Cihan University Sulaimani
Faculty of Engineering
Architectural Department

CIHAN UNIVERSITY SULAIMANI

Chapter Two:
Simple Stress

Prepared by: Diyari B. Hussein
M.Sc. In Structural Engineering

2022 - 2023

1

SI Units
(International system of units):

Force:(N), Moment:(N.m), Stress:(N/m^2)

1 N/m^2 = 1 Pascal(Pa)

PREFIXES:

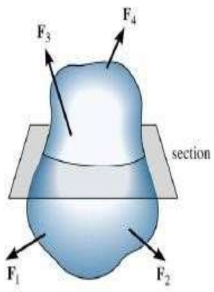
10^9 :Giga (G)	**10^{-9} : nano (η)**
10^6 :Mega (M)	**10^{-6}: micro (μ)**
10^3 :Kilo (K)	**10^{-3}: milli (m)**

INTRODUCTION:

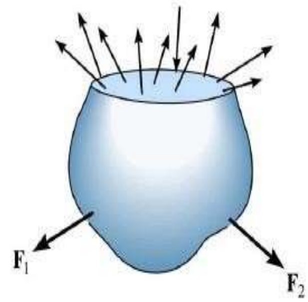
- *When an external force acts on a body, the body tends to undergo some deformation.*
- *Due to cohesion between the molecules , the body resists deformation.*
- *This resistance by which material of the body oppose the deformation is known as “Strength of Materials “*

- “ *Strength of Materials* (SOM) “ is an extension of “ *Engineering Mechanics* “; *i.e* “*Statics* “ *which you already studied*.
- In Engineering Mechanics bodies are assumed to be rigid and in equilibrium.
- In SOM bodies are in equilibrium but no longer rigid ……… Deformations are of great interest.

Analysis of Internal Forces



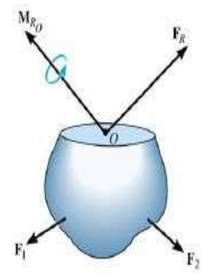
(a)



(b)

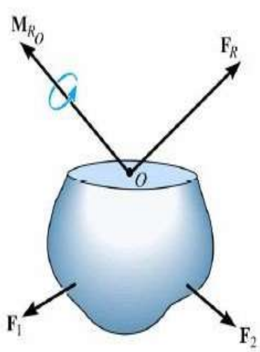
Apply Equilibrium
at this stage

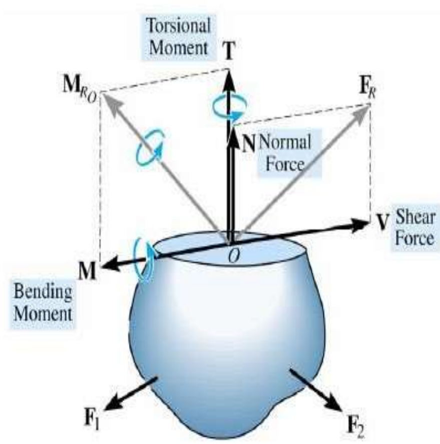
Point O is often chosen as
the centroid of the
sectioned area



(c)

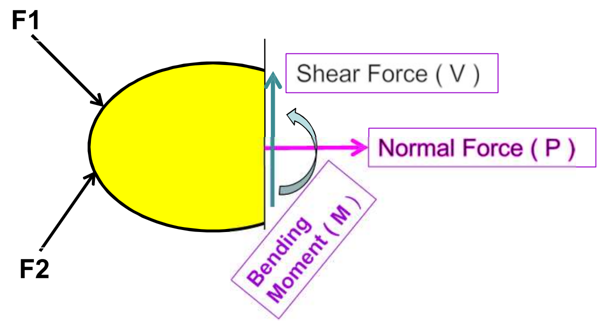
•Three Dimensional Loading:





Three Dimensional Loading

6



Two Dimensional Loading

7

Internal Forces:

For **three** dimensional loading ***four*** types of internal loadings can be defined:

1- Normal force, P : This force acts perpendicular to the cross-sectional area.

2. Shear force, V : This force lies in the plane of the area (parallel or tangent).

3. **Torsional moment, T :** This torque is developed when the external loads tend to **twist (or rotate)** one segment of the body with respect to the other.

4. Bending moment, M:

it is developed when the external loads tend to **bend** the body.

