elastic range.

The region from P to R is called the plastic range.

- **YIELD POINT:** Yield point is the point at which the material will have an appreciable elongation or yielding without any increase in load.
- **ULTIMATE STRENGTH:** The maximum ordinate in the stress-strain diagram is the ultimate strength or tensile strength.
- **RAPTURE STRENGTH:** Is the strength of the material at rupture. This is also known as the **breaking strength.**

- Metallic engineering materials are classified as either **ductile** or **brittle** materials.
- A ductile material is one having relatively large tensile strains up to the point of rupture like structural steel and aluminum.
- A brittle material has a relatively small strain up to the point of rupture like cast iron and concrete.
- An arbitrary strain of 0.05 mm/mm is frequently taken as the dividing line between these two classes

AXIAL DEFORMATION:

In the linear portion of the stress – strain diagram, the stress is proportional to strain and is given by

$\sigma = E \epsilon$

$P / A = E (\delta / L)$

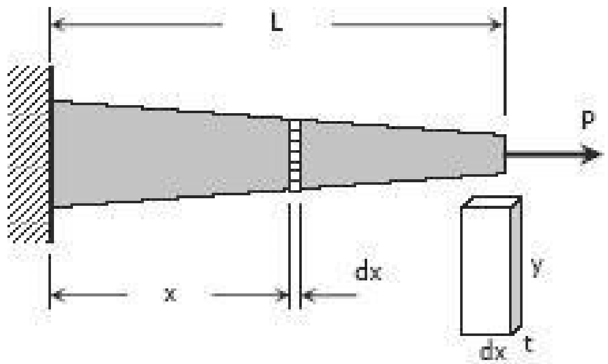
$$\delta = \frac{P L}{A E}$$

To use this formula, the load must be axial,

the bar must have a uniform cross - sectional area, and

the stress must not exceed the proportional limit.

If however, the cross-sectional area is not uniform, the axial deformation can be determined by considering a **differential length and applying integration.**



$$\delta = \frac{P}{E}\int_0^L \frac{dx}{A}$$

Example (1):

A steel rod having a cross-sectional area of 300 mm^2 and a length of 150 m is suspended vertically from one end.

It supports a tensile load of 20 kN at the lower end. Neglecting its own weight and taking E = 200 GPa, find the elongation of the rod.

Solution:

$$\delta = \frac{P\ L}{A\ E}$$

$$\delta = \frac{(20 \times 10^3) N . (150) m}{(300 \times 10^{-6})\, m^2 . (200 \times 10^9) N/m^2}$$

$$= 0.05\ m$$

$$= \mathbf{50\ mm}$$

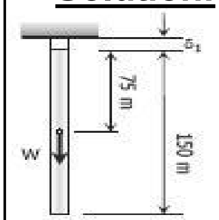
Example (2):

A steel rod having a cross-sectional area of 300 mm^2 and a length of 150 m is suspended vertically from one end.

It supports a tensile load of 20 kN at the lower end.

If the unit mass of steel is 7850 kg/m^3 and E = 200 GPa, find the total elongation of the rod.

Solution:



Let δ = total elongation

δ_1 = elongation due to its own weight

δ_2 = elongation due to applied load

$\delta = \delta_1 + \delta_2$

$\delta_1 = \frac{PL}{AE}$ P=W=(Volume)(Unit mass)(g) = (A x L)(Unit mass)(g) =[(300x10 -6)(150)](7850)(9.81) =3465.3825 N

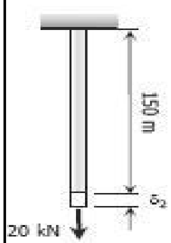
Where: ; Mid height = 150/2 = 75 m 75 000 mm

L = 75(1000) = 75 000 mm

A = 300 mm^2

E = 200 000 MPa

$$\delta_1 = \frac{3465.3825\,(75000)}{300\,(200\,000)} = 4.33 \text{ mm}$$



$$\delta_2 = \frac{PL}{AE}$$

Where: P = 20 kN = 20 000 N

L = 150 m = 150 000 mm

A = 300 mm^2

E = 200 000 MPa

$$\delta_2 = \frac{20000(150000)}{300(200000)} = 50 \text{ mm}$$

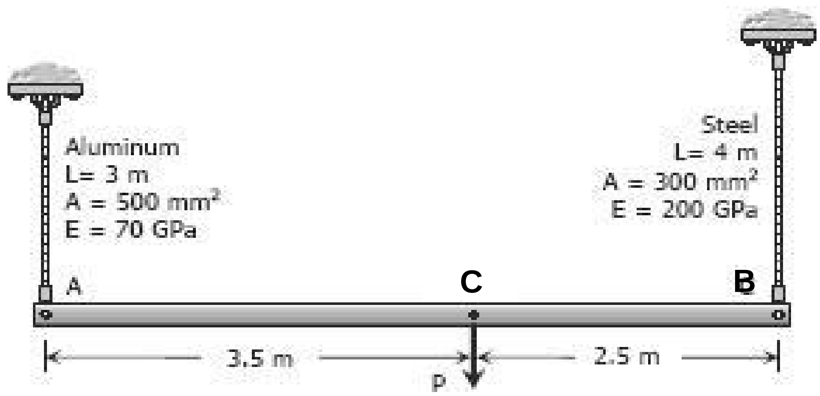
Total elongation:

