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CONCRETE DESIGN I

CHAPTER 2

Analysis and Design of Singly Reinforced Beams

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Chapter Two

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CHAPTER 2 ANALYSIS AND DESIGN OF SINGLY REINFORCED BEAMS

2.1 Notations

M_u: Ultimate bending Moment (Factored Applied Load)

M_n: Nominal Moment Capacity (Internal Moment Resistance)

M_d: Design Moment Capacity ($M_d = \varphi M_n \geq M_u$)

2.2 Summary

$Elasticity\ of\ Concrete \qquad E_c = 4700\sqrt{f_c'} \quad MPa$

$Elasticity\ of\ Reinforcing\ steel \qquad E_s = 200000 \quad MPa$

$Poission\ ratio\ of\ concrete \ \cong 0.2$

$Poission\ ratio\ of\ steel \ \cong 0.3$

$$Modular\ ratio, n = \frac{E_s}{E_c}$$

$Modulus\ of\ rupture = 0.62\sqrt{f_c'}$

$Maximum\ strain\ in\ concrete, \quad \varepsilon_{cu} = 0.003\ unitless(ACI318M - 14{:}22.2.2.1)$

$$Yield\ strain\ of\ reinforcing\ streel \quad \varepsilon_y = \frac{f_y}{E_s}$$

$$Yield\ strain\ of\ grade\ 420\ MPa\ \ (Grade\ 60\ ksi) = \frac{420}{200\ 000} \cong 0.002 \quad unitless$$

$Minimum\ allowable\ strain\ in\ tensile\ reinforcement\ in\ beams: 0.004$

$Stress\ in\ steel\ at\ any\ level\ of\ loading,\ f_s = \varepsilon_s E_s\ \ (before\ yielding)$

2.3 Introduction to Bending Theory

When a beam is subjected to bending moments, bending strains are produced. **Under positive moment**, compressive strains are produced in the top of the beam and tensile strains are produced in the bottom. These *strains* produce *stresses* in the beam, **compression in the top**, and **tension in the bottom**. Bending members must therefore be able to resist both tensile and compressive stresses.

For a concrete flexural member (beam, wall, slab, and so on) to have any significant load-carrying capacity, its basic inability to resist tensile stresses must be overcome. **By embedding reinforcement in the tension zones**, a reinforced concrete member is created. When properly designed, and constructed, members composed of these materials perform very adequately when subjected to flexure. [4]

2.4 Behavior under Load – Tension Reinforcement [4]

The load on the beam increases from zero to the magnitude that would cause failure.

a) **Very Small Load (Less than modulus of rupture, $f_r = 0.62\sqrt{f'_c}$)**

At very small loads, assuming that the concrete has not cracked, <u>both concrete and steel</u> will resist the tension, and concrete alone will resist the compression. (See figure 7)

b) **Moderate Loads**

At moderate loads, the tensile strength of the concrete will be exceeded, and the concrete will crack; hairline cracks. Because the <u>concrete cannot transmit any tension across a crack</u>, the steel bars will then resist the entire tension. The stress distribution at or near a cracked section then becomes as shown in Figure 7. This stress pattern exists up to approximately a concrete stress f_c of about $f'_c/2$. The concrete compressive stress **is still assumed to be proportional** to the concrete strain.

c) Further Load Increase (Ultimate Applied Load)

The compressive strains and stresses will increase; they will cease to be proportional, however, and some nonlinear stress curve will result on the compression side of the beam. This stress curve above the neutral axis will be essentially the same shape as the concrete stress-strain curve. The stress and strain distribution that exists at or near the ultimate load is shown in the figure 7.

Eventually, the ultimate capacity of the beam will be reached and the beam will fail.