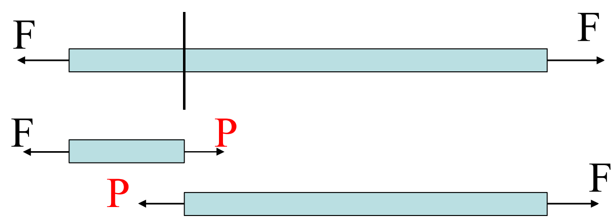
For coplanar loading (two dimensional) , only **Normal Force (P), Shear Force (V), and Bending Moment (M) components will exist at the section as shown before.**

- In Engineering Mechanics, we've looked at the forces applied to **rigid** bodies (In equilibrium)
- Bodies **deform** when loads are applied to them
- The deformation of bodies is a result of *strain*, which is the response of a material to *stress*
- Stress is the **intensity of loading per unit area**
- Loads applied to a body causes stress, which causes a strain in the material, which is measured as deformation of the body
- Stress may be: - Axial or Normal or Direct
  - Shear
  - Bearing

Axial or Normal or Direct Stress:

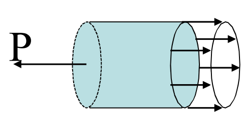
- Let's take a look at a truss member in tension:

- Cut through the member into two parts:



- The **external forces (F)** that come from the external loads are balanced by **internal forces (P)**.
- In the case shown before, the internal forces (P) are clearly equal to (F) in static equilibrium.
- The internal forces must be resisted by the material of the member, otherwise the member will fail. The forces (P) are distributed over areas within the body as **STRESS** , a measure of the force intensity:

$$\sigma = P/A$$



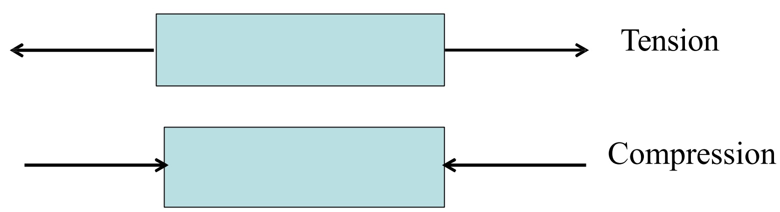
σ = Normal stress

P = Internal force

A = Cross-sectional area

- Normal stress may be tensile [σ_t]or compressive [σ_c] and result from forces acting perpendicular to the plane of the cross-section

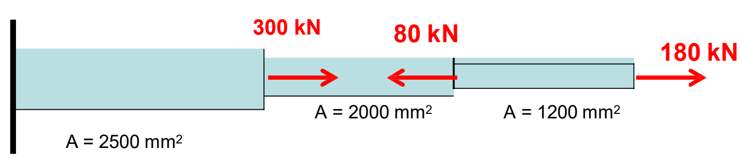
Units: Usually N/m^2 (Pa), kN/mm^2, MPa, GPa



13 SOM (Spring 2023)

Example (1):

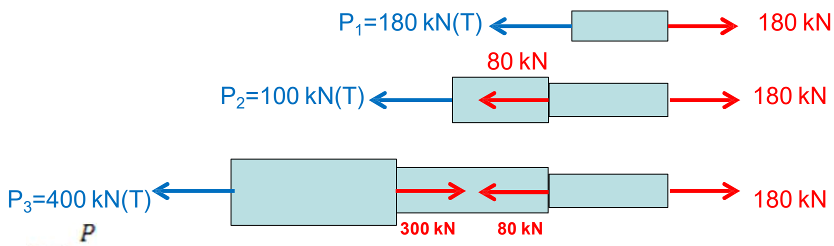
A rod of variable cross section fixed at one end is subjected to axial forces as shown. Determine the axial stress in each section and the maximum axial (Normal) stress in the rod.



Solution:

Draw the FBD for each portion as the 1st step then calculate the stresses in each portion.

14 SOM (Spring 2023)



$$\sigma = \frac{P}{A}$$

$$\sigma_1 = \frac{P_1}{A_1} = \frac{180 \times 10^3\ (N)}{1200 \times 10^{-6}\ (m^2)} = 150 \times 10^6\ N/m^2 = 150 \times 10^6\ Pa = 150\ MPa\ (T)$$

$$\sigma_2 = \frac{P_2}{A_2} = \frac{100 \times 10^3\ (N)}{2000 \times 10^{-6}\ (m^2)} = 50 \times 10^6\ N/m^2 = 50 \times 10^6\ Pa = 50\ MPa\ (T)$$

$$\sigma_3 = \frac{P_3}{A_3} = \frac{400 \times 10^3\ (N)}{2500 \times 10^{-6}\ (m^2)} = 160 \times 10^6\ N/m^2 = 160 \times 10^6\ Pa = 160\ MPa\ (T)$$

Maximum normal stress is **160 MPa (T)**

Example (2) :

A hollow steel tube with an inside diameter of 100 mm must carry a tensile load of 400 kN.