$\delta = 4.33 + 50 = 54.33$ mm

Example (3):

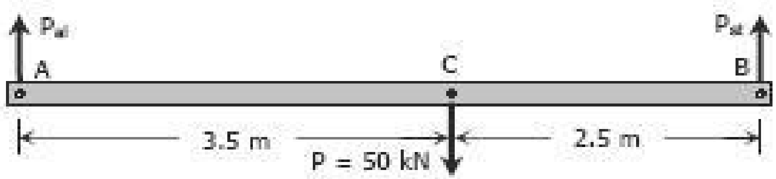
The rigid bar AB, attached to two vertical rods as shown in the Fig, is horizontal before the load P is applied.

Determine the deflection of each rod if P = 50 kN



Solution:

Free body diagram:



For aluminum:

$[\Sigma M_B = 0]$ $\quad 6P_{al} = 2.5(50)$

$P_{al} = 20.83$ kN

$\left[\delta = \frac{PL}{AE}\right]_{al}$ $\quad \delta_{al} = \frac{20.83(3)1000^2}{500(70000)}$

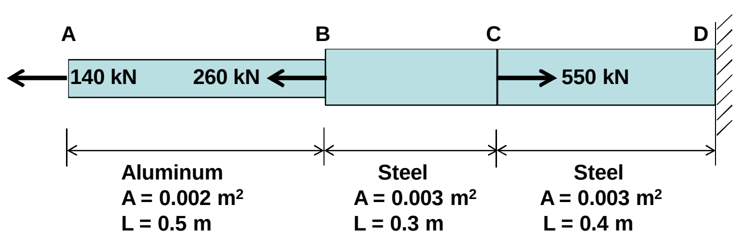
$\delta_{al} = 1.78$ mm

Example (4):

Steel and aluminum rods are rigidly attached at point B of a bar, as shown.

Determine the deformation of the composite bar due to the axial loads applied.

Use E_{st} = **200 GPa**, E_{al} = **70 GPa**



Solution:

From FBDs

P_{AB}= 140 kN (T)

P_{BC}= 400 kN (T)

P_{CD}= -150 kN (C)

$$\delta = \sum \frac{PL}{AE} = \frac{140x10^{3} x0.5}{0.002x70x10^{9}} + \frac{400x10^{3} x0.3}{0.003x200x10^{9}} - \frac{150x10^{3} x0.4}{0.003x200x10^{9}} =$$

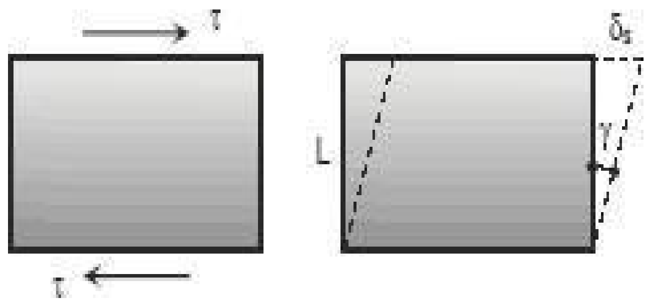
$$0.6x10^{-3}\, m = 0.6\, mm$$

WORKING STRESS, ALLOWABLE STRESS, AND FACTOR OF SAFETY:

- Working stress is defined as the actual stress of a material under a given loading.
- The maximum safe stress that a material can carry is termed as the allowable stress.
- The allowable stress should be limited to values not exceeding the proportional limit.
- The allowable stress is mostly taken as the ultimate strength divided by a factor of safety.
- The ratio of this ultimate strength to allowable strength is called the **factor of safety**

Shearing Deformation:

- Shearing forces cause shearing deformation. An element subject to shear does not change in length but undergoes a change in shape.



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- The change in angle at the corner of an original rectangular element is called the **shear strain** and is expressed as:

$$\gamma = \frac{\delta_s}{L}$$

The ratio of the shear stress (τ) and the shear strain (γ) is called the **modulus of elasticity in shear** or **modulus of rigidity** and is denoted as **G**, in GPa.

$$G = \frac{\tau}{\gamma}$$

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The relationship between the shearing deformation and the applied shearing force is:

$$\delta_s = \frac{VL}{A_s G} = \frac{\tau L}{G}$$

Shear strain is the change in the right angle. It is dimensionless and is measured in **radians.**

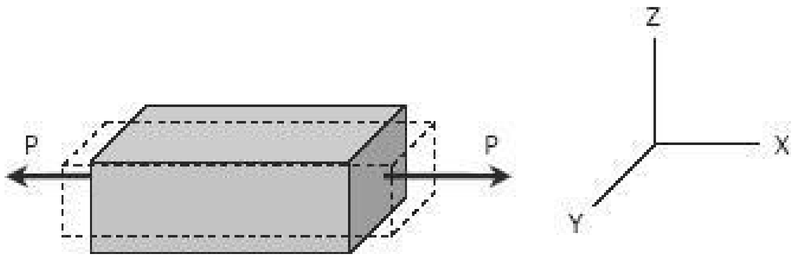
Poisson's Ratio:

- When a bar is subjected to a tensile loading there is an increase in length of the bar in the direction of the applied load

-but there is also a decrease in a lateral dimension perpendicular to the load.