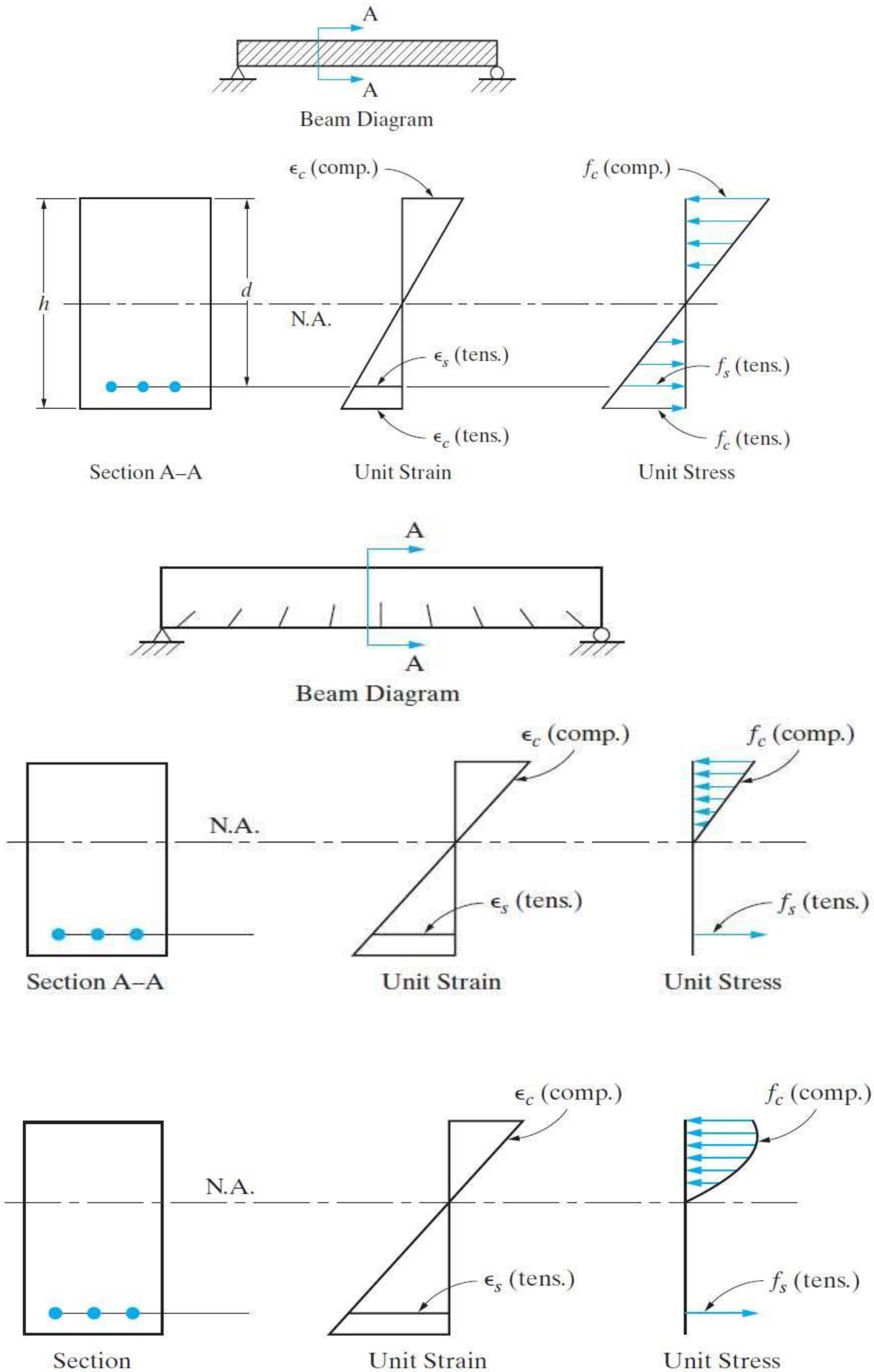


Figure 7: Flexural Behavior under small, moderate and ultimate loads

2.5 Design Assumptions [4]

The development of the strength design approach depends on the following basic assumptions:

1. A plane section before bending remains a plane section after bending.
2. Stresses and strains are approximately proportional only up to moderate loads (assuming that the concrete stress does not exceed approximately $f_c^{'}/2$). When the load is increased, and approaches an ultimate load, stresses and strains are no longer proportional. Hence the variation in concrete stress is no longer linear.
3. In calculating the ultimate moment capacity of a beam, the tensile strength of the concrete is neglected.
4. The maximum usable concrete compressive strain at the extreme fiber is assumed equal to 0.003.
5. The steel is assumed to be uniformly strained to the strain that exists at the level of the centroid of the steel.
6. The bond between the steel and concrete is perfect and no slip occurs.