Determine the outside diameter of the tube if the stress is limited to 120 MN/m^2.

Solution:-

$P = \sigma A$

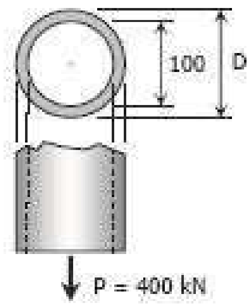
where:

$P = 400\text{ kN} = 400\,000\text{ N}$

$\sigma = 120\text{ MPa}$

$A = \frac{1}{4}\pi D^2 - \frac{1}{4}\pi(100^2)$

$= \frac{1}{4}\pi(D^2 - 10\,000)$



thus,

$$400\,000 = 120[\tfrac{1}{4}\pi(D^2 - 10\,000)]$$

$$400\,000 = 30\pi D^2 - 300\,000\pi$$

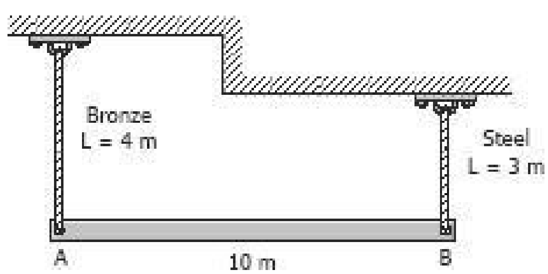
$$D^2 = \frac{400\,000 + 300\,000\pi}{30\pi}$$

$$D = 119.35\text{ mm}$$

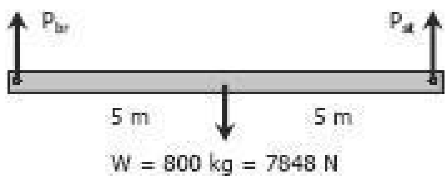
Example (3) :

A homogeneous 800 kg bar AB is supported at either end by a cable as shown in the Fig.

Calculate the area of each cable if the stress is not to exceed 90 MPa in bronze and 120 MPa in steel.



Solution:



By symmetry:

$$P_{br} = P_{st} = \frac{1}{2}(7848)$$
$$= 3924 \text{ N}$$

For bronze cable:

$$P_{br} = \sigma_{br} A_{br}$$
$$3924 = 90 A_{br}$$
$$A_{br} = 43.6 \text{ mm}^2$$

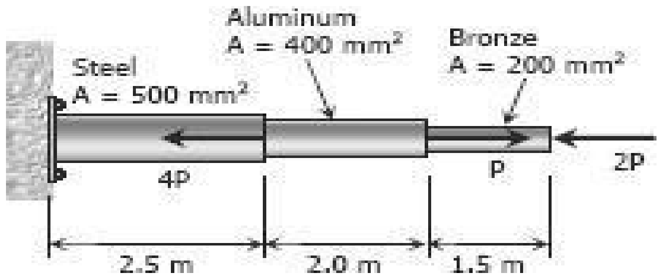
For steel cable:

$$P_{st} = \sigma_{st} A_{st}$$
$$3924 = 120 A_{st}$$
$$A_{st} = 32.7 \text{ mm}^2$$

Example (4) :

An aluminum rod is rigidly attached between a steel rod and a bronze rod as shown in the Fig.

Axial loads are applied at the positions indicated. Find the maximum value of P that will not exceed a stress in steel of 140 MPa, in aluminum of 90 MPa, or in bronze of 100 MPa



Solution:

For bronze:
$\sigma_{br} A_{br} = 2P$
$100(200) = 2P$
$P = 10\ 000$ N

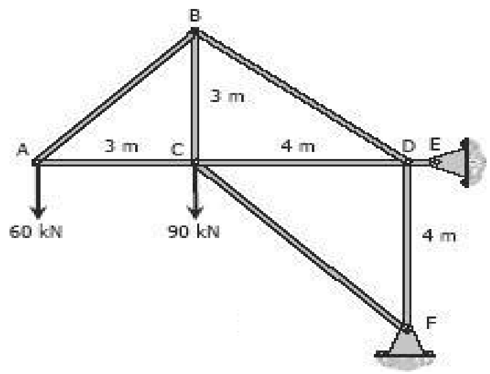
For aluminum:
$\sigma_{al} A_{al} = P$
$90(400) = P$
$P = 36\ 000$ N
For Steel:
$\sigma_{st} A_{st} = 5P$
$P = 14\ 000$ N