-The ratio of the lateral deformation to the longitudinal deformation is called the **Poisson's ratio** and is denoted by [ν].

-For most steel, it lies in the range of 0.25 to 0.3, and 0.20 for concrete.



$$\nu = -\frac{\varepsilon_y}{\varepsilon_x} = -\frac{\varepsilon_z}{\varepsilon_x}$$

The negative sign indicates a decrease in the transverse dimension with respect to the longitudinal dimension

Thermal Stress:

Temperature changes cause the body to expand or contract. The amount (δ_T), is given by:

$$\delta_T = \alpha L(T_f - T_i) = \alpha L \, \Delta T$$

where:

$\propto$: is the coefficient of thermal expansion in m/m°C,

L : is the length in meter, and

T_i and T_f are the initial and final temperatures, in °C.

For steel, $\propto = 11.25 \times 10^{-6}$ / °C.

- If temperature deformation is permitted to occur freely, no load or stress will be induced in the structure.
- In some cases where temperature deformation is not permitted, an internal stress is created.
- This internal stress is termed as **Thermal stress.**

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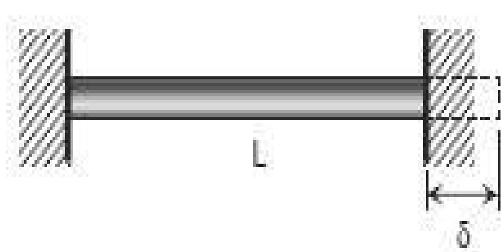
For a homogeneous rod placed between unyielding [Fixed] supports as shown, the thermal stress is computed as:



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- deformation due to temperature changes:

$$\delta_T = \alpha L \, \Delta T$$

- deformation due to equivalent axial stress:

$$\delta_P = \frac{PL}{AE} = \frac{\sigma L}{E}$$

$$\delta_T = \delta_P$$

$$\alpha L \, \Delta T = \frac{\sigma L}{E}$$

$$\boxed{\sigma = E \, \alpha \, \Delta T}$$

Note:

As the temperature rises above the normal, the rod will be in compression, and if the temperature drops below the normal, the rod is in tension.

Example (1) :

A steel rod is stretched between two rigid walls and carries a tensile load of 5000 N at 20°C.

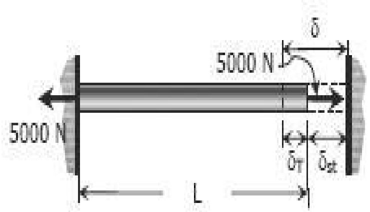
If the allowable stress is not to exceed 130 MPa at (- 20°C),

what is the minimum diameter of the rod?

Assume $\propto$ = 11.7 µm/(m·°C) and

E = 200 GPa.

Solution:



$$\delta = \delta_T + \delta_{st}$$

$$\frac{\sigma L}{E} = \alpha L(\Delta T) + \frac{PL}{AE}$$

$$\sigma = \alpha E(\Delta T) + \frac{P}{A}$$

$$130 = (11.7 \times 10^{-6})(200\,000)(40) + \frac{5000}{A}$$

$$A = \frac{5000}{36.4} = 137.36 \text{ mm}^2$$

$$\tfrac{1}{4}\pi d^2 = 137.36; \quad d = 13.22 \text{ mm}$$

Example (2) :

Steel railroad reels 10 m long are laid with a clearance of 3 mm at a temperature of 15°C.

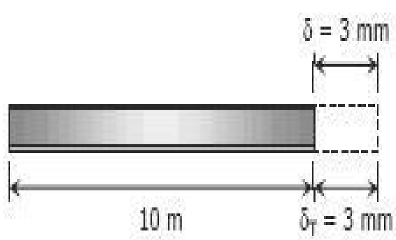
At what temperature will the rails just touch?

What stress would be induced in the rails at that temperature if there were no initial clearance?

Assume ∝ = 11.7 μm/(m·°C) and

E = 200 GPa.

Solution:



Temperature at which $\delta_T = 3$ mm:

$$\delta_T = \alpha L(\Delta T)$$

$$\delta_T = \alpha L(T_f - T_i)$$

$$3 = (11.7 \times 10^{-6})(10\,000)(T_f - 15)$$

$$T_f = 40.64°C$$

Required stress:

$$\delta = \delta_T$$

$$\frac{\sigma \cancel{L}}{E} = \alpha \cancel{L}(\Delta T)$$

$$\sigma = \alpha E(T_f - T_i)$$

$$\sigma = (11.7 \times 10^{-6})(200\ 000)(40.64 - 15)$$

$$\sigma = \mathbf{60\ MPa}$$