2.6 Equivalent Stress Distribution

For purposes of simplification and practical application, a fictitious but equivalent rectangular concrete stress distribution was proposed by Whitney and subsequently adopted by the ACI Code. The ACI Code also stipulates that other compressive stress distribution shapes may be used, provided results are in substantial agreement with comprehensive test results. Because of the simplicity of the rectangular shape, however, it has become the more widely used fictitious stress distribution for design purposes.

With respect to this equivalent stress distribution as shown in Figure 10 Equivalent Stress Distribution , the average stress intensity is taken as $0.85\ f_c'$ and is assumed to act over the upper area of the beam cross section defined by the width b and a depth of a. The magnitude of a may be determined by

$$a = \beta_1 c$$

where

c = distance from the outer compressive fiber to the neutral axis

β_1 = a factor that is a function of the strength of the concrete

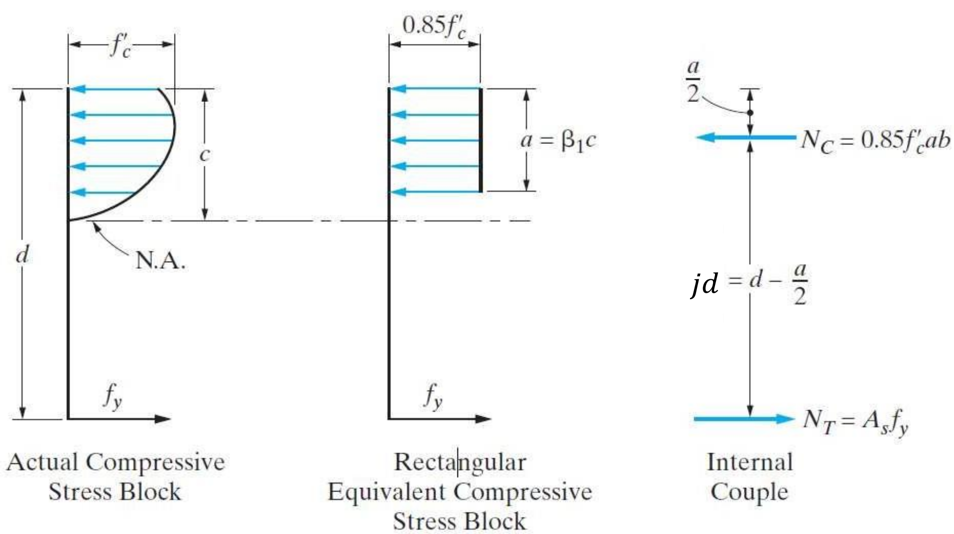


Figure 10 Equivalent Stress Distribution , [4]