For safe P, use $P = 10\ 000$ N = **10 kN**

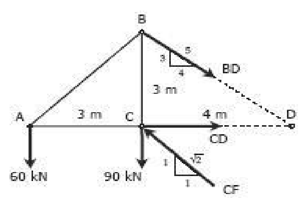
Example (5) :

Find the stresses in members BC, BD, and CF for the truss shown in Fig. Indicate the tension or compression.

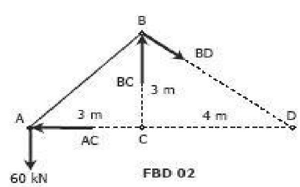
The cross sectional area of each member is 1600 mm^2.



Solution:



FBD 01



FBD 02

For member BD: (See FBD 01)

$\Sigma M_C = 0$

$3(\frac{4}{5}BD) = 3(60)$

$BD = 75$ kN Tension

$BD = \sigma_{BD} A$

$75\ (1000) = \sigma_{BD}\ (1600)$

$\sigma_{BD} = 46.875$ MPa (Tension)

For member CF: (See FBD 01)

$\Sigma M_D = 0$

$4(\frac{1}{\sqrt{2}}CF) = 4(90) + 7(60)$

$CF = 195\sqrt{2}$

$= 275.77$ kN Compression

$CF = \sigma_{CF} A$

$275.77\ (1000) = \sigma_{CF}\ (1600)$

$\sigma_{CF} = 172.357$ MPa (Compression)

For member BC: (See FBD 02)

$\Sigma M_D = 0$

$4BC = 7(60)$

$BC = 105$ kN Compression

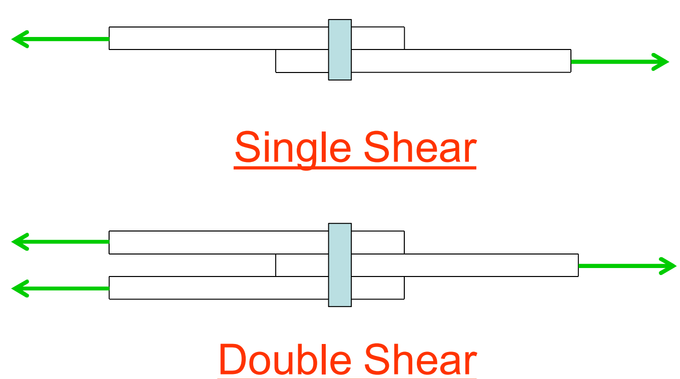
$BC = \sigma_{BC} A$

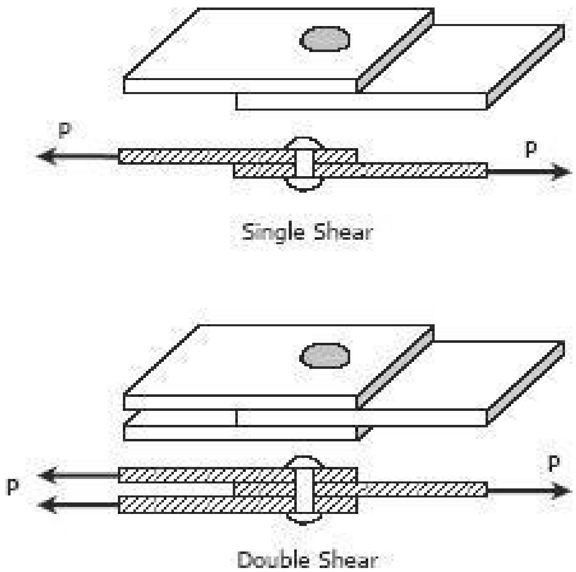
$105\ (1000) = \sigma_{BC}\ (1600)$

$\sigma_{BC} = 65.625$ MPa (Compression)

Shear Stress:

- Shear stresses are produced by equal and opposite parallel forces not acting on the same line.
- The forces tend to make one part of the material slide over the other part.
- Shear stress is tangential to the area over which it acts.





$$\text{Shear Stress} = \frac{\text{Shear Force}}{\text{Shear Area}}$$

OR ; $\tau = V / A$

If the area resisting the shear force is ONE area its called *Single Shear*, *and if the resisting area is Two its called* *Double Shear*.