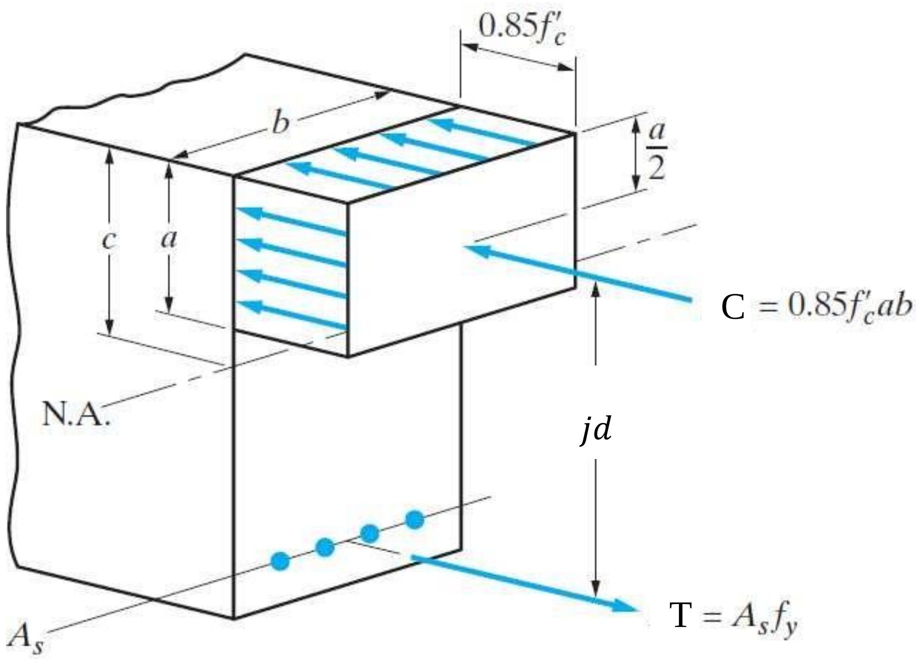


Figure 11 Equivalent Stress Block - [4]

$Equilibrium\ equation$

$C = T$

$$0.85\, f_c'\, a\, b = A_s \quad f_y \qquad \rightarrow \qquad a = \frac{A_s\, f_y}{0.85\, f_c'\, b}$$

$M_n = C\, jd = T\, jd$

$$M_n = 0.85\, f_c'\, a\, b \ \left(d - \frac{a}{2}\right)$$

$$\boldsymbol{M_n = A_s\, f_y\, \left(d - \frac{a}{2}\right)}$$

This is the nominal internal moment resistance that should be multiplied by a reduction factor (ϕ) and the result should be equal or greater than the applied factored bending moment.

$$\boxed{M_d = \phi\, M_n \geq M_u}$$

2.7 Flexural Strength of Rectangular Beams – Procedure

Given: M_u **and Span**

Required: Design of Beams (width, depth and reinforcements)

Procedure:

1. Estimate the depth of the beam (h) from ACI318M-14: Table 9.3.1.1

Table 1 Minimum depth of beams

Support condition	Minimum h[1]
Simply supported	$\ell/16$
One end continuous	$\ell/18.5$
Both ends continuous	$\ell/21$
Cantilever	$\ell/8$

[1]Expressions applicable for normalweight concrete and Grade 420 reinforcement. For other cases, minimum h shall be modified in accordance with 9.3.1.1.1 through 9.3.1.1.3, as appropriate.

2. Estimate the width of the beam: $b \cong (0.5\ to\ 0.67)\ of\ h$
3. Calculate the effective depth: $d = h - cover - stirrup - db/2$
4. Assume jd ($jd = 0.9$ for singly reinforced beams)
5. Determine the required area of tensile reinforcement

$$A_{s,req} = \frac{M_u}{\phi\, f_y\, jd}$$

6. Find the required number of bars (roundup to the nearest number):

$$n = \frac{A_{s,req}}{A_b}$$

7. Check clear bar spacing:

$$s = \frac{b - 2 * cover - 2 * stirrup - n\, d_b}{n-1} \geq \left(25mm\ ,\ d_b\ ,\ \frac{4}{3}d_a\right)$$

8. Calculate the provided area of reinforcement

$$A_{s,prov} = n * A_b$$

9. Check Minimum Reinforcement, $A_{s,min}$

$$A_{s,prov} \geq A_{s,min} = \begin{cases} \dfrac{\sqrt{f'_c}}{4\, f_y}\, b_w\, d \\[2ex] \dfrac{1.4}{f_y}\, b_w\, d \end{cases}$$

10. Compute the depth of compression block

$$a = \frac{A_s\, f_y}{0.85\, f'_c\, b}$$

11. Compute the depth of Neutral axis

$$c = \frac{a}{\beta_1}$$

Table 2 Values of β1 equivalent rectangular concrete stress distribution (ACI318m-14: Table 22.2.2.4.3)

f_c', MPa	β_1	
$17 \leq f_c' \leq 28$	0.85	(a)
$28 < f_c' < 55$	$0.85 - \dfrac{0.05(f_c' - 28)}{7}$	(b)
$f_c' \geq 55$	0.65	(c)