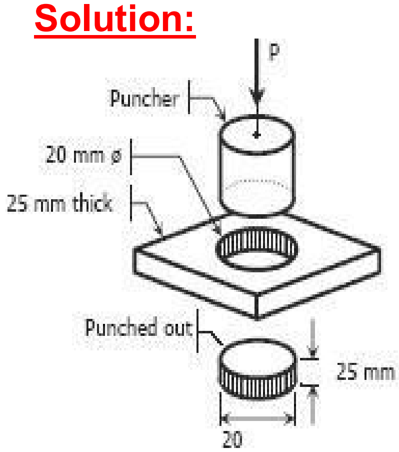
So in Double Shear case

$$\tau = V / 2A$$

Example (1) :

What force is required to punch a 20-mm-diameter hole in a plate that is 25 mm thick? The shear strength is 350 MPa

Solution:



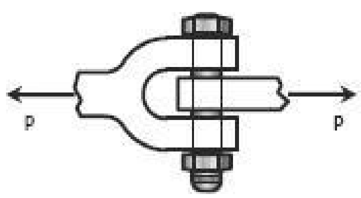
The resisting area is the shaded area along the perimeter and the shear force V is equal to the punching force P.

$$V = \tau A$$
$$P = 350[\pi(20)(25)]$$
$$= 549\,778.7 \text{ N}$$
$$= 549.8 \text{ kN}$$

Example (2) :

Find the diameter of a bolt that can be used in the clevis shown in the Fig. if P = 400 kN. The shearing strength of the bolt is 300 MPa.

Solution:



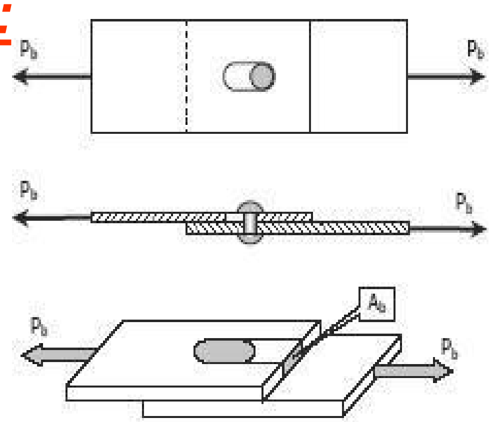
The bolt is subject to double shear.

$V = \tau A$

$400(1000) = 300[2(\frac{1}{4}\pi d^2)]$

$d = 29.13$ mm

Bearing Stress:



-Bearing stress is the contact pressure between the separate bodies.

-It differs from compressive stress, as it is an internal stress caused by compressive forces.

$$\sigma_b = \frac{P_b}{A_b}$$

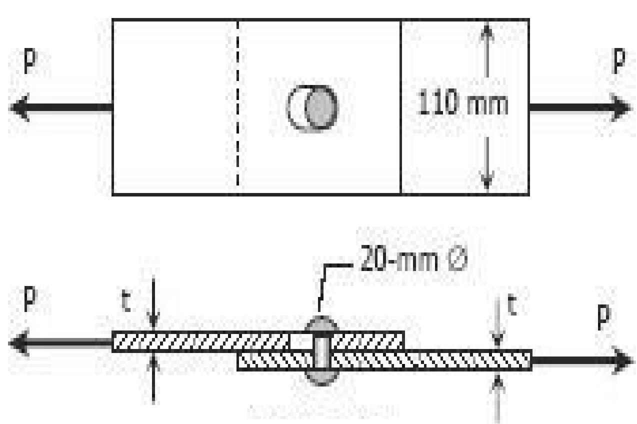
Example (1):

In the Fig. shown assume that a 20-mm-diameter rivet joins the plates that are each 110 mm wide.

The allowable stresses are 120 MPa for bearing in the plate material and 60 MPa for shearing of rivet. Determine:

(a)the minimum thickness of each plate; and

(b)the largest average tensile stress in the plates.



Solution:

(a) From shearing of rivet:

$P = \tau A_{rivets}$

$= 60[\frac{1}{4}\pi(20^2)]$

$= 6000\pi$ N

From bearing of plate material:

$P = \sigma_b A_b$

$6000\pi = 120(20t)$

$t = 7.85$ **mm**

(b) Largest average tensile stress in the plate:

$P = \sigma A$

$6000\pi = \sigma[7.85(110 - 20)]$

$\sigma = 26.67$ **MPa**