B. Check Mode of Failure (Stress Controlled)

Reinforcement Ratio:

Minimum reinforcement: for shirinkage and temperature

$$\rho_{min} = \frac{1.4}{f_y} \qquad Mpa$$

Actual reinforcement:

$$\rho_{act} = \frac{A_s}{b\,d} \qquad Mpa$$

Ballaced reinforcement:

$$\rho_b = 0.85\ \beta_1\ \frac{f'_c}{f_y}\ \frac{600}{600 + f_y} \qquad Mpa$$

$$\rho_{max} = 0.75\,\rho_b$$

Three cases of reinforcement:

1. Over reinforced Beams ($\rho_{act} > \rho_{max}$) (Brittle Failure)

   Concrete crushes before yielding of reinforcing steel (No warning before collapse)

2. Under reinforced Beams ($\rho_{act} < \rho_{max}$) (Ductile Failure)

   Steel yields before crushing the concrete (warning before collapse)

3. Balanced Failure ($\rho_{act} = \rho_b$)

   Steel yields simultaneously with the crushing of concrete.

According to ACI318, beam sections should be under reinforced to have a ductile behavior and avoid sudden failure.

12. Calculate Nominal Moment Capacity, M_n

$$M_n = A_c f_y (d - \frac{a}{2})$$

13. Calculate the Design Moment Capacity, M_d

$$\boxed{M_d = \emptyset\, M_n = \emptyset\, A_s f_y\ (d - \frac{a}{2})}$$

14. Check Whether the Design Capacity is larger than the Applied Factored Moment

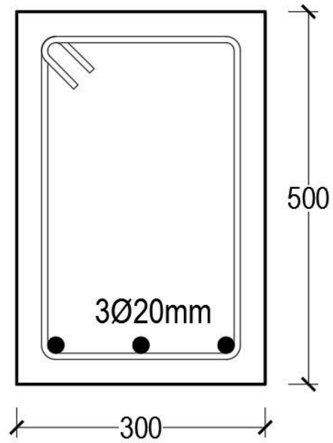
$$M_d \geq M_u$$

15. If step 14 has not been satisfied!?

2.8 Analysis of Singly Reinforced Beams - Examples

2.8.1 Example 1

Compute the positive nominal moment capacity for the rectangular reinforced concrete beam shown in the figure.



Given:

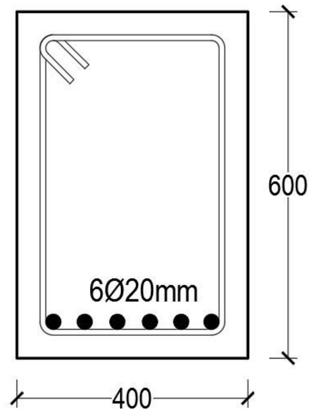
b= 300mm; h= 500mm, Reinforcement: $3\phi20$mm;

Stirrups: $\phi10$mm, $f'_c = 28\ MPa$; $f_y = 420\ MPa$

SOLUTION

2.8.2 Example 2

Calculate the design positive moment capacity of the rectangular beam shown in the figure using the given data.



Given:

$f'_c = 25\ MPa$; $f_y = 420\ MPa$, Stirrups: $\phi 10$mm

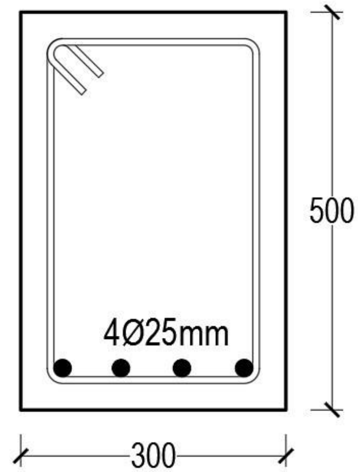
SOLUTION

2.8.3 Example 3

GIVEN:

b= 300 mm; h= 500mm, Reinforcement: $4\phi25$mm;

Stirrups: $\phi10$mm, $f'_c = 21\ MPa$; $f_y = 420\ MPa$



REQUIRED:

Design Positive Moment Capacity

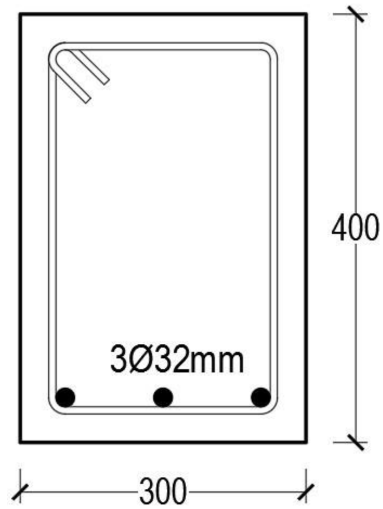
SOLUTION

2.8.4 Example 4

Compute the design moment capacity of the section shown below. Use the given data.

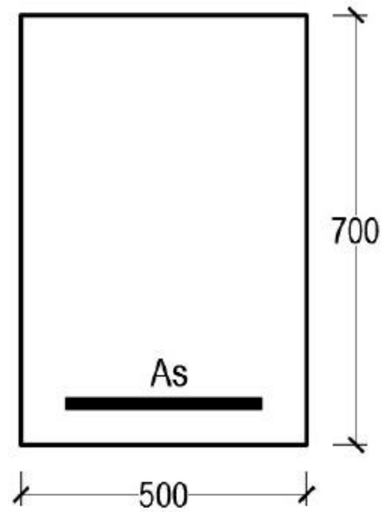
Stirrups: ϕ10mm, $f_c' = 21\ MPa$; $f_y = 420\ MPa$

SOLUTION



2.9 Design of Singly Reinforced Beams – Design Examples

2.9.1 Example 5



Given:

Mu = 820 kN.m

b= 500 mm; h= 700mm,

Stirrups: $\phi 10$mm, $f_c' = 35\ MPa$; $f_y = 420\ MPa$

Required:

Design the beam reinforcement

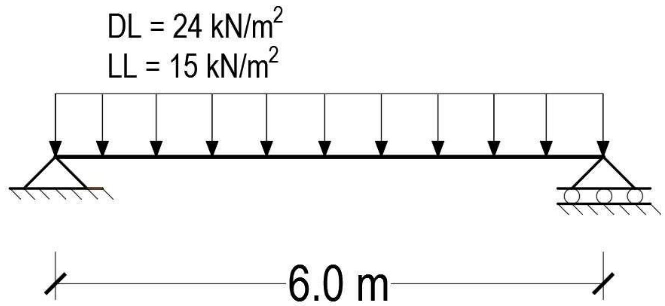
SOLUTION

2.9.1 Example 6

Design the simply supported beam for the given loading and data.

Stirrups: $\phi 10$mm, $f_c' = 25\ MPa$; $f_y = 420\ MPa$

SOLUTION



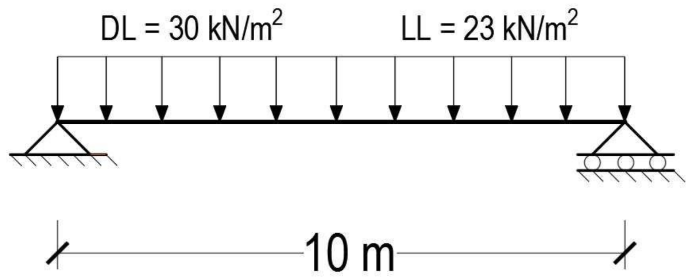
2.9.2 Example 7 (Homework)

Design the simply supported beam to carry the applied load using the given data.

b= 300 mm; h= 500mm, Reinforcement: $4\phi25$mm;

Stirrups: $\phi10$mm, $f_c' = 21\ MPa$; $f_y = 420\ MPa$

SOLUTION



2.10 Homework: will be given during the lectures

2.10.1 Homework

2.10.2 Homework

2.11 References

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[11] A. M. Ibrahim, M. S. Mahmood, and Q. W. Ahmed, *Design of Reinforced Concrete Structures*, First. Baghdad, 2